The synchronization of a fractional order hyperchaotic system based on passive control

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This paper investigates the synchronization of a fractional order hyperchaotic system using passive control. A passive controller is designed, based on the properties of a passive system. Then the synchronization between two fractional order hyperchaotic systems under different initial conditions is realized, on the basis of the stability theorem for fractional order systems. Numerical simulations and circuitry simulations are presented to verify the analytical results.

Keywords: fractional order hyperchaos, passive control, numerical simulation, circuitry simulation

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1. Introduction

Fractional calculus is a mathematical topic with more than 300 years of history, but the applications of fractional calculus in physics and engineering are just a recent focus of interest. There exist some well-known fractional order systems, such as the fractional order Chua's circuit,[1] the fractional order Genesio Tesi system,[2] the fractional order hyperchaotic Rossler system,[3] the fractional order 'jerk' model,[4,5] and so on.[6−8] The dynamics of fractional order chaotic systems have been proved to be useful in physics, engineering and control processing.[9−11] It is very important to further explore the control and synchronization of these fractional order hyperchaotic systems for the engineering applications. The synchronization of fractional order hyperchaotic systems has a practical value for science and can be applied to many areas such as secure communication and information processing.

Since the phenomenon of chaotic synchronization was first revealed by Pecora and Carroll,[12] a wide variety of methods have been proposed for the synchronization of chaotic systems. They include linear and nonlinear feedback control,[13] active control,[14] backstepping control,[15] adaptive control,[16] impulsive control,[17,18] and so on.[19−24]

Recently, the concept of passive control of nonlinear systems has attracted new interest in nonlinear system. Passivity is part of a broader and more general theory of dissipativity. The main idea of passivity theory is that the passive properties of system can keep the system internally stable. In order to make the system stable, we can design a controller which renders the closed loop system passive by using passivity theory. Wen[25] applied the passivity technique to design the linear feedback controller to control the Lorenz system. Wang and Liu[26] applied passivity-based control to design the controller to control a unified chaotic system. During the last decade the passivity theory played an important role in designing an asymptotically stabilizing controller for a nonlinear system.[27,28]

In this paper, based on the stability theorem for fractional order systems and the properties of a passive system, the synchronization between two fractional order hyperchaotic systems is studied. The essential conditions are derived under which the erroneous dynamical system could be equivalent to a passive system and globally asymptotically stabilized at a zero equilibrium point via smooth state feedback. Compared with other methods, the passive control has many advantages, such as clear physical interpretation, ease in implementation and less control effort being required.

The rest of this paper is organized as follows. In Section 2, a novel smooth four-dimensional continuous-time fractional order hyperchaotic system has been obtained by utilizing the approximation of $1/s^{\alpha}$ with $\alpha = 0.9$ in the Laplace domain. The numerical simulations of this fractional order hyperchaotic system are realized. In Section 3, the properties of a passive system are introduced. In Section 4, a brief

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description of the stability theorem for fractional order systems is introduced and the passive control is applied to synchronization between two fractional order hyperchaotic systems. In Section 5, numerical and circuitry simulations are given for illustration and verification. Finally, some concluding remarks and comments are presented.

2. A hyperchaotic system and its fractional order realization

The hyperchaotic system, which is put forward by Liu chongxin,[29] is described by

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = a(y-x), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -kxz + bx + v, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = dxy - cz, \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = -hy. \end{cases} \tag{1}$$

With $a = 10$, $b = 40$, $c = 2.5$, $d = 4$, $h = 5$, and $k = 1$, the fractional hyperchaotic strange attractors are obtained as shown in Fig. 1, where the initial values are set to be (0.1, 0.2, 0.3, 0.4).

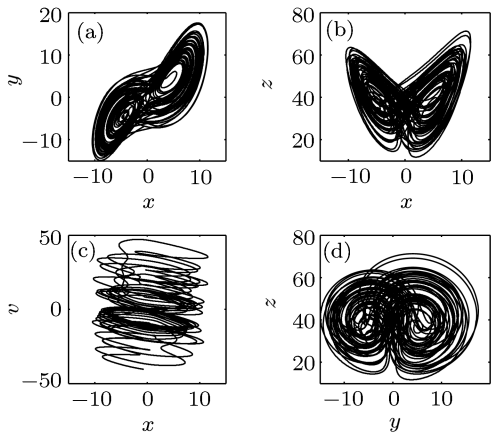


Fig. 1. Phase plane strange attractors for (a) x–y phase plane, (b) x–z phase plane, (c) x–v phase plane, and (d) y–z phase plane.

Then consider the fractional order system. The standard derivative is replaced by a fractional derivative as follows:

$$\begin{cases} \dfrac{\mathrm{d}^{\alpha}x}{\mathrm{d}t^{\alpha}} = a(y-x), \\ \dfrac{\mathrm{d}^{\alpha}y}{\mathrm{d}t^{\alpha}} = -kxz + bx + v, \\ \dfrac{\mathrm{d}^{\alpha}z}{\mathrm{d}t^{\alpha}} = dxy - cz, \\ \dfrac{\mathrm{d}^{\alpha}v}{\mathrm{d}t^{\alpha}} = -hy, \end{cases} \tag{2}$$

where x, y, z, and v are state variables and a, b, c, d, h, and k are system parameters.

When $\alpha = 0.9$, the differential equation can be expressed as

$$\begin{cases} \dfrac{\mathrm{d}^{0.9}x}{\mathrm{d}t^{0.9}} = a(y-x), \\ \dfrac{\mathrm{d}^{0.9}y}{\mathrm{d}t^{0.9}} = -kxz + bx + v, \\ \dfrac{\mathrm{d}^{0.9}z}{\mathrm{d}t^{0.9}} = dxy - cz, \\ \dfrac{\mathrm{d}^{0.9}v}{\mathrm{d}t^{0.9}} = -hy. \end{cases} \tag{3}$$

Let us consider the Riemann–Liouville definition which is well known for the fractional differential operator. The Riemann–Liouville definition is given as

$$\frac{\mathrm{d}^{\alpha}f(t)}{\mathrm{d}t^{\alpha}} = \frac{1}{\Gamma(n-\alpha)}\frac{\mathrm{d}^{n}}{\mathrm{d}t^{n}}\int_{0}^{t}\frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}\mathrm{d}\tau, \tag{4}$$

where $n-1 < \alpha < n$, n is an integer number and $\Gamma(\cdot)$ is the gamma function. Another alternative definition of the Riemann–Liouville function was reported by Caputo as follows:

$$\frac{\mathrm{d}^{\alpha}f(t)}{\mathrm{d}t^{\alpha}} = \frac{1}{\Gamma(n-\alpha)}\int_{0}^{t}\frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}\mathrm{d}\tau. \tag{5}$$

Considering all the initial conditions to be zero, the Laplace transform of the Riemann–Liouville functional derivative is given as

$$L\left\{\frac{\mathrm{d}^{\alpha}f(t)}{\mathrm{d}t^{\alpha}}\right\} = s^{\alpha}L\left\{f(t)\right\}. \tag{6}$$

The fractional integral operator of order "α" can be represented by the transfer function $F(s) = 1/s^{\alpha}$ in the frequency domain.

In Ref. [30], an effective algorithm is developed to approximate the fractional order transfer functions. From Table 1 in Ref. [4], we can obtain an approximation of $1/s^{0.9}$ with an error of about 2 dB as follows:

$$F(s) = \frac{2.2675(s+1.292)(s+215.4)}{(s+0.01292)(s+2.154)(s+359.4)}. \tag{7}$$

We utilize these approximations to realize the fractional order hyperchaotic system by using SIMULINK. Figure 2 shows the fractional order hyperchaotic phase diagram with $a = 10$, $b = 40$, $c = 2.5$, $d = 4$, $h = 5$, and $k = 1$.

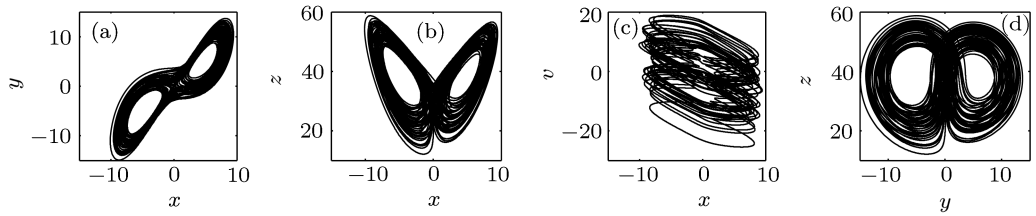


Fig. 2. Fractional order phase plane strange attractors. (a) x–y phase plane, (b) x–z phase plane, (c) x–v phase plane, and (d) y–z phase plane.

3. The theory of passive control

The physical meaning of the passive system is that the energy of the nonlinear system can be increased only through the supply from an external source. In other words, a passive system cannot store more energy than that supplied externally. The passive system is a naturally stable system.[31,32]

Consider the following differential equation[26,31,32]

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \tag{8}$$

where $x \in R^n$ is the state variable, $f(x)$ and $g(x)$ are the smooth vector fields, $u(t) \in U$ is the external input, $f(0) = 0$ and $h(x)$ is a smooth mapping.

At first, we review three definitions of the passive system.

Definition 1 The system (8) is passive if the following two conditions are satisfied:

1) $f(x)$ and $g(x)$ exist and are smooth vector fields, $h(x)$ is a smooth mapping.

2) For any $t \geq 0$, there is a real value β satisfying the inequality

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau \geq \beta, \tag{9}$$

or there are a real value β and $\rho > 0$ satisfying the inequality

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau + \beta \geq \int_0^t \rho y^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau. \tag{10}$$

Definition 2 The system (8) is passive if there is a continuous differentiable nonnegative function $V(x) : X \to R$, $V(0) = 0$, named the storage function, or continuous positive-definite storage functions $V(x)$ and $S(x)$, such that $\forall x \in X$,

$$V(x) - V(x_0) \leq \int_0^t y^{\mathrm{T}}(s)u(s)\mathrm{d}s; \tag{11}$$

or

$$V(x) - V(x_0) = \int_0^t y^{\mathrm{T}}(s)u(s)\mathrm{d}s - \int_0^t S(x(s))\mathrm{d}s. \tag{12}$$

Definition 3 The system (8) is a minimum phase system if $L_{\mathrm{g}}h(0)$ is nonsingular and $x = 0$ is one of asymptotically stabilized equilibrium points of $f(x)$.

Theorem 1[26] Let $z = \varphi(x)$ and the system (8) will be changed into the following generalized form

$$\begin{cases} \dot{z} = f_0(z) + p(z, y)y, \\ \dot{y} = b(z, y) + a(z, y)u, \end{cases} \tag{13}$$

where $a(z, y)$ is nonsingular for any (z, y).

If the system (8) is a minimum phase system and has a relative degree $[1, 1, \ldots]$ at $x = 0$, the system (13) will be equivalent to a passive system and asymptotically stabilized at an equilibrium point through the local feedback control as follows:

$$u = a(z, y)^{-1}\Big[- b^{\mathrm{T}}(z, y) - \frac{\partial}{\partial y}W(z)p(z, y) - \alpha y + v\Big]. \tag{14}$$

4. Synchronization between two identical fractional order hyperchaotic systems using passive control

First, consider some stability theorems for fractional systems.[33]

Consider the following autonomous system

$$\frac{\mathrm{d}X}{\mathrm{d}t} = A(X)X \tag{15}$$

and its fractional form

$$\frac{\mathrm{d}^{\alpha}X}{\mathrm{d}t^{\alpha}} = A(X)X, \tag{16}$$

where $X(x_1, x_2, \ldots) \in R^n (n \in N)$.

Lemma 1 For fractional system (16), if the arbitrary eigenvalues λ of matrix $A(X)$ satisfy $|\arg(\lambda)| \geq \alpha\pi/2$, it is of stability.

Theorem 2 If the system (15) is of asymptotic stability, in the range of state variable X (except the origin), all the real part of $A(X)$'s eigenvalues are not more than 0.

From Lemma 1 and Theorem 2, we can obtain that if the system (15) is of asymptotic stability, the fractional system (16) is also of asymptotic stability. In other words, if the integral order system is of stability, then the fractional system (the order is less than 1) is also of stability.

Then consider the synchronization between two identical fractional order hyperchaotic systems using passive control.

The nonlinear differential equations that describe this fractional hyperchaotic system, as a drive system, are

$$\begin{cases} \dfrac{\mathrm{d}^\alpha x_1}{\mathrm{d}t^\alpha} = a(x_2 - x_1), \\ \dfrac{\mathrm{d}^\alpha x_2}{\mathrm{d}t^\alpha} = -kx_1x_3 + bx_1 + x_4, \\ \dfrac{\mathrm{d}^\alpha x_3}{\mathrm{d}t^\alpha} = dx_1x_2 - cx_3, \\ \dfrac{\mathrm{d}^\alpha x_4}{\mathrm{d}t^\alpha} = -hx_2. \end{cases} \tag{17}$$

The response system is described by the following equations, (u is controller)

$$\begin{cases} \dfrac{\mathrm{d}^\alpha y_1}{\mathrm{d}t^\alpha} = a(y_2 - y_1), \\ \dfrac{\mathrm{d}^\alpha y_2}{\mathrm{d}t^\alpha} = -ky_1y_3 + by_1 + y_4 + u, \\ \dfrac{\mathrm{d}^\alpha y_3}{\mathrm{d}t^\alpha} = dy_1y_2 - cy_3, \\ \dfrac{\mathrm{d}^\alpha y_4}{\mathrm{d}t^\alpha} = -hy_2. \end{cases} \tag{18}$$

The erroneous dynamical fractional system between the drive system (17) and response system (18) can be expressed as

$$\begin{cases} \dfrac{\mathrm{d}^\alpha e_1}{\mathrm{d}t^\alpha} = a(e_2 - e_1), \\ \dfrac{\mathrm{d}^\alpha e_2}{\mathrm{d}t^\alpha} = -k(e_1e_3 + x_1e_3 + e_1x_3) + be_1 + e_4 + u, \\ \dfrac{\mathrm{d}^\alpha e_3}{\mathrm{d}t^\alpha} = d(e_1e_2 + x_1e_2 + e_1x_2) - ce_3, \\ \dfrac{\mathrm{d}^\alpha e_4}{\mathrm{d}t^\alpha} = -he_2, \end{cases} \tag{19}$$

where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$, and $e_4 = y_4 - x_4$. The aim is to determine the controller u for stabilizing the erroneous dynamical system (19) at a zero equilibrium point.

According to the stability theorems for fractional systems as mentioned above, we consider the integer order erroneous dynamical system

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1), \\ \dot{e}_2 = -k(e_1e_3 + x_1e_3 + e_1x_3) + be_1 + e_4 + u, \\ \dot{e}_3 = d(e_1e_2 + x_1e_2 + e_1x_2) - ce_3, \\ \dot{e}_4 = -he_2. \end{cases} \tag{20}$$

Our object is to design a smooth control u for the system (20) to make the closed-loop system passive. Then, the integer order erroneous dynamical system (20) will be asymptotically stabilized at zero equilibrium points. There are two steps stated below.

First step we will prove that the controlled system (20) is a minimum phase.

Let $z_1 = e_1$, $z_2 = e_3$, $z_3 = e_4$, and $y = e_2$. The system (20) can be rewritten as

$$\begin{cases} \dot{z}_1 = a(y - z_1), \\ \dot{z}_2 = d(z_1y + x_1y + z_1x_2) - cz_2, \\ \dot{z}_3 = -hy, \\ \dot{y} = -k(z_1z_2 + x_1z_2 + z_1x_3) + bz_1 + z_3 + u; \end{cases} \tag{21}$$

or in other form

$$\begin{cases} \dot{z} = f_0(z) + p(z,y)y, \\ \dot{y} = b(z,y) + a(z,y)u. \end{cases} \tag{22}$$

So we have

$$f_0(z) = \begin{bmatrix} -az_1 \\ dx_2z_1 - cz_2 \\ 0 \end{bmatrix}, \tag{23}$$

$$p(z,y) = \begin{bmatrix} a \\ dz_1 + dx_1 \\ -h \end{bmatrix}, \tag{24}$$

$$b(z,y) = -k(z_1z_2 + x_1z_2 + z_1x_3) + bz_1 + z_3, \tag{25}$$

$$a(z,y) = 1. \tag{26}$$

Choose a storage function

$$V(z,y) = W(z) + \frac{1}{2}y^2, \tag{27}$$

where

$$W(z) = \frac{dM^2}{8ac}z_1^2 + \frac{1}{2d}z_2^2 + \frac{1}{2}z_3^2,$$

which is the Lyapunov function with $W(0) = 0$.

The zero dynamics of the system (21) describes the internal dynamics, which is consistent with external constraint $y = 0$,

$$\dot{z} = f_0(z). \tag{28}$$

Considering Eq. (28), we can obtain

$$\frac{\mathrm{d}W(z)}{\mathrm{d}t} = \frac{\partial W(z)}{\partial z}\dot{z}$$

$$
\begin{aligned}
&= \frac{\partial W(z)}{\partial z} f_0(z) \\
&= \left[\frac{dM^2}{4ac} z_1, \frac{1}{d} z_2, z_3\right] \begin{bmatrix} -az_1 \\ dx_2 z_1 - cz_2 \\ 0 \end{bmatrix} \\
&= -\frac{dM^2}{4c} z_1^2 + x_2 z_1 z_2 - \frac{c}{d} z_2^2 \\
&= -\frac{c}{d}\left(z_2 - \frac{d}{2c} x_2 z_1\right)^2 + \frac{d}{4c} x_2^2 z_1^2 - \frac{dM^2}{4c} z_1^2 \\
&= -\frac{c}{d}\left(z_2 - \frac{d}{2c} x_2 z_1\right)^2 - \left(\frac{dM^2}{4c} - \frac{dx_2^2}{4c}\right) z_1^2.
\end{aligned} \tag{29}
$$

With each chaotic or hyperchaotic system having its own bound, the following inequality can be obtained

$$
|x_2| \leq x_{2\max} = M, \tag{30}
$$

then

$$
\frac{dM^2}{4c} - \frac{dx_2^2}{4c} \geq 0,
$$

the parameters c, d is positive and the following inequality is obtained

$$
\frac{\partial W(z)}{\partial z} f_0(z) \leq 0. \tag{31}
$$

The zero dynamics of the controlled system (21) is Lyapunov-stable, namely, the controlled system (21) is a minimum phase.

Second step we design a smooth control u for the system (21) to make the closed-loop system passive. The derivative of $V(z, y)$ along the trajectory of the controlled system (21) is

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} V(z, y) &= \frac{\partial W(z)}{\partial z} \dot{z} + y\dot{y} \\
&= \frac{\partial W(z)}{\partial z} f_0(z) + \frac{\partial W(z)}{\partial z} p(z, y) y \\
&\quad + yb(z, y) + ya(z, y)u.
\end{aligned} \tag{32}
$$

As the system (21) is a minimum phase, the following inequalities are obtained:

$$
\frac{\partial W(z)}{\partial z} f_0(z) \leq 0, \tag{33}
$$

so

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} V(z, y) &\leq \frac{\partial W(z)}{\partial z} p(z, y) y + yb(z, y) \\
&\quad + ya(z, y)u.
\end{aligned} \tag{34}
$$

According to Theorem 1, we select the feedback control in the following form

$$
\begin{aligned}
u &= a(z, y)^{-1}\left[-b^{\mathrm{T}}(z, y) - \frac{\partial}{\partial y} W(z) p(z, y) - \alpha y + v\right] \\
&= -\left(\frac{dM^2}{4c} + b\right) e_1 + (k-1) e_3 (e_1 + x_1) \\
&\quad + (h-1) e_4 + k x_3 e_1 + \alpha e_2 + v,
\end{aligned} \tag{35}
$$

where α is a positive constant and v is an external signal which is connected with the reference input. Substituting Eq. (35) into the inequality (34), we have

$$
\frac{\mathrm{d}}{\mathrm{d}t} V(z, y) \leq -\alpha y^2 + vy. \tag{36}
$$

Performing integration on both sides of the inequality (36) yields

$$
\begin{aligned}
&V(z, y) - V(z_0, y_0) \\
&\leq -\int_0^t \alpha y(\tau)^2 \mathrm{d}\tau + \int_0^t v(\tau) y(\tau) \mathrm{d}\tau.
\end{aligned} \tag{37}
$$

For $V(z, y) \geq 0$, let $V(z_0, y_0) = \mu$, then the inequality (37) can be rewritten as

$$
\begin{aligned}
\int_0^t v(\tau) y(\tau) \mathrm{d}\tau + \mu &\geq \int_0^t \alpha y(\tau)^2 \mathrm{d}\tau + V(z, y) \\
&\geq \int_0^t \alpha y(\tau)^2 \mathrm{d}\tau.
\end{aligned} \tag{38}
$$

According to Definition 1, the system (21) is a passive system. Because $W(z)$ is radially unbounded, it follows from Eq. (27) that $V(z, y)$ is also radially unbounded, so that a closed-loop system is in a bounded state stable for $[z^{\mathrm{T}}, y]^{\mathrm{T}}$. This means that we can use the local feedback control described in the form (35) to regulate the system (21) to an equilibrium point.

When we let $v = 0$, the erroneous dynamical system (20) will be asymptotically stabilized at a zero equilibrium point and the two hyperchaotic systems under different initial conditions will be synchronized with each other.

Now, we have proved that the integer erroneous dynamical system (20) is of stability at a zero equilibrium point with the controller u. From Theorem 2, we obtain that all the real parts of $A(e)$'s eigenvalues are not more than 0. Then the arbitrary eigenvalues λ of matrix $A(e)$ satisfy Lemma 1. In other words, the fractional erroneous dynamical system (19) is also of stability at a zero equilibrium point. The controlled response system (18) is synchronized with the drive system (17).

5. Simulation research

In this section, we design an electronic oscillation circuit by using the Pspice to realize the fractional order hyperchaotic system and obtain the self-synchronization of the fractional order hyperchaotic system by using SIMULINK.

First, the designed circuit, which is shown in Fig. 3, consists of operational amplifiers, multipliers, resistors and capacitors. For Op-Amps AD741 is used and AD633 is selected for multiplier; $R_1 = R_2 = R_8 = R_{15} = 1$ kΩ, $R_3 = R_4 = R_{11} = 10$ kΩ, $R_5 = R_6 = R_{12} = 1$ kΩ, $R_7 = 2.5$ kΩ, $R_9 = R_{10} = 100$ kΩ, $R_{13} = R_{18} = R_{19} = 1$ kΩ, $R_{14} = 160$ kΩ, $R_{16} = 400$ kΩ, $R_{17} = R_{22} = 10$ kΩ, $R_{20} = 2$ kΩ, $R_{21} = 15$ kΩ; $C_1 = C_2 = C_3 = C_4 = 1$ μF. The four state variables x, y, z, and v are respectively obtained from the voltage outputs of V_{C_1}, V_{C_2}, V_{C_3}, and V_{C_4}. When C_1, C_2, C_3, and C_4 are replaced by fractional order block, the fractional order hyperchaotic system with $\alpha = 0.9$ is realized, $R_a = 62.84$ MΩ, $R_b = 250$ kΩ, $R_c = 2.5$ kΩ, $C_a = 1.23$ μF, $C_b = 1.835$ μF, $C_c = 1.1$ μF. The circuitry simulation result of the phase diagram is shown in Fig. 4.

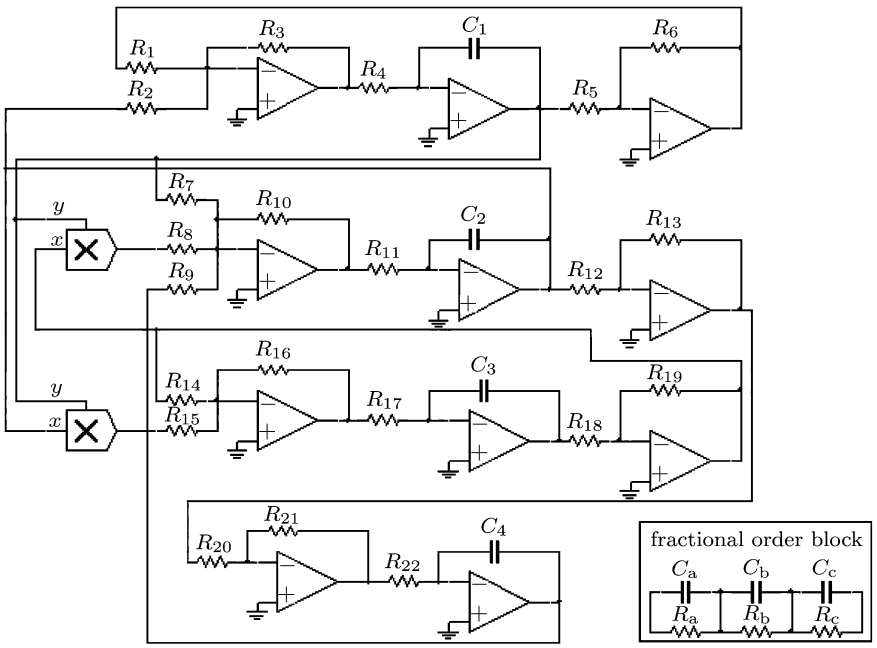


Fig. 3. Analog electronic implementation of hyperchaotic system, Op-Amps: AD741, multiplier: AD633.

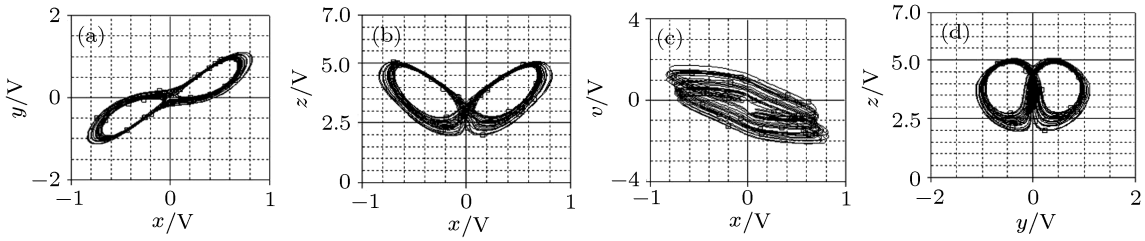


Fig. 4. phase plane of fractional order hyperchaotic system by using Pspice for (a) x–y phase plane. (b) x–z phase plane, (c) x–v phase plane, and (d) y–z phase plane.

Secondly, the numerical simulation using SIMULINK proves the conclusion that is mentioned in Section 4. Choose initial conditions: $x_1(0) = 1$, $x_2(0) = 2$, $x_3(0) = 3$, and $x_4(0) = 4$ for the driving system (17) and $y_1(0) = 0.1$, $y_2(0) = 0.1$, $y_3(0) = 0.3$, and $y_4(0) = 0.4$ for the response system (18). After 50 s, the controller u is added to the response system. As expected, one can observe that the trajectories of the response system asymptotically approach those of the drive system as illustrated in Figs. 5(a)–5(d) and the norm $||e||$ of erroneous vector eventually converges to zero after the controller is activated, as shown in Fig. 5(e). The simulation result implies that the two identical fractional hyperchaotic systems are synchronized with each other and validates the effectiveness of the proposed method.

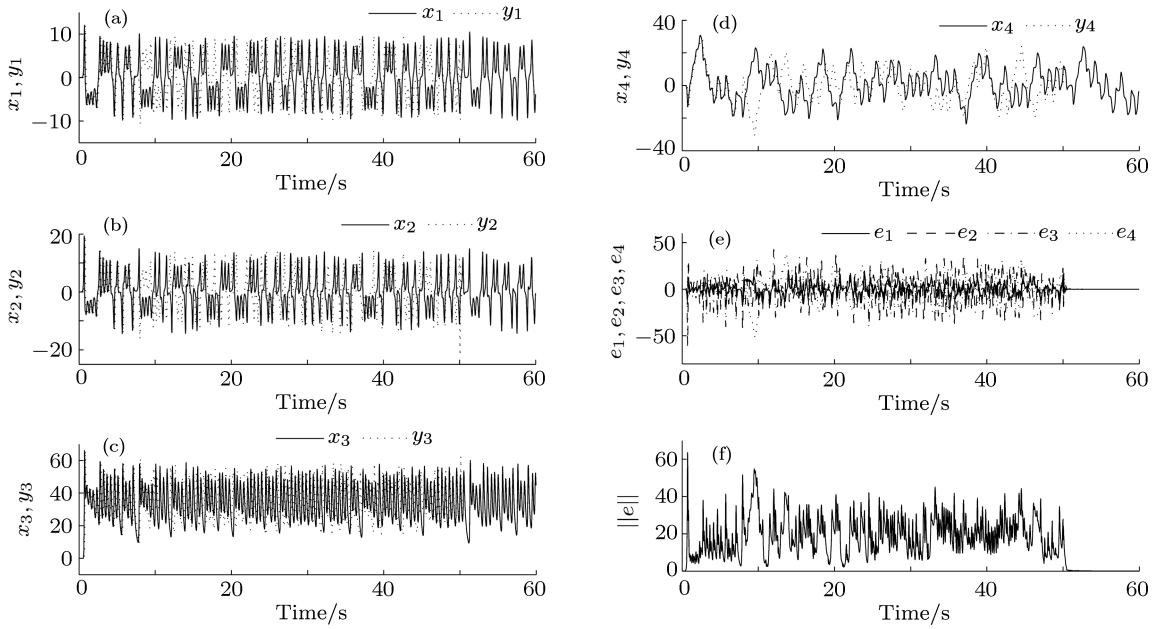


Fig. 5. Synchronization between two identical fractional order hyperchaotic systems. (a) Time waveforms of the states x_1 (solid) and y_1 (dashed); (b) Time waveforms of the states x_2 (solid) and y_2 (dashed); (c) Time waveforms of the states x_3 (solid) and y_3 (dashed); (d) Time waveforms of the states x_4 (solid) and y_4 (dashed); (e) Error dynamical system states e_1, e_2, e_3, and e_4; (f) Time waveform of the norm $\|e\|$ of error vector.

6. Conclusions

Based on the stability theorem for fractional systems and the properties of a passive system, a feedback controller for the synchronization of the two fractional order hyperchaotic systems is derived. The effectiveness of the proposed synchronization method has been demonstrated by numerical simulation. It is expected that both integral and fractional order hyperchaotic system will benefit in chaos control, security communication and many other engineering practical applications.

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