Soliton excitations and chaotic patterns for the (2+1)-dimensional Boiti–Leon–Pempinelli system*

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By improved projective equation approach and a linear variable separation approach, a new family of exact solutions of the (2+1)-dimensional Boiti–Leon–Pempinelli (BLP) system is derived. Based on the derived solitary wave solution, some dromion and solitoff excitations and chaotic behaviours are investigated.

Keywords: BLP system, exact solutions, dromion and solitoff excitations, chaotic behaviours

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1. Introduction

It is well known that the nonlinear partial differential equations (NPDEs) are widely used to describe complex phenomena in various fields of sciences, such as physics, biology, chemistry, etc.[1−5] Therefore, seeking exact solutions of NPDE is very important and significant in the nonlinear sciences. In the past few decades, a great effort has been made for this task and many powerful methods have been presented, such as the sine–cosine method, the Jacobi elliptic function method, the tanh-function method, the bilinear method, the standard Painlevé truncated expansion, the method of "coalescence of eigenvalue" or "wavenumbers", the homotopy-perturbation method, the hyperbolic function method, the variable separation method, the (G'/G)-expansion method and the projective equation approach, etc.[6−20] Among these methods, the projective equation approach is one of the most effectively straightforward algebraic methods for constructing exact solutions of NPDE.[21−26] In this paper, via a projective equation and a special ansatz, we find some new exact solutions of the (2+1)-dimensional Boiti–Leon–Pempinelli (BLP) system. Based on the derived solitary wave solution, we obtain some dromion and solitoff excitations and investigate the chaotic behaviours using a known chaotic system. The (2+1)-dimensional BLP system is

$$\begin{aligned} u_{ty} &= (u^2 - u_x)_{xy} + 2v_{xxx}, \\ v_t &= v_{xx} + 2uv_x. \end{aligned} \tag{1}$$

The integrability of the above BLP system was established in Ref. [27]. In Ref. [28], it was shown that the BLP system was Hamiltonian and it was also pointed out that by a certain transformation the sin-Gordon equation or the sinh-Gordon equation can be derived from the BLP model. Soliton-like and multisoliton-like solutions of BLP system have been discussed by Lu and Zhang.[29]

2. New exact solutions to the BLP system

In the usual projective equation approach, the basic idea of the algorithm is as follows: Consider a given nonlinear partial differential equation with independent variables $x = (x_0 = t, x_1, x_2, \ldots, x_m)$, and one dependent variable u, in the form

$$P(u, u_t, u_{x_i}, u_{x_i x_j}, \ldots) = 0, \tag{2}$$

where P is in general a polynomial function of its arguments and the subscripts denote the partial derivatives. One assumes its solution to be in the form of

$$u = A(x) + \sum_{i=1}^{n} B_i(x)\phi^i(q(x)) \tag{3}$$

with

$$\phi' = \sigma\phi + \phi^2, \tag{4}$$

where σ is a constant and the prime denotes differentiation with respect to q. To determine u explicitly, one may substitute Eqs. (3) and (4) into the given NPDE, collect the coefficients of the polynomials in ϕ, then

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eliminate each coefficient to derive a set of partial differential equations of A, B_i, and q and solves this system of partial differential equations to obtain A, B_i, and q. Finally, as equation (4) is known to possess the solution

$$\phi = \frac{-\sigma \mathrm{e}^{\sigma q + c\sigma}}{\mathrm{e}^{\sigma q + c\sigma} - 1} \quad \sigma \neq 0, \tag{5}$$

by some appropriate transformation for Eq. (5), we can obtain the following solitary wave solutions of Eq. (4):

$$\phi = \begin{cases} -\frac{1}{2}\sigma\left[1 + \tanh\left(\frac{1}{2}\sigma q\right)\right], & \sigma \neq 0, \\ -\frac{1}{2}\sigma\left[1 + \coth\left(\frac{1}{2}\sigma q\right)\right], & \sigma \neq 0. \end{cases} \tag{6}$$

Substituting A_i, B_i, q, and Eq. (6) into Eq. (3), one obtains the exact solutions to the given NPDE.

In order to find some new solutions of the BLP system, we try to improve the above method. We rewrite ansatz (3) as

$$\begin{aligned} u =\ & A(x) + \sum_{i=1}^{n} B_i(x)\phi^i(q(x)) \\ & + C_i(x)\phi^{i-1}(q(x))\sqrt{\sigma\phi + \phi^2(q(x))}. \end{aligned} \tag{7}$$

Now we apply the improved projective equation approach to Eq. (1). By the balancing procedure, ansatz (7) becomes

$$\begin{aligned} u =\ & f(x,y,t) + g(x,y,t)\phi(q(x,y,t)) \\ & + h(x,y,t)\sqrt{\sigma\phi(q(x,y,t)) + \phi^2(q(x,y,t))}, \\ v =\ & F(x,y,t) + G(x,y,t)\phi(q(x,y,t)) \\ & + H(x,y,t)\sqrt{\sigma\phi(q(x,y,t)) + \phi^2(q(x,y,t))}, \end{aligned} \tag{8}$$

where f, g, h, F, G, H, and q are functions of (x, y, t) to be determined. Substituting Eqs. (8) and (4) into Eq. (1) and collecting the coefficients of the polynomials of ϕ, then setting each coefficient to be zero, we have

$$\begin{aligned} f =\ & -\frac{1}{4}\frac{-2q_t + 2q_{xx} + \sigma q_x^2}{q_x}, \quad g = -\frac{1}{2}q_x, \\ h =\ & \frac{1}{2}q_x, \quad G = -\frac{1}{2}q_y, \quad H = \frac{1}{2}q_y, \\ F =\ & -\frac{1}{4}[q_x^3 q_{xxy}\sigma - 2q_x q_{xy} q_{xxx} + 2q_x q_{xxy} q_t \\ & + 2q_x q_{xy} q_{xt} + 2q_{xxxy} q_x^2 - 2q_{xyt} q_x^2 \\ & + 2q_x q_{ty} q_{xx} + 4q_{xy} q_{xx}^2 - 4q_{xx} q_{xy} q_t \\ & - 4q_{xx} q_{xxy} q_x]/q_x^2 \end{aligned} \tag{9}$$

with

$$q = \chi(x,t) + \varphi(y), \tag{10}$$

where $\chi \equiv \chi(x,t)$ and $\varphi \equiv \varphi(y)$ are two arbitrary variable separation functions of (x,t) and of y, respectively. Based on the solutions of Eq. (4), one thus obtains an explicit solution of Eq. (1).

$$u_1 = \frac{1}{2}\frac{-\chi_{xx} + \chi_t}{\chi_x} + \frac{1}{4}\chi_x\sigma\left\{\tanh\left(\frac{1}{2}\sigma(\chi+\varphi)\right) + \sqrt{-1 + \tanh\left(\frac{1}{2}\sigma(\chi+\varphi)\right)^2}\right\}, \tag{11}$$

$$v_1 = \frac{1}{4}\sigma\varphi_y\left\{1 + \tanh\left(\frac{1}{2}\sigma(\chi+\varphi)\right) + \sqrt{-1 + \tanh\left(\frac{1}{2}\sigma(\chi+\varphi)\right)^2}\right\}, \tag{12}$$

$$u_2 = \frac{1}{2}\frac{-\chi_{xx} + \chi_t}{\chi_x} + \frac{1}{4}\chi_x\sigma\left\{\coth\left(\frac{1}{2}\sigma(\chi+\varphi)\right) + \sqrt{-1 + \coth\left(\frac{1}{2}\sigma(\chi+\varphi)\right)^2}\right\}, \tag{13}$$

$$v_2 = \frac{1}{4}\sigma\varphi_y\left\{1 + \coth\left(\frac{1}{2}\sigma(\chi+\varphi)\right) + \sqrt{-1 + \coth\left(\frac{1}{2}\sigma(\chi+\varphi)\right)^2}\right\}, \tag{14}$$

with two arbitrary functions being $\chi(x,t)$ and $\varphi(y)$.

3. Soliton excitations in the BLP system

In this section, we mainly discuss the solitoff excitations of the BLP system. Due to the arbitrariness of the functions $\chi(x,t)$ and $\varphi(y)$ included in the above cases, the physical quantities u and v may possess rich structures when the functions $\chi(x,t)$ and $\varphi(y)$ are chosen appropriately. In the following discussion, we merely analyse their potential u_{2y} or v_{2x} determined by Eq. (13) or (14) and rewrite them in a simple form,

namely

$$\begin{aligned} U &\equiv u_{2y} \\ &= v_{2x} \\ &= -\frac{1}{8}\left\{\chi_x\varphi_y\sigma^2\mathrm{csch}\left(\frac{1}{2}\sigma(\chi+\varphi)\right)\right. \\ &\quad \left.\times\left[\mathrm{csch}\left(\frac{1}{2}\sigma(\chi+\varphi)\right)+\coth\left(\frac{1}{2}\sigma(\chi+\varphi)\right)\right]\right\}. \end{aligned} \tag{15}$$

According to the solution U of Eq. (15), we first discuss its dromion and solitoff excitations. For instance, if we choose χ and φ as

$$\chi = 1+\mathrm{sech}(x+t), \quad \varphi = 1+\tanh(y), \tag{16}$$

and

$$\chi = 1+\exp(x+t), \quad \varphi = 1+\exp(y), \tag{17}$$

we can obtain a dromion excitation and a solitoff excitation for the physical quantity U of Eq. (15) presented in Figs. 1(a) and 1(b) at time $t = 0$.

Generally, the interactions between solitons may be regarded as completely elastic, while for some specific cases, the interactions between solitons are nonelastic. For example, if we choose χ and φ as

$$\begin{aligned} \chi &= 1+10\exp(x+t)+8\mathrm{sech}(x-t), \\ \varphi &= 1+10\exp(y), \end{aligned} \tag{18}$$

we can obtain two solitons with nonelastic behaviour. Figure 2 shows an evolutional profile corresponding to

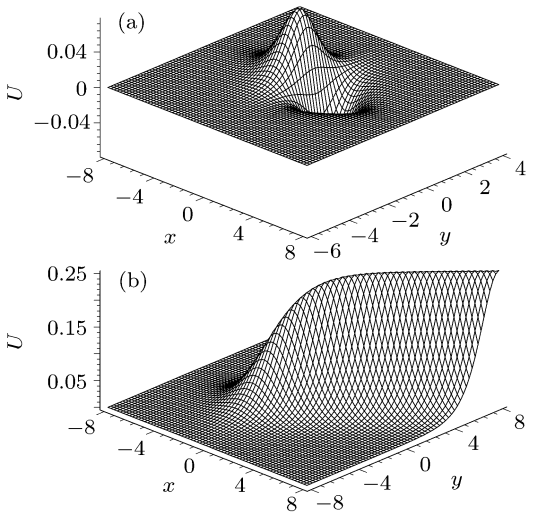


Fig. 1. A plot of dromion and solitoff structures for the physical quantity U given by Eq. (15) with the choice of expressions (16) and (17) at time $t = 0$.

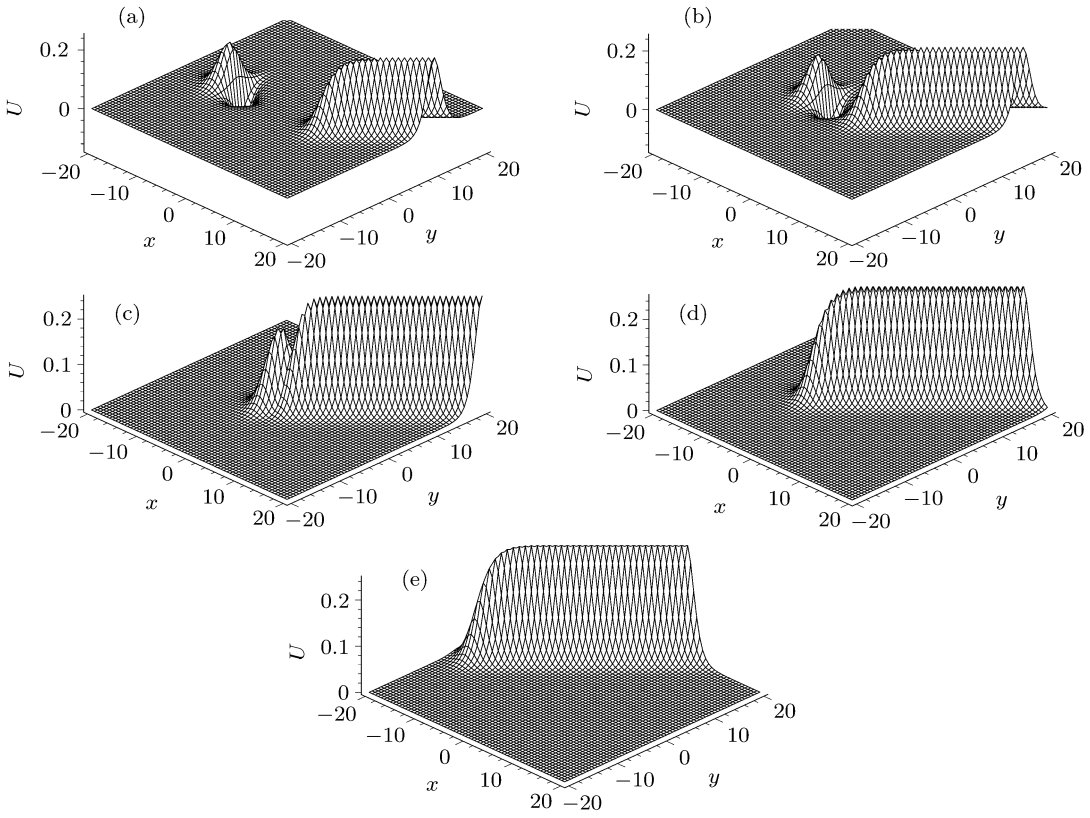


Fig. 2. Evolutional profiles of two solitons fusing into one soliton for the solution U expressed by Eq. (15) with condition (18) at different times (a) $t = -10$, (b) $t = -5$, (c) $t = 0$, (d) $t = 5$, and (e) $t = 10$.

the physical quantity U of the dromion and the solitoff expressed by Eq. (15), exhibiting a fusion phenomenon for the two solitons. From Fig. 2, we can clearly see that the two solitons fuse to one soliton finally.

4. Chaotic behaviours of the BLP system

Just like solitons, chaos is another important part of nonlinear science. It has been widely applied in many natural sciences. In this section, we mainly discuss some localized coherent excitations with chaotic behaviour in the (2+1)-dimensional BLP system.

4.1. Novel four-wing shape chaotic attractor

Recently, Liu *et al.*[30] have introduced a new chaotic system (LXGN) of three-dimensional quadratic autonomous ordinary differential equations

$$m_\xi = 2.4m - nl, \quad n_\xi = -6n + ml,$$
$$l_\xi = -4l - 2.8m + mn, \tag{19}$$

where m, n, and l are functions of ξ ($\xi = (x + ct)$ or $\xi = y$). A novel four-wing shape chaotic attractor for LXGN system (19) is depicted in Fig. 3 when

$$m(0) = 0.01, \quad n(0) = 0.01, \quad l(0) = 0.01. \tag{20}$$

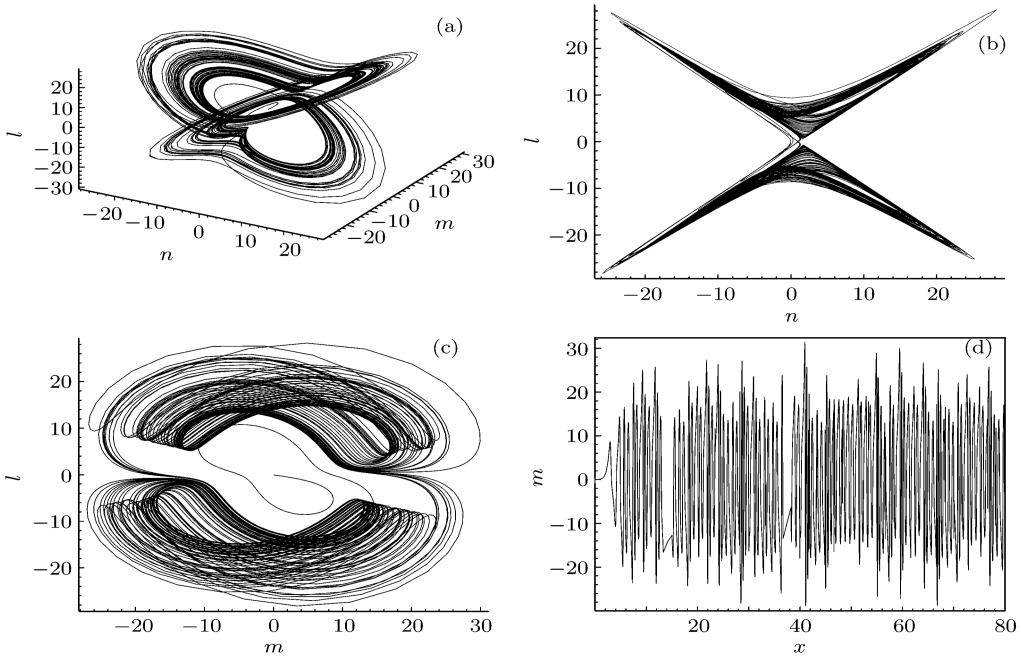


Fig. 3. (a) A novel four-wing shape attractor plot of chaotic LXGN system (19) with initial condition (20). (b) Two-dimensional phase diagram of n, l. (c) Two-dimensional phase diagram of m, l. (d) A typical plot of the chaotic solution m of Eq. (19) related to panel (a).

4.2. Chaotic patterns

If the functions χ and/or φ are assumed to be solutions of a chaotic dynamical system, we can derive some localized excitations with chaotic behaviour. For example, χ is defined by a solution of the LXGN system (19) as

$$\chi = 1 + m(x + ct), \quad \varphi = 1 + \exp(y), \tag{21}$$

where $m(x + ct)$ is a solution of LXGN system (19) with initial condition (20). By this choice, the dromion localized in all directions is changed into a chaotic line soliton, which presents chaotic behaviour in the x direction though still localized in y direction. Figure 4(a) shows the corresponding plot of the chaotic line soliton for the field U of Eq. (15) with parameter $c = 1$ at time $t = 0$.

Furthermore, if χ and φ are both selected as chaotic solutions of the LXGN system, the field U of Eq. (15) will behave chaotically in all directions and will yield a chaotic pattern. For example, χ and φ may be chosen respectively as

$$\chi(x) = 1 + m(x + ct) \ \text{ and } \ \varphi(y) = 1 + m(y), \tag{22}$$

where $m(x + ct)$ and $m(y)$ are the solutions of LXGN system (19) with the initial condition (20). Figure 4(b) shows a plot of the special chaotic pattern for the field U expressed by Eq. (15) with the condition (22) at time $t = 0$.

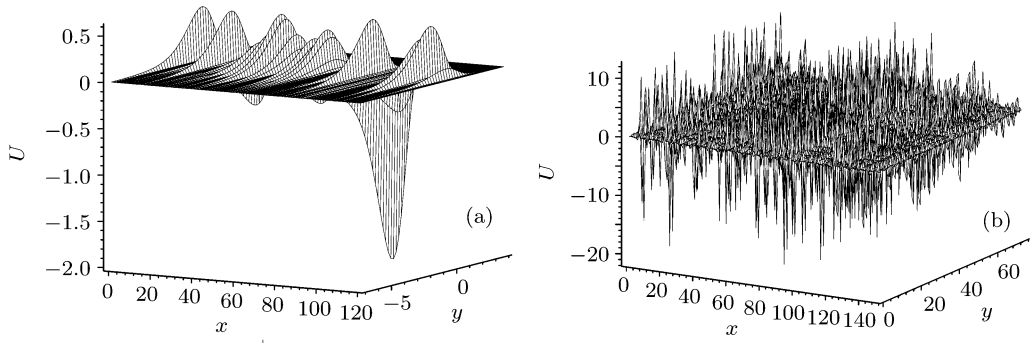


Fig. 4. (a) Plot of the chaotic line soliton for the field U determined by Eq. (19) with condition (21) and $c = 1,\ t = 0$. (b) Plot of the chaotic patten for the field U expressed by Eq. (15) with condition (22) and $c = 1,\ t = 0$.

5. Summary and discussion

Via an improved projective equation approach and a linear variable separation approach, we found some new exact solutions of the (2+1)-dimensional Boiti–Leon–Pempinelli system. Based on the derived solitary wave solution, we investigate the dromion and solitoff excitations and using a known LXGN chaotic system, we obtain some chaotic patterns of the BLP system, which are different from the ones of the previous work.

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