Effect of unequal injection rates on asymmetric exclusion processes with junction*

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In this paper, we investigate the effect of unequal injection rates on totally asymmetric simple exclusion processes (TASEPs) with a 2-input 1-output junction and parallel update. A mean-field approach is developed to deal with the junction that connects two sub-chains and the single main chain. We obtain the stationary particle currents, density profiles and phase diagrams. Interestingly, we find that the number of stationary-state phases is changeable depending on the value of α_1 (α_1 is the injection rate on the first sub-chain). When $\alpha_1 > 1/3$, there are seven stationary-state phases in the system, however when $\alpha_1 < 1/3$, only six stationary-state phases exist in the system. The theoretical calculations are shown to be in agreement with Monte Carlo simulations.

Keywords: totally asymmetric simple exclusion process, mean-field approach, Monte Carlo simulations, junction

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1. Introduction

Totally asymmetric simple exclusion process (TASEP) as a basic model has been widely used to study many non-equilibrium systems.[1−7] Recently, parallel two-lane TASEP has been studied, for example, TASEP with coupling to two reservoirs,[4] TASEP with symmetric and asymmetric couplings[8,9] as well as the motion of kinks in a two-lane model in Ref. [10].

More recently, the TASEP on lattice chains with junctions has been studied in Refs. [11]–[15]. Junctions in these models are regarded as bottlenecks which can lead to traffic jams. It has been proposed that some human diseases can be caused by traffic congestion of molecular motors.[16,17] Additional fuel consumption and environment pollution are also results of vehicle traffic congestion. Reference [13] introduced a wide range of possible applications of junctions and investigated the dynamics of synchronous TASEP with a multi-input multi-output junction. However, the injection rates of all sub-chains were assumed to be equal. In reality, these injection rates may be different, e.g., different road priorities in vehicular traffic or different bandwidth requirements in internet traffic. Therefore, it would be interesting to study the effect of unequal injection rates on such systems.

In this paper, we investigate the effect of unequal injection rates on synchronous TASEP on lattices with a 2-input 1-output junction. We obtain particle currents, density profiles and phase diagrams via a simple approximate theory and extensive Monte Carlo simulations. The domain wall theory[18] is used to calculate density profiles at phase boundaries. Extensive Monte Carlo computer simulations are supportive to the theoretical results.

This paper is organized as follows. In Section 2, we describe the model and investigate the stationary properties by a simple approximate theory. In Section 3, theoretical and simulation results are presented. Finally, we give summary and conclusions in Section 4.

2. Model and simple approximate theory

A synchronous TASEP on lattices with the junction under unequal injection rates is shown in Fig. 1. There are three equal-size chains (i.e. two parallel sub-chains and one main chain); each of which contains L sites. Particles can enter sub-chain 1 or 2 at rate α_1 or α_2 if the first site at the corresponding sub-chain is empty. Two sub-chains merge together at site $L+1$ and form main chain 3. The particle can leave the system at rate β from the right end of the main chain

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(see Fig. 1). Inside the lattice chains the particle can move only one step forward if the neighbouring site is empty.

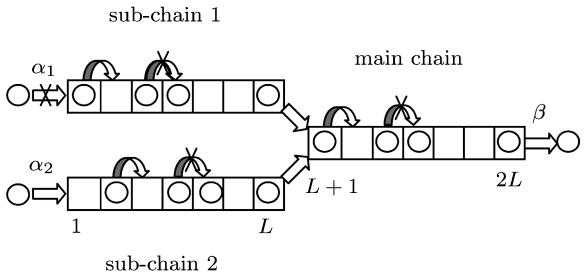


Fig. 1. Schematic diagram of the model for an unequal injection rate synchronous TASEP on a lattice with a junction.

When $\alpha_2 = 0$ (or $\alpha_1 = 0$), the model reduces to a normal TASEP on a single lattice, for which the full description of the stationary properties has been obtained.[19,20,21] We briefly recall these exactly theoretical results of this simplest model.

For $\alpha < \beta \leq 1$, the system is found to be in a low density (LD) phase, we can obtain

$$J = \rho, \quad \rho = \rho_1 = \frac{\alpha}{1+\alpha}, \quad \rho_N = \frac{\alpha}{\beta\,(1+\alpha)}, \tag{1}$$

where J is the system current, ρ is the bulk density and ρ_1 (ρ_N) is the particle density at the first (last) site.

For $\beta < \alpha \leq 1$, the stable stationary state for the system is a high density (HD) phase, we can obtain

$$J = 1-\rho, \quad \rho = \rho_N = \frac{1}{1+\beta},$$
$$\rho_N = 1 - \frac{\beta}{\alpha\,(1+\beta)}. \tag{2}$$

Finally, for $\beta = \alpha = 1$, the stationary state for the system is in a maximal current (MC) phase, we can obtain

$$J = 0.5, \quad \rho = 0.5. \tag{3}$$

Three stationary phases can exist in each lattice chain (i.e. LD, HD or MC); since the unequal injection rates for each sub-chain, the total number of possible stationary phases of the system is equal to $3^3 = 27$. According to the current conservation through the system, we obtain

$$J_3 = J_1 + J_2. \tag{4}$$

From Eq. (4), we can see that the MC phase cannot exist in either of two sub-chains, since the maximal possible current in the system is no more than 0.5. Therefore, the total number of possible stationary phases of system is reduced to $2 \times 2 \times 3 = 12$.

Each of the two sub-chains can be seen as a synchronous TASEP with injection rates α_1 and α_2, respectively and ejection rate $\beta_{\rm eff}$, while the main chain can be seen as a synchronous TASEP with injection rate $\alpha_{\rm eff}$ and ejection rate β (see Fig. 2). $\alpha_{\rm eff}$ and $\beta_{\rm eff}$ can be expressed as

$$\alpha_{\rm eff} = 2\rho_N, \quad \beta_{\rm eff} = 1 - \rho_{N+1}. \tag{5}$$

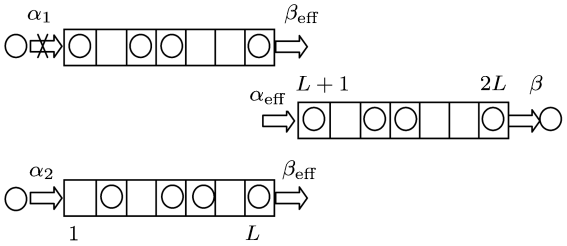


Fig. 2. Model mapping three homogeneous TASEPs with different injection rates and ejection rates.

Due to the symmetry of α_1 and α_2, we study only the case of $\alpha_1 < \alpha_2$. The existence of different stationary phases can be investigated as follows.

The (LD/LD/LD) phase, which can be specified by the following conditions:

$$\alpha_1 < \beta_{\rm eff} \leq 1, \quad \alpha_2 < \beta_{\rm eff} \leq 1, \quad \alpha_{\rm eff} < \beta \leq 1. \tag{6}$$

The stationary currents are given by

$$J_1 = \frac{\alpha_1}{1+\alpha_1}, \quad J_2 = \frac{\alpha_2}{1+\alpha_2}, \quad J_3 = \frac{\alpha_{\rm eff}}{1+\alpha_{\rm eff}}. \tag{7}$$

Using Eq. (4) for currents, the effective rate $\alpha_{\rm eff}$ can be expressed in terms of the entrance rates α_1 and α_2 as

$$\alpha_{\rm eff} = \frac{\alpha_1 + \alpha_2 + 2\alpha_1\alpha_2}{1-\alpha_1\alpha_2}. \tag{8}$$

Thus $\alpha_{\rm eff}$ always satisfies the conditions in expression (6) and we can obtain

$$\alpha_2 < \frac{1-\alpha_1}{1+3\alpha_1}. \tag{9}$$

Thus the system is in the (LD/LD/LD) phase when

$$\alpha_2 < \frac{1-\alpha_1}{1+3\alpha_1}, \quad \alpha_1 < 1, \quad \beta > \frac{\alpha_1 + \alpha_2 + 2\alpha_1\alpha_2}{1-\alpha_1\alpha_2}. \tag{10}$$

For the (LD/LD/HD) phase the conditions of existence can be written as

$$\alpha_1 < \beta_{\rm eff} \leq 1, \quad \alpha_2 < \beta_{\rm eff} \leq 1, \quad \beta < \alpha_{\rm eff} \leq 1. \tag{11}$$

The stationary currents are given by

$$J_1 = \frac{\alpha_1}{1+\alpha_1}, \quad J_2 = \frac{\alpha_2}{1+\alpha_2}, \quad J_3 = \frac{\beta}{1+\beta}. \quad (12)$$

Because the particle current is stationary, equation (4) implies that

$$\beta = \frac{\alpha_1 + \alpha_2 + 2\alpha_1\alpha_2}{1-\alpha_1\alpha_2}. \quad (13)$$

Thus the system is in the (LD/LD/HD) phase when

$$\alpha_2 < \frac{1-\alpha_1}{1+3\alpha_1}, \quad \alpha_1 < 1, \quad \beta = \frac{\alpha_1 + \alpha_2 + 2\alpha_1\alpha_2}{1-\alpha_1\alpha_2}. \quad (14)$$

The (LD/LD/MC) phase, the allowed parameters for this phase are specified by

$$\alpha_1 < \beta_{\text{eff}} \leq 1, \quad \alpha_2 < \beta_{\text{eff}} \leq 1, \quad \alpha_{\text{eff}} = \beta = 1. \quad (15)$$

The stationary currents are given by

$$J_1 = \frac{\alpha_1}{1+\alpha_1}, \quad J_2 = \frac{\alpha_2}{1+\alpha_2}, \quad J_3 = 0.5. \quad (16)$$

Because the particle current is stationary, equation (4) implies that

$$\alpha_2 = \frac{1-\alpha_1}{1+3\alpha_1}. \quad (17)$$

Thus the system is in the (LD/LD/MC) phase when

$$\alpha_2 = \frac{1-\alpha_1}{1+3\alpha_1}, \quad \alpha_1 < 1, \quad \beta = 1. \quad (18)$$

For the (LD/HD/LD) phase, we have

$$\alpha_1 < \beta_{\text{eff}} \leq 1, \quad \beta_{\text{eff}} < \alpha_2 \leq 1, \quad \alpha_{\text{eff}} < \beta \leq 1. \quad (19)$$

The stationary currents are given by

$$J_1 = \frac{\alpha_1}{1+\alpha_1}, \quad J_2 = \frac{\beta_{\text{eff}}}{1+\beta_{\text{eff}}}, \quad J_3 = \frac{\alpha_{\text{eff}}}{1+\alpha_{\text{eff}}}. \quad (20)$$

Because the particle current is conserved, equation (4) implies that

$$\alpha_{\text{eff}} = \frac{\alpha_1 + \beta_{\text{eff}} + 2\alpha_1\beta_{\text{eff}}}{1-\alpha_1\beta_{\text{eff}}}. \quad (21)$$

According to Eq. (5), we can obtain

$$\beta_{\text{eff}} = 1 - \rho_{L+1} = \frac{1}{1+\alpha_{\text{eff}}}. \quad (22)$$

However, these two equations have no real solutions together and therefore the (LD/HD/LD) phase does not exist in the system.

The (LD/HD/HD) phase is determined from the following conditions:

$$\alpha_1 < \beta_{\text{eff}} \leq 1, \quad \beta_{\text{eff}} < \alpha_2 \leq 1, \quad \beta < \alpha_{\text{eff}} \leq 1. \quad (23)$$

The stationary currents are given by

$$J_1 = \frac{\alpha_1}{1+\alpha_1}, \quad J_2 = \frac{\beta_{\text{eff}}}{1+\beta_{\text{eff}}}, \quad J_3 = \frac{\beta}{1+\beta}. \quad (24)$$

Because the particle current is stationary, equation (4) indicates that

$$\beta_{\text{eff}} = \frac{\beta - \alpha_1}{1+2\alpha_1+\beta\alpha_1}. \quad (25)$$

It can be seen that β_{eff} always satisfies the conditions in expression (23), we can have

$$\alpha_1 > \frac{\beta-\alpha_2}{1+2\alpha_2+\beta\alpha_2}, \quad \alpha_2 < \frac{\sqrt{1+\beta\,(1+\beta)}-1}{2+\beta}. \quad (26)$$

Thus the system is in the (LD/HD/HD) phase when

$$\alpha_1 > \frac{\beta-\alpha_2}{1+2\alpha_2+\beta\alpha_2}, \quad \alpha_2 < \frac{\sqrt{1+\beta\,(1+\beta)}-1}{2+\beta}, \quad \beta < 1. \quad (27)$$

For the (LD/HD/MC) phase, the allowed parameters are given by

$$\alpha_1 < \beta_{\text{eff}} \leq 1, \quad \beta_{\text{eff}} < \alpha_2 \leq 1, \quad \alpha_{\text{eff}} = \beta = 1. \quad (28)$$

The stationary currents are given by

$$J_1 = \frac{\alpha_1}{1+\alpha_1}, \quad J_2 = \frac{\beta_{\text{eff}}}{1+\beta_{\text{eff}}}, \quad J_3 = 0.5. \quad (29)$$

According to Eq. (4), we obtain

$$\beta_{\text{eff}} = \frac{1-\alpha_1}{1+3\alpha_1}. \quad (30)$$

β_{eff} should satisfy the conditions in expression (28), and we can have

$$\alpha_1 > \frac{1-\alpha_2}{1+3\alpha_2}, \quad \alpha_2 < \frac{1}{3}. \quad (31)$$

Thus the system is in the (LD/HD/MC) phase when

$$\alpha_1 > \frac{1-\alpha_2}{1+3\alpha_2}, \quad \alpha_2 < \frac{1}{3}, \quad \beta = 1. \quad (32)$$

For the (HD/HD/LD) phase, the allowed parameters are given by

$$\beta_{\text{eff}} < \alpha_1 \leq 1, \quad \beta_{\text{eff}} < \alpha_2 \leq 1, \quad \alpha_{\text{eff}} < \beta \leq 1. \quad (33)$$

The stationary currents are given by

$$J_1 = \frac{\beta_{\text{eff}}}{1+\beta_{\text{eff}}}, \quad J_2 = \frac{\beta_{\text{eff}}}{1+\beta_{\text{eff}}}, \quad J_3 = \frac{\alpha_{\text{eff}}}{1+\alpha_{\text{eff}}}. \quad (34)$$

Because the particle current is stationary, according to Eqs. (2), (4), and (5) we can have

$$\alpha_{\text{eff}} = \pm\sqrt{2}. \quad (35)$$

Since $\alpha_{\rm eff}$ cannot satisfy the conditions in expression (33), the (HD/HD/LD) phase cannot exist.

For the (HD/HD/HD) phase, the allowed parameters are given by

$$\beta_{\rm eff} < \alpha_1 \leq 1, \quad \beta_{\rm eff} < \alpha_2 \leq 1, \quad \beta < \alpha_{\rm eff} \leq 1. \tag{36}$$

The stationary currents are given by

$$J_1 = \frac{\beta_{\rm eff}}{1+\beta_{\rm eff}}, \quad J_2 = \frac{\beta_{\rm eff}}{1+\beta_{\rm eff}}, \quad J_3 = \frac{\beta}{1+\beta}. \tag{37}$$

According to Eq. (4), we have

$$\beta_{\rm eff} = \frac{\beta}{2+\beta}. \tag{38}$$

One can see that $\beta_{\rm eff}$ always satisfies the conditions in expression (36), we can have

$$\alpha_1 > \frac{\beta}{2+\beta}, \quad \alpha_2 > \frac{\beta}{2+\beta}. \tag{39}$$

Thus the system is in the (HD/HD/HD) phase when

$$\alpha_1 > \frac{\beta}{2+\beta}, \quad \alpha_2 > \frac{\beta}{2+\beta}, \quad \beta < 1. \tag{40}$$

The (HD/HD/MC) phase is determined from the following conditions:

$$\beta_{\rm eff} < \alpha_1 \leq 1, \quad \beta_{\rm eff} < \alpha_2 \leq 1, \quad \alpha_{\rm eff} = \beta = 1. \tag{41}$$

The stationary currents are given by

$$J_1 = \frac{\beta_{\rm eff}}{1+\beta_{\rm eff}}, \quad J_2 = \frac{\beta_{\rm eff}}{1+\beta_{\rm eff}}, \quad J_3 = 0.5. \tag{42}$$

According to Eq. (4), we obtain

$$\beta_{\rm eff} = \frac{1}{3}. \tag{43}$$

Thus $\beta_{\rm eff}$ always satisfies the conditions in expression (41), we can have

$$\alpha_1 > \frac{1}{3}, \quad \alpha_2 > \frac{1}{3}. \tag{44}$$

Thus the system is in the (HD/HD/MC) phase when

$$\alpha_1 > \frac{1}{3}, \quad \alpha_2 > \frac{1}{3}, \quad \beta = 1. \tag{45}$$

From the above analyses, one can see that there are seven possible phases in this system. The calculated phase diagram is shown in Fig. 3.

It can be seen that the phase diagram includes two sections; one is the main bulk phases as the phases (LD/LD/LD), (LD/HD/HD), and (HD/HD/HD); the other is the phase boundaries as the phases (LD/LD/MC), (LD/LD/HD), (LD/HD/MC), and (HD/HD/MC) (see Fig. 3).

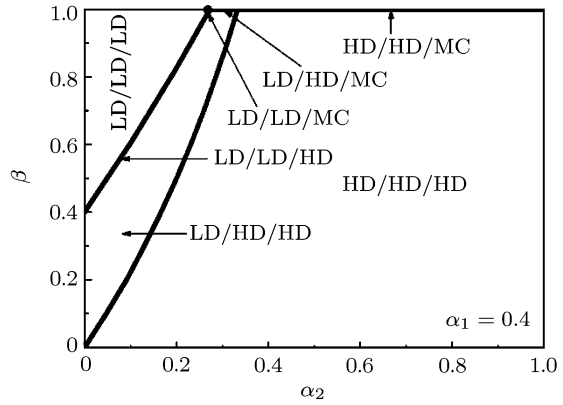


Fig. 3. Phase diagram for the synchronous TASEP with the junction under $\alpha_1 = 0.4$.

When $\alpha_1 > 1/3$, with α_1 increasing, the regions of the phases (LD/LD/LD) and (LD/LD/HD) shrink, while the regions of the phases (LD/HD/HD) and (LD/HD/MC) expand, but the regions of the phases (HD/HD/MC) and (HD/HD/HD) do not change (see Fig. 4) as compared with Fig. 3.

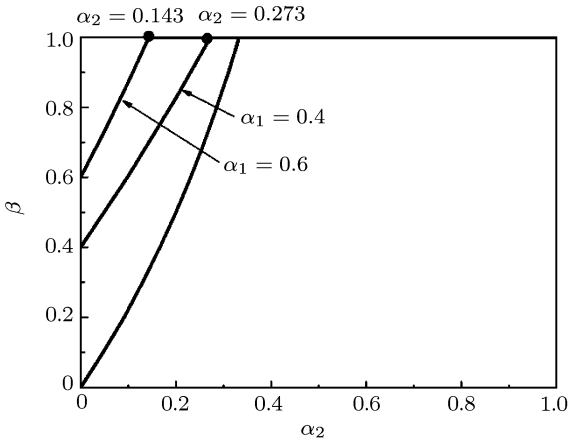


Fig. 4. Phase boundaries (i.e., the LD/LD/HD phase) for $\alpha_2 = 0.4$ and 0.6.

When $\alpha_1 < 1/3$, we can find that there is an interesting phenomenon, that is, the total number of stationary phases reduces to six, i.e. the (LD/HD/MC) phase vanishes. With α_1 decreasing, the regions of the phases (LD/LD/LD) and (LD/LD/HD) expand, while the regions of the phases (LD/HD/HD), (HD/HD/MC), and (HD/HD/HD) shrink. When $\alpha_1 = 0$, the phase diagram becomes the one of the normal synchronous TASEP (see Fig. 5(b)). We note that the existence of a threshold has been reported by Wang *et al.*[4] In Ref. [4], TASEP with a 1-input-2-output junction and random update is investigated.

In that junction, particles hop with a reduced rate at branching points. When the reduce rate is larger than the threshold, the number of phases reduces from 5 to 4.[4]

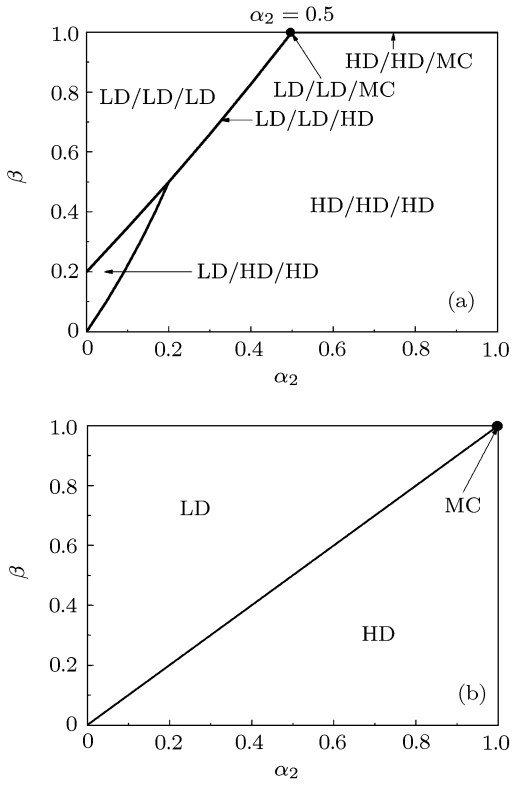


Fig. 5. Evolution of the phase (i.e., the LD/LD/HD phase) boundaries with different injection rates (a) $\alpha_1 = 0.2$ and (b) $\alpha_1 = 0$.

3. Monte Carlo simulations and discussion

We perform extensive computer Monte Carlo simulations to check the validity of our approximate theory. Here, the injection rate of sub-chain 1 is fixed, i.e. $\alpha_1 = 0.4$. Each of two sub-chain and main chain both include 1000 sites. In simulations, stationary density profiles are obtained by averaging 10^8 samplings at each site. The first 10^8 time steps are discarded as transients.

The density profiles for the main bulk phases are shown in Fig. 6. It is found that mean-field analyses are in agreement with Monte Carlo simulations well (see Fig. 6).

The density profiles of the phase boundaries can be calculated by domain wall (DW) theory[14] (see Fig. 7). These results are shown to be in agreement with the computer simulations.

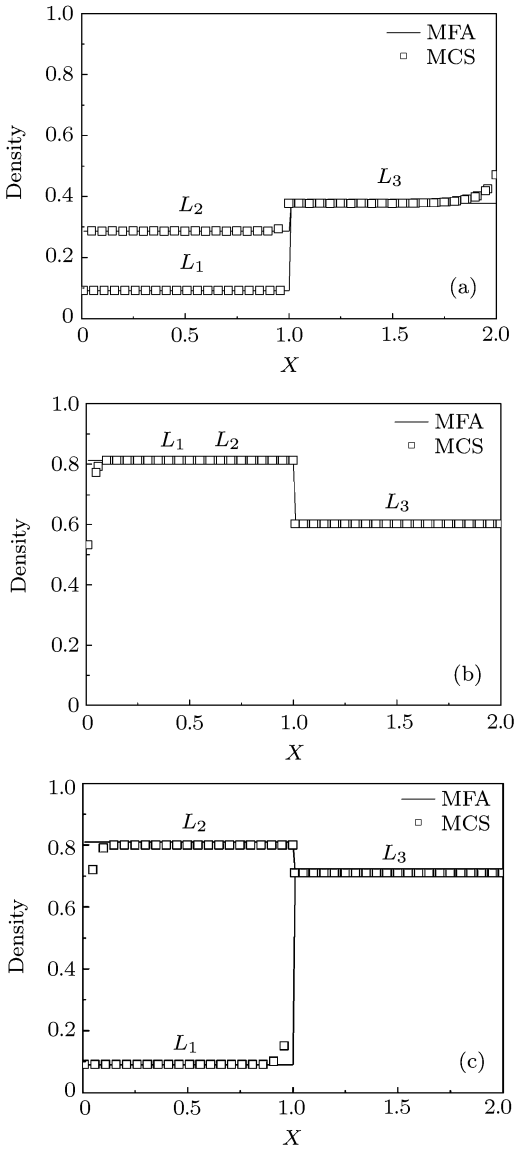


Fig. 6. Density profiles obtained from mean-field approximation (MFA) and Monte Carlo simulations (MCS) when $\alpha_1 = 0.4$. Panel (a) shows the phase (LD/LD/LD) with $\alpha_2 = 0.1$ and $\beta = 0.8$; panel (b) displays the phase (HD/HD/HD) with $\alpha_2 = 0.6$ and $\beta = 0.6$; and panel (c) exhibits the phase (LD/HD/HD) with $\alpha_2 = 0.1$ and $\beta = 0.4$.

The analysis of the stationary properties of the effect of unequal injection rates on synchronous totally asymmetric simple exclusion process is shown to be in good agreement with the theoretical predictions (see Figs. 6 and 7). However, due to the ignorance of the correlation, there are still some discrepancies between the theoretical calculations and simulation results, especially around $X = 1.0$ and $X = 0.0$. Nevertheless, the phase boundaries, stationary currents and bulk density profiles are not affected qualitatively.

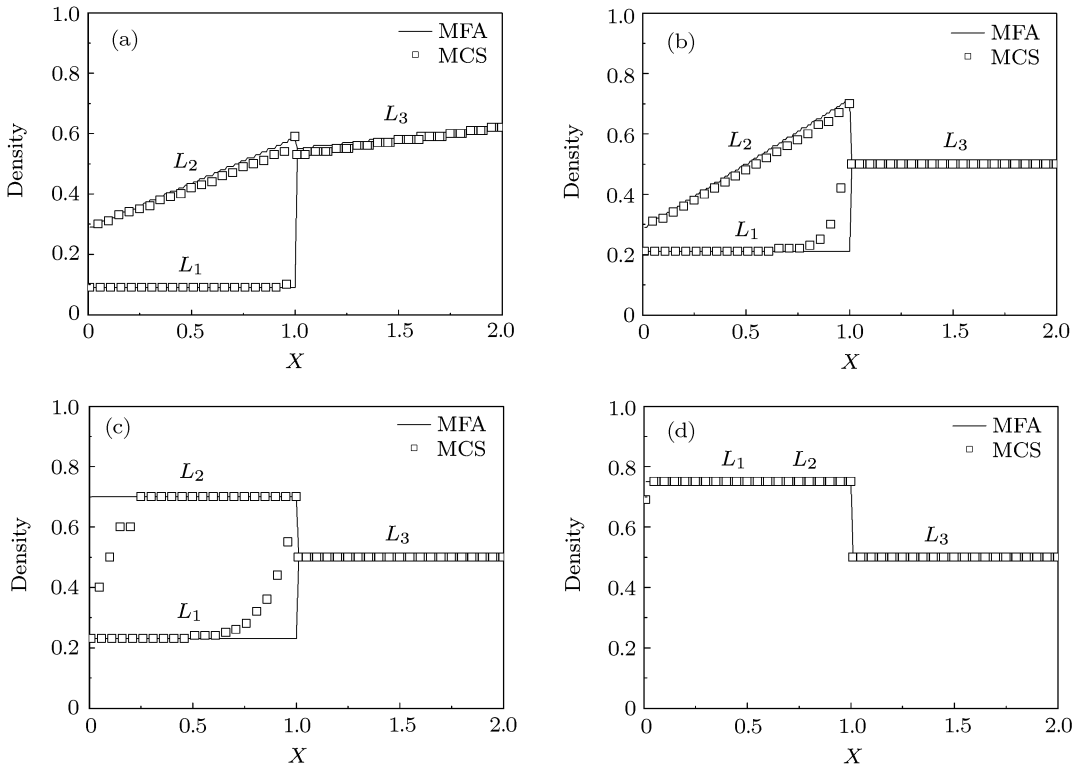


Fig. 7. Density profiles obtained by the domain wall theory and Monte Carlo simulations when $\alpha_1 = 0.4$. Panel (a) shows the (LD/LD/HD) phase with $\alpha_2 = 0.1$ and $\beta = 0.604$; panel (b) displays the (LD/LD/MC) phase with $\alpha_2 = 0.273$ and $\beta = 1.0$; panel (c) exhibits the (LD/HD/MC) phase with $\alpha_2 = 0.3$ and $\beta = 1.0$, and panel (d) indicates the phase (HD/HD/MC) with $\alpha_2 = 0.6$ and $\beta = 1.0$.

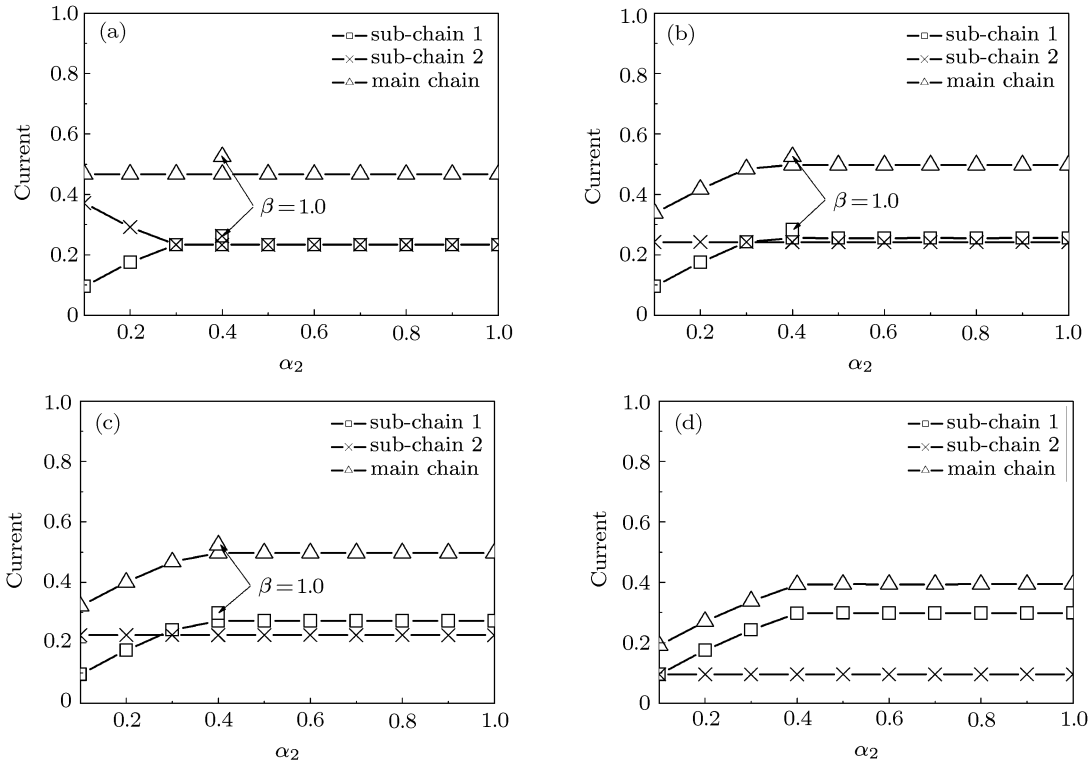


Fig. 8. Particle currents each as a function of the entrance rate α_2. In panel (a) the exit rate is fixed, i.e., $\beta = 0.8$ and the inject rate of sub-chain 1 is fixed, i.e., $\alpha_1 = 0.6$; in panel (b) the exit rate is fixed, i.e., $\beta = 0.9$ and the inject rate of sub-chain 1 is fixed, i.e., $\alpha_1 = 0.3$; in panel (c) the exit rate is fixed, i.e., $\beta = 0.9$ and the inject rate of sub-chain 1 is fixed, i.e., $\alpha_1 = 0.273$; in panel (d) the exit rate is fixed, i.e., $\beta = 0.6$ and the inject rate of sub-chain 1 is fixed, i.e., $\alpha_1 = 0.1$.

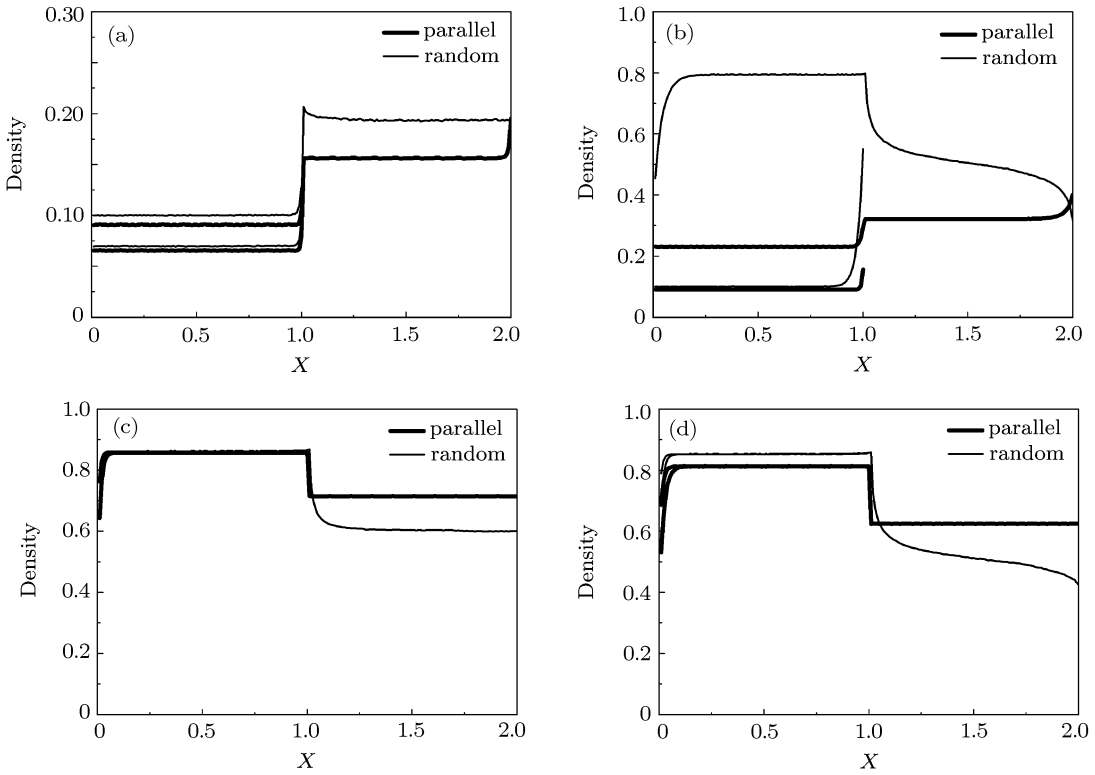


Fig. 9. Density profiles in Monte Carlo simulations with parallel and random updates, with (a) $\alpha_1 = 0.07$, $\alpha_2 = 0.1$, and $\beta = 0.8$; (b) $\alpha_1 = 0.07$, $\alpha_2 = 0.3$, and $\beta = 0.8$; (c) $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, and $\beta = 0.4$; (d) $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, and $\beta = 0.6$.

The differences in density profile between the synchronous update scheme and the random update scheme are shown in Fig. 9. In Fig. 9(a), the two systems are in the (LD/LD/LD) phase when $\alpha_1 = 0.07$, $\alpha_2 = 0.1$, and $\beta = 0.8$. When α_2 is increased to 0.3 but α_1 and β keep unchanged, the system with the synchronous update scheme is still in the (LD/LD/LD) phase, while the phase of the system with random update scheme changes to the (HD/LD/MC) phase (see Fig. 9(b)). Similarly, both systems in the (HD/HD/HD) phase are shown in Fig. 9(c) when $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, and $\beta = 0.4$. While β is increased to 0.6 but α_1 and α_2 keep unchanged, the phase of the system with random update scheme changes to the (HD/HD/MC) phase, while the other system stays in the (HD/HD/HD) phase (see Fig. 9(d)). This result still exhibits that different update procedures can lead to different traffic dynamics, which has been indicated in Refs. [22] and [23].

4. Summary and conclusions

With the help of a simple approximate theory and extensive Monte Carlo simulations we investigated the stationary properties of synchronous TASEP on lattices with a 2-input 1-output junction under unequal injection rates. It is found that the phase diagram of the system consists of seven stationary phases. There are two different types of phase transition in this system. On the coexistence line between (LD/LD/LD) and (LD/HD/HD) there are density jumps in all lattice segments that correspond to the first-order phase transitions. At the phase boundary between (HD/HD/HD) and (HD/HD/MC) the density profile changes continually in the lattice main chain. At the phase boundary between (LD/HD/HD) and (HD/HD/HD) the density profile changes continually in the lattice sub-chain 2. However, the phase transition between (LD/LD/LD) and (HD/HD/MC) has quite an unusual mixed character: a first-order transformation in the lattice sub-chain 1 and sub-chain 2 and a continuous change in lattice sub-chain 2 and main chain. This phase behaviour is significantly different from that of the TASEP on the lattices with junctions.

When $\alpha_1 > 1/3$, the number of possible phases reduces from 7 to 6. We also found that the lattice main chain is always in the HD phase (see Fig. 4). When

sub-chains 1 and 2 are both in the HD phase, the density profiles are equal, since in this phase the density is determined by exit rate $\beta_{\rm eff}$. A comparison of density profile between the system with synchronous update scheme and the system with random update scheme is presented.

References

[1] Derrida B 1998 *Phys. Rep.* **301** 65

[2] Schütz G M 2003 *J. Phys.* A: *Math. Gen.* **40** R339

[3] Du H F, Yuan Y M, Hu M B, Wang R, Jiang R and Wu Q S 2010 *J. Stat. Mech.* **P03014**

[4] Wang X, Jiang R, Hu M B, Nishinari K and Wu Q S 2009 *Int. J. Mod. Phys.* C **20** 967

[5] Shaw L B, Zia R K P and Lee K H 2003 *Phys. Rev.* E **68** 021910

[6] Klummp S and Lipowsky R 2003 *J. Stat. Phys.* **113** 233

[7] John A, Schadschneider A, Chowdhury D and Nishinari K 2004 *J. Theor. Biol.* **231** 279

[8] Pronina E, Kolomeisky A B 2004 *J. Phys.* A: *Math. Gen.* **37** 9907

[9] Pronina E and Kolomeisky A B 2006 *Physica* A: *Stat. Mech. Appl.* **372** 12

[10] Mitsudo T and Hayakawa H 2005 *J. Phys.* A **38** 3087

[11] Pronina E, Kolomeisky A B 2005 *J. Stat. Mech.* **P07010**

[12] Brankov J, Pesheva N and Bunzarova N 2004 *Phys. Rev.* E **69** 066128

[13] Liu M and Wang R 2009 *Physica* A **388** 4068

[14] Xiao S, Cai J J, Wang R L, Liu M Z and Liu F 2009 *Chin. Phys.* B **18** 5103

[15] Xiao S, Cai J J, Liu F and Liu M Z 2010 *Chin. Phys.* B **19** 090202

[16] Goldstein L S B 2001 *Proc. Natl. Acad. Sci. USA* **98** 6999

[17] Hurd D D and Saxton W M 1996 *Genetics* **144** 1075

[18] Kolomeisky A B, Schütz G M, Kolomeisky E B and Straley J P 1998 *J. Phys.* A: *Math. Gen.* **31** 6911

[19] Tilstra L G and Ernst M H 1998 *J. Phys.* A: *Math. Gen.* **31** 5033

[20] Gier J de and Nienhuis B 1999 *Phys. Rev.* E **59** 4899

[21] Evans M R, Rajewsky N and Speer E R 1999 *J. Stat. Phys.* **95** 45

[22] Rajewsky N, Santen L, Schadschneider A and Schreckenberg M 1998 *J. Stat. Phys.* **92** 151

[23] Jiang R, Wang R, Hu M B, Jia B and Wu Q S 2007 *J. Phys.* A: *Math. Theor.* **40** 9213