Resonance response of a single-degree-of-freedom nonlinear vibro-impact system to a narrow-band random parametric excitation*

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The resonant response of a single-degree-of-freedom nonlinear vibro-impact oscillator with a one-sided barrier to a narrow-band random parametric excitation is investigated. The narrow-band random excitation used here is a bounded random noise. The analysis is based on a special Zhuravlev transformation, which reduces the system to one without impacts, thereby permitting the applications of random averaging over "fast" variables. The averaged equations are solved exactly and an algebraic equation of the amplitude of the response is obtained for the case without random disorder. The methods of linearization and moment are used to obtain the formula of the mean-square amplitude approximately for the case with random disorder. The effects of damping, detuning, restitution factor, nonlinear intensity, frequency and magnitude of random excitations are analysed. The theoretical analyses are verified by numerical results. Theoretical analyses and numerical simulations show that the peak response amplitudes will reduce at large damping or large nonlinear intensity and will increase with large amplitude or frequency of the random excitations. The phenomenon of stochastic jump is observed, that is, the steady-state response of the system will jump from a trivial solution to a large non-trivial one when the amplitude of the random excitation exceeds some threshold value, or will jump from a large non-trivial solution to a trivial one when the intensity of the random disorder of the random excitation exceeds some threshold value.

Keywords: nonlinear vibro-impact system, resonance response, random averaging method, stochastic jump

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1. Introduction

Many non-smooth factors arise naturally in engineering applications, such as impacts, collisions, dry frictions and so on. A considerable number of research activities have focused on non-smooth dynamical systems in recent years. For deterministic non-smooth systems, the dynamics of a periodically forced impact system was investigated in Ref. [1]. Bifurcations and chaos of two-degree-of-freedom linear vibro-impact systems were explored by the Pioncaré map in Refs. [2]–[6]. The particular bifurcations unique to non-smooth systems have been examined extensively in Refs. [7]–[12]. In practice, engineering structures are often subjected to time-dependent loadings of stochastic nature, such as the natural phenomena due to wind gusts, earthquakes, ocean waves and random disturbance or noise which always exists in a physical system. The influence of random excitation on the dynamical behaviour of an impact dynamical system has caught the attention of many researchers. Many effective methods have been developed, e.g., linearization method,[13] quasistatic approach method,[14,15] Markov process method,[16,17] stochastic averaging method,[18,19] variable transformation method,[20] energy balance method,[21] mean impact Poincaré map method,[22] and numerical simulation method.[23] In Ref. [24], the authors reviewed and summarized the existing methods, results and literature available for solving problems of stochastic vibro-impact systems. However, most researches focused on responses of linear (here, "linear" means that the differential equation of motion between impacts is linear) impact oscillator under wide-band random excitations. Few focused on the responses of linear,[25] especially nonlinear[26,27] impact oscillators under narrow-band random excitations. In many physical applications, vibro-impact systems are subjected to narrow-band random processes.[24] A clear

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example of such systems may be found in ocean engineering, where ocean wave excitation may be considered as a white noise only to the first approximation; a model of a narrow-band random process as a harmonic process with random phase modulation should be much more realistic. Therefore, in comparison with the researches on wide-band random excitations, researches on narrow-band random excitations should also be important but will be more difficult, especially for the reason that nonlinear effects will be more prominent near the resonance region. In Ref. [25], Dimentberg and Iourtchenko discussed the subharmonic response of a linear impact system with a rigid one-sided barrier under a special kind of narrow-band random excitation—sinusoidal force with disorder or random phase modulation, in detail. However, as they pointed out, for certain applications the model of a filtered Gaussian white noise, rather than one used in Ref. [25], may be more suitable for the basic narrow-band random excitation. In Refs. [26] and [27], the authors of these papers discussed the subharmonic response of a nonlinear impact system with a rigid one-sided barrier under sinusoidal force with disorder[26] and filtered Gaussian white noise,[27] and generalized the results of Dimentberg and Iourtchenko. The narrow-band excitations used in Refs. [25]–[27] are additional ones. Rare researches focused on the responses of nonlinear impact oscillator under narrow-band random parametric excitations. In the present paper, the resonance response of a single-degree-of-freedom nonlinear vibro-impact oscillator with a one-sided barrier, which is slightly offset from the system equilibrium position and under a special kind of narrow-band random parametric excitation—bounded noise process, is investigated. The impact considered here is an instantaneous one with restitution factor e.

The rest of the present paper is organized as follows. In Section 2, the mean-square amplitude of the response is obtained through the Zhuravlev transformation and by stochastic averaging method. In Section 3, the direct numerical simulations verify the analytical result. Finally, the conclusions are presented in Section 4.

2. System description and theoretical analyses

Consider a single-degree-of-freedom nonlinear vibro-impact oscillator as a random parametric excitation, then we will have

$$\begin{cases} \ddot{y} + 2\beta\dot{y} + \Omega^2[1+\xi(t)]y + \alpha y^3 = 0, & y > -\Delta, \\ \dot{y}_+ = -e\dot{y}_-, & y = -\Delta, \end{cases} \tag{1}$$

where dot indicates the differentiation with respect to time t, β represents the damping coefficient, Ω represents the natural frequency of the degenerated system, α represents the intensity of the nonlinear term, Δ represents the distance from the system static equilibrium position to the single rigid barrier and $0 < e \leq 1$ is the restitution factor that is a known parameter of impact loss, subscripts "minus" and "plus" refer to the values of response velocity just before and after the instantaneous impacts. Thus $\dot{y}_+$ and $\dot{y}_-$ are actually rebound and impact velocities of the mass, respectively. They have the same magnitudes whenever $e = 1$. Therefore, this special case is the case of elastic impact, whereas in case $e < 1$, some impact losses are observed. $\xi(t)$ reflects a bounded noise process, which was introduced by Rong *et al.*[28] and expressed as

$$\xi(t) = h\cos(\Omega_1 t + \sigma W(t) + \theta), \tag{2}$$

where $h > 0$ and $\Omega > 0$ are the amplitude and the frequency of the random excitation respectively, $W(t)$ reflects a standard Wiener process and θ a uniformly distributed random number in a range of 0–2π. $\sigma\dot{W}(t)$ is the random disorder which describes random temporal deviation of the excitation frequency from its expectation or mean Ω_1. The process $\xi(t)$ has the following power spectral density:[29]

$$S(\omega) = \frac{\sigma^2(\omega^2 + \Omega_1^2 + \frac{1}{4}\sigma^2)h^2}{2[(\omega+\Omega_1)^2 + \frac{1}{4}\sigma^2][(\omega-\Omega_1)^2 + \frac{1}{4}\sigma^2]}. \tag{3}$$

This process will be assumed to be of narrow-band, which is clearly seen in the case where $\sigma \to 0$. We assume that $\sigma \ll \Omega_1$ in this paper.

Periodic-in-time excitation with a "random disorder", or random phase modulation appears in structural dynamics with traveling loadings and/or structures, having a certain imperfect spatial periodicity in certain problems of aeroelasticity.[30] Thus, a so-called slug flow of a two-phase fluid in a heat exchanger tube with alternating slugs of, say, steam and water may lead to timewise variation of the tube apparent mass, and therefore to the parametric excitation of the lateral vibration in the case of a straight tube.

When there is no impact, $\alpha = 0$ and $\sigma = 0$, system (1) reduces to a damped Mathieu equation. It is known that the resonance response will take place when the frequency of the harmonic excitation

equals doubled the natural frequency of the system, i.e., $\Omega_1 = 2\Omega$. Therefore the resonance response near the region $\Omega_1 \approx 2\Omega$ is also of most interest for the case where impact exists, $\alpha \neq 0$ and $\sigma \neq 0$.

Following Zhuravlev,[21] the non-smooth transformation of state variable is introduced as follows:

$$y = |x| - \Delta, \quad \dot{y} = \dot{x}\,\mathrm{sgn}\,x, \tag{4}$$

where $\mathrm{sgn}\,x$ is the signum function so that $\mathrm{sgn}\,x = 1$ for $x > 0$ and $\mathrm{sgn}\,x = -1$ for $x < 0$. Obviously, this transformation makes the transformed velocity $\dot{x}$ continuous at the impact instant (i.e., $x = 0$) in a special case of elastic impact (i.e., $e = 1$), thereby reducing the problem into one without velocity jump. However, this is not the case for a general vibro-impact system with impact loss. The jump of the transformed velocity $\dot{x}$ becomes proportional to $1 - e$ instead of $1 + e$ for the jump of original velocity $\dot{y}$. This jump may be included in the transformed differential equation of motion by using the Dirac delta function $\delta(x)$. Since $x(t_*) = 0$ at the impact instant t_* and $\Delta(t - t_*) = |\dot{x}|\,\delta(x)$, the impulse becomes

$$(\dot{x}_+ - \dot{x}_-)\delta(t - t_*) = (1 - e)\dot{x}\,|\dot{x}|\,\delta(x).$$

The transformed equation of motion can be written by substituting Eq. (4) into Eq. (1) as

$$\begin{aligned}\ddot{x} + \Omega^2 x = {} & -2\beta\dot{x} + \Delta\Omega^2\mathrm{sgn}\,x \\ & - (1 - e)\dot{x}\,|\dot{x}|\,\Delta(x) - \alpha(|x| - \Delta)^3\mathrm{sgn}\,x \\ & - h\Omega^2(|x| - \Delta)\mathrm{sgn}\,x\cos(\Omega_1 t + \sigma W(t) + \theta).\end{aligned} \tag{5}$$

Thus, the original impact system (1) is reduced into "common" vibration system (5) without impact. The term $(1-e)\dot{x}\,|\dot{x}|\,\delta(x)$ on the right-hand side of Eq. (5) describes the impact loss of the system, which can be regarded as an impulsive damping term. The transformed equation (5) permits rigorous analytical studies by the asymptotic method of averaging over the period, as long as coefficients α, β, Δ, h, σ and $1 - e$ are all small and proportional to a small parameter. Moreover, only the resonance response will be considered, i.e., frequency Ω_1 of the random excitation is near the resonance frequency 2Ω, $\Omega_1 \approx 2\Omega$. The detuning parameter μ is defined as $\mu = \Omega - \frac{1}{2}\Omega_1$. And μ is also assumed to be small and proportional to a small parameter. In order to use the method of random averaging,[30] the following transformation is used:

$$\begin{cases} x(t) = A(t)\sin\Phi(t), & \dot{x}(t) = \Omega A(t)\cos\Phi(t), \\ \Phi(t) = \frac{1}{2}\Omega_1 t + \varphi(t), & \psi(t) = \sigma W(t) + \theta. \end{cases} \tag{6}$$

Notice that equation (5) can be transformed to the following pair of first-order equations:

$$\begin{cases} \dot{A} = \dfrac{\cos\Phi}{\Omega}\Big[-2\beta\Omega A\cos\Phi + \Delta\Omega^2\mathrm{sgn}(\sin\Phi) - (1-e)\Omega^2 A\cos\Phi\,|A\cos\Phi|\,\Delta(A\sin\Phi) \\ \qquad -\alpha(|A\sin\Phi| - \Delta)^3\mathrm{sgn}(\sin\Phi) - h\Omega^2(|A\sin\Phi| - \Delta)\mathrm{sgn}(\sin\Phi)\sin(2\Phi - 2\varphi + \psi)\Big], \\ \dot{\varphi} = \mu - \dfrac{\sin\Phi}{\Omega A}\Big[-2\beta\Omega A\cos\Phi + \Delta\Omega^2\mathrm{sgn}(\sin\Phi) - (1-e)\Omega^2 A\cos\Phi\,|A\cos\Phi|\,\Delta(A\sin\Phi) \\ \qquad -\alpha(|A\sin\Phi| - \Delta)^3\mathrm{sgn}(\sin\Phi) - h\Omega^2(|A\sin\Phi| - \Delta)\mathrm{sgn}(\sin\Phi)\sin(2\Phi - 2\varphi + \psi)\Big], \\ \mathrm{d}\psi = \sigma\mathrm{d}W. \end{cases} \tag{7}$$

Under the foregoing assumption that damping, intensity of nonlinear term, impact losses and excitation terms are all small, the right-hand side of Eq. (7) is proportional to a small parameter, and the derivatives $\dot{A}$, $\dot{\varphi}$ and $\dot{\psi}$ are all small, A, φ and ψ all describe slowly varying random processes with time t. According to the definition of $\Phi(t)$, one has

$$\Phi(t) = \tfrac{1}{2}\Omega_1 t + \varphi(t), \quad \dot{\Phi} = \tfrac{1}{2}\Omega_1 + \dot{\varphi},$$

where $\varphi(t)$ is a slowly varying random function of time and the derivative $\dot{\varphi}$ is small, while $\dot{\Phi} \approx \Omega_1/2$ is not small and Φ describes a fast varying random process. By averaging over the fast state variable Φ, the following shortened equations can be obtained

$$\begin{cases} \dot{A} = -\Big(\beta + \dfrac{1-e}{\pi}\Omega\Big)A - \dfrac{h}{4}\sin(2\varphi - \psi)A, \\ \dot{\varphi} = \Big(\mu - \dfrac{3\alpha\Delta^2}{2\Omega}\Big) - \dfrac{h}{4}\cos(2\varphi - \psi) \\ \qquad -\dfrac{2\Delta}{\pi\Omega A}(\Omega^2 + \alpha\Delta^2) - \dfrac{\alpha}{\Omega}\Big(\dfrac{4}{\pi}\Delta A - \dfrac{3}{8}A^2\Big). \end{cases} \tag{8}$$

Equation (8) shows that the difference between elastic impact ($e = 1$) and inelastic impact ($e < 1$) is that

inelastic impact increases the damping of the system from β to $\beta+[(1-e)/\pi]\Omega$. Denote

$$\begin{cases} \bar{\beta}=\beta+\dfrac{1-e}{\pi}\Omega, \\ \bar{\mu}=\mu-\dfrac{3\alpha\Delta^2}{2\Omega}, \\ \rho=\dfrac{4\Delta}{\pi\Omega}(\Omega^2+\alpha\Delta^2), \\ \Theta(t)=2\varphi(t)-\psi(t), \end{cases} \tag{9}$$

then equation (8) can be transformed into the following Itô equations:

$$\begin{cases} \mathrm{d}A=\Big(-\bar{\beta}-\dfrac{h\sin\Theta}{4}\Big)A\mathrm{d}t, \\ \mathrm{d}\Theta=\Big(2\bar{\mu}-\dfrac{h}{2}\cos\Theta-\dfrac{\rho}{A}-\dfrac{8\alpha\Delta}{\pi\Omega}A+\dfrac{3\alpha}{4\Omega}A^2\Big)\mathrm{d}t \\ \qquad -\sigma\mathrm{d}W. \end{cases} \tag{10}$$

Consider first the steady-state response for the case of a perfect periodicity excitation without random disorder at $\sigma=0$. Equation (10) reduces to

$$\begin{cases} \dfrac{\mathrm{d}A}{\mathrm{d}t}=\Big(-\bar{\beta}-\dfrac{h\sin\Theta}{4}\Big)A, \\ \dfrac{\mathrm{d}\Theta}{\mathrm{d}t}=2\bar{\mu}-\dfrac{h}{2}\cos\Theta-\dfrac{\rho}{A}-\dfrac{8\alpha\Delta}{\pi\Omega}A+\dfrac{3\alpha}{4\Omega}A^2. \end{cases} \tag{11}$$

Let $A=A_0$, $\Theta=\Theta_0\mathrm{d}A(t)/\mathrm{d}t=0$ and $\mathrm{d}\Theta(t)/\mathrm{d}t=0$, then we will obtain the steady-state solutions of Eq. (11) as follows:

$$\begin{cases} -2\bar{\beta}=\dfrac{h\sin\Theta_0}{2}, \\ 2\bar{\mu}-\dfrac{\rho}{A_0}-\dfrac{8\alpha\Delta}{\pi\Omega}A_0+\dfrac{3\alpha}{4\Omega}A_0^2=\dfrac{h}{2}\cos\Theta_0. \end{cases} \tag{12}$$

Squaring and adding Eq. (12) yield the frequency-response equation

$$\Big[2\bar{\mu}-\frac{\rho}{A_0}-\frac{8\alpha\Delta}{\pi\Omega}A_0+\frac{3\alpha}{4\Omega}A_0^2\Big]^2=\frac{h^2}{4}-4\bar{\beta}^2. \tag{13}$$

Equation (13) can be solved numerically for given parameters of the system. Since the left-hand side of Eq. (13) is nonnegative, the necessary condition that equation (13) has real solutions is $h>4\bar{\beta}$, which means that system (11) will have non-trivial steady-state solution only when the amplitude of the random excitation h exceeds the threshold value $4\bar{\beta}$. Equation (13) has more than one steady-state solution in some parameter areas. The stability of these responses can be examined by introducing some perturbation terms as

$$A(t)=A_0+A_1(t),\ \Theta(t)=\Theta_0+\Theta_1(t), \tag{14}$$

where A_0 and Θ_0 are governed by Eqs. (12) and (13) and A_1 and Θ_1 are perturbation terms. Substituting Eq. (14) into Eq. (11) and neglecting the nonlinear terms, one obtains the following linearization of modulation equation (11) at A_0 and Θ_0:

$$\begin{cases} \dfrac{\mathrm{d}A_1}{\mathrm{d}t}=-\dfrac{h\cos\Theta_0}{4}\Theta_1, \\ \dfrac{\mathrm{d}\Theta_1}{\mathrm{d}t}=\Big(\dfrac{\rho}{A_0^2}-\dfrac{8\alpha\Delta}{\pi\Omega}+\dfrac{3\alpha}{2\Omega}A_0\Big)A_1+\dfrac{h\sin\Theta_0}{2}\Theta_1. \end{cases} \tag{15}$$

The eigenvalues of the coefficient matrix of system (15) are

$$\lambda_{1,2}=-\bar{\beta}\pm\sqrt{\bar{\beta}^2-\frac{h\cos\Theta_0}{4}\Big(\frac{\rho}{A_0^2}-\frac{8\alpha\Delta}{\pi\Omega}+\frac{3\alpha}{2\Omega}A_0\Big)}. \tag{16}$$

Therefore the necessary and sufficient condition of the stability of the steady-state solutions A_0 and Θ_0 is that the real parts of the eigenvalues $\lambda_{1,2}$ are all less than zero, i.e.,

$$\frac{h\cos\Theta_0}{4}\Big(\frac{\rho}{A_0^2}-\frac{8\alpha\Delta}{\pi\Omega}+\frac{3\alpha}{2\Omega}A_0\Big)>0. \tag{17}$$

Next, we determine the steady-state response of system (10) in the stochastic excitation case with random disorder at $\sigma\neq0$. Note that an exact analytical study of system (10) seems impossible due to the nonlinear nature. Thus, approximate solutions of the second-order moments of the resonance response are proposed. As the intensity of the random disorder σ is a small parameter, the solutions of Eq. (10) can be represented by Eq. (14). Substituting Eq. (14) into Eq. (10) and neglecting the nonlinear terms, one obtains the following linearization of the modulation equations (10) at A_0 and Θ_0:

$$\begin{cases} \mathrm{d}A_1=-\dfrac{h\cos\Theta_0}{4}\Theta_1\mathrm{d}t, \\ \mathrm{d}\Theta_1=\Big[\Big(\dfrac{\rho}{A_0^2}-\dfrac{8\alpha\Delta}{\pi\Omega}+\dfrac{3\alpha}{2\Omega}A_0\Big)A_1+\dfrac{h\sin\Theta_0}{2}\Theta_1\Big]\mathrm{d}t \\ \qquad -\sigma\mathrm{d}W. \end{cases} \tag{18}$$

Equation (18) is a linear Itô equation. According to Itô rule, the steady-state moments of A_1 and Θ_1 can be obtained by the moment method.[31] For steady-state moments, one has $\dfrac{\mathrm{d}\langle A_1\rangle}{\mathrm{d}t}=\dfrac{\mathrm{d}\langle\Theta_1\rangle}{\mathrm{d}t}=0$, where $\langle\cdot\rangle$ denotes the expectation value. Taking expectation on both sides of Eq. (18), one obtains

$$\langle A_1\rangle=\langle\Theta_1\rangle=0. \tag{19}$$

Although the procedure can be easily extended to predict response moments of any order, only second

moments $\langle A_1^2\rangle$, $\langle A_1\Theta_1\rangle$, $\langle\Theta_1^2\rangle$ will be considered here. From Eq. (18), one obtains

$$\begin{cases} \dfrac{\mathrm{d}\langle A_1^2\rangle}{\mathrm{d}t} = -\dfrac{h\cos\Theta_0}{2}\langle A_1\Theta_1\rangle, \\ \dfrac{\mathrm{d}\langle A_1\Theta_1\rangle}{\mathrm{d}t} = -\dfrac{h\cos\Theta_0}{4}\langle\Theta_1^2\rangle \\ \qquad\qquad + \Big(\dfrac{\rho}{A_0^2} - \dfrac{8\alpha\Delta}{\pi\Omega} + \dfrac{3\alpha}{2\Omega}A_0\Big)\langle A_1^2\rangle \\ \qquad\qquad + \dfrac{h\sin\Theta_0}{2}\langle A_1\Theta_1\rangle, \\ \dfrac{\mathrm{d}\langle\Theta_1^2\rangle}{\mathrm{d}t} = 2\Big(\dfrac{\rho}{A_0^2} - \dfrac{8\alpha\Delta}{\pi\Omega} + \dfrac{3\alpha}{2\Omega}A_0\Big)\langle A_1\Theta_1\rangle \\ \qquad\qquad + h\sin\Theta_0\langle\Theta_1^2\rangle + \sigma^2. \end{cases} \tag{20}$$

For steady-state moments, one has

$$\frac{\mathrm{d}\langle A_1^2\rangle}{\mathrm{d}t} = \frac{\mathrm{d}\langle A_1\Theta_1\rangle}{\mathrm{d}t} = \frac{\mathrm{d}\langle\Theta_1^2\rangle}{\mathrm{d}t} = 0. \tag{21}$$

Substituting Eq. (21) into Eq. (20), one obtains the following solutions of Eq. (18):

$$\begin{cases} \langle A_1\Theta_1\rangle = 0, \quad \langle\Theta_1^2\rangle = \dfrac{\sigma^2}{4\bar{\beta}}, \\ \langle A_1^2\rangle = \dfrac{2\bar{\mu} - \dfrac{\rho}{A_0} - \dfrac{8\alpha\Delta}{\pi\Omega}A_0 + \dfrac{3\alpha}{4\Omega}A_0^2}{\dfrac{\rho}{A_0^2} - \dfrac{8\alpha\Delta}{\pi\Omega} + \dfrac{3\beta}{2\Omega}A_0}\dfrac{\sigma^2}{8\bar{\beta}}. \end{cases} \tag{22}$$

From Eqs. (14), (19), and (22), one has

$$\begin{cases} \langle A\rangle = A_0, \\ \langle A^2\rangle = A_0^2 + \langle A_1^2\rangle \\ \qquad = A_0^2 + \dfrac{2\bar{\mu} - \dfrac{\rho}{A_0} - \dfrac{8\alpha\Delta}{\pi\Omega}A_0 + \dfrac{3\alpha}{4\Omega}A_0^2}{\dfrac{\rho}{A_0^2} - \dfrac{8\alpha\Delta}{\pi\Omega} + \dfrac{3\beta}{2\Omega}A_0}\dfrac{\sigma^2}{8\bar{\beta}}, \end{cases} \tag{23}$$

where $\langle A_1^2\rangle$ represents the influence of the random disorder on the steady-state response and will increase as the intensity of the random disorder σ increases from Eq. (23).

3. Numerical simulation

In this section, the analytical results will be shown and compared with the direct numerical results. All the direct numerical simulations obtained by using the Monte–Carlo method are based on the original system dominated by Eq. (1), which gives a powerful validation of analytical results. For the method of numerical simulation, readers can refer to references by Zhu[32] and Shinozuka.[33] The governing equation (1) is numerically integrated by the fourth order Runge–Kutta algorithm between impacts, which is valid until the first encounter with the barriers, that is, until the equality $y = \Delta$ is satisfied. The impact condition $\dot{y}_+ = -e\dot{y}_-$ is then imposed by using the numerical solution $\dot{y}_-$. This results in the rebound velocity $\dot{y}_+$, thereby providing the initial values for the next step numerical calculation. In fact, the numerical solution $y(t)$ will be approximately equal to Δ ($y(t) \approx \Delta$) and some interpolation techniques should be used to determine the exact impact time $t = t_*$ such that $y(t_*) = \Delta$. In the present paper, the linear interpolation method is used to determine the exact impact time $t = t_*$ and impact velocity $\dot{y}_- = \dot{y}(t_* - 0)$. In the following numerical simulation, the parameters in system (1) are chosen as $\Delta = 0.1$ and $\Omega = 1$. From Eq. (6), one has $A^2 = x^2 + \dot{x}^2/\Omega^2$. And the mean-square response amplitudes of A_0^2 and $\langle A^2\rangle$ are calculated by the time averaging for the response sample of $x^2 + \dot{x}^2/\Omega^2$ in numerical simulation.

We first consider the perfect periodic excitation in the case where $\sigma = 0$, $h = 2.0$, $e = 0.9$, $\beta = 0.1$, $\alpha = 0.02, 0.04$. The variations of the steady-state response A_0 with Ω_1 are shown in Fig. 1. For comparison, the theoretical results given by Eq. (13) are also shown in Fig. 1. Figure 1 shows that the deterministic response predicted by the averaging method is in good agreement with that obtained by numerical simulations. It can be seen from Fig. 1 that the response amplitude will increase as Ω_1 increases and will decrease strongly as α increases. So nonlinearity should

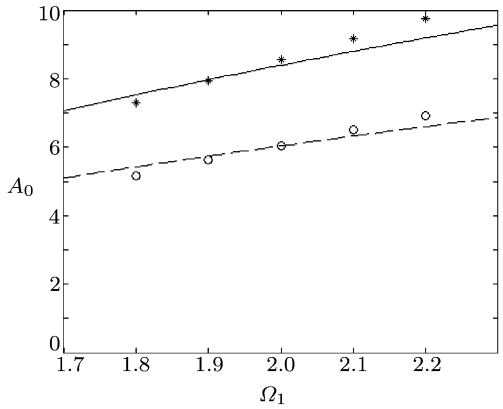


Fig. 1. Frequency response of system (1) for $\sigma = 0$, $h = 2.0$, $\beta = 0.1$, and $e = 0.9$. The solid line and dash line are the theoretical solutions for $\alpha = 0.02$ and 0.04 respectively, while the symbols $*$ and $\circ$ represent the numerical solutions for $\alpha = 0.02$ and 0.04 respectively.

be considered in the analysis of impact system. Compared with the accuracy of the numerical solution, the accuracy of the analytical solution is reduced a little in the case of large detuning. This may be partly because of some inaccuracies of the random averaging method at large detuning.

Next, we determine the effect of the random disorder on primary response in the case where $\sigma \neq 0$. The variations of the steady-state response $\langle A^2 \rangle$ with Ω_1 in the case $h = 2.0$, $e = 0.9$, $\beta = 0.1$, $\sigma = 0.1$, $\alpha = 0.02, 0.04$ are shown in Fig. 2. For comparison, the theoretical results given by Eq. (23) are also shown in Fig. 2. Similarly, the high accuracy of the analytical method can also be claimed for the case under non-perfect periodic excitation as can be seen from Fig. 2. Once again, the response amplitude will increase as Ω_1 increases and will decrease strongly as α increases. The accuracy of the analytical solution is reduced a little in the case of large detuning.

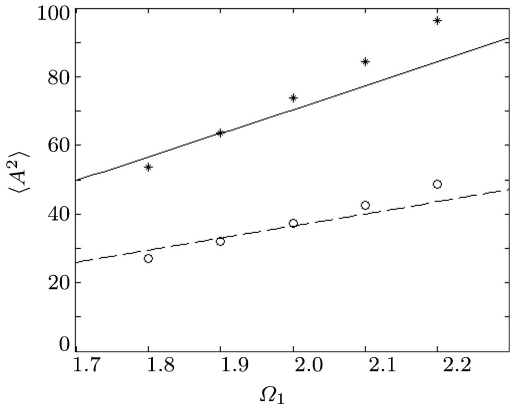


Fig. 2. Frequency response of system (1) for $h = 2.0$, $\beta = 0.1$, $\sigma = 0.1$, and $e = 0.9$. The solid and dash lines are the theoretical solutions for $\alpha = 0.02$ and 0.04 respectively, while the symbols $*$ and $\circ$ represent the numerical solutions for $\alpha = 0.02$ and 0.04 respectively.

Now we consider the effects of β and h on the mean-square amplitude $\langle A^2 \rangle$ of the system. Variations of the steady-state response $\langle A^2 \rangle$ with h are shown in Fig. 3 for the case where $e = 0.9$, $\Omega_1 = 2.1$, and $\beta = 0.1, 0.2$. For comparison, the theoretical results given by Eq. (23) are also shown in Fig. 3. It can be seen from Fig. 3 that the response amplitude will decrease as damping β increases, which is in accordance with the physical intuition. A new and interesting phenomenon can be observed from Fig. 3. $\langle A^2 \rangle$ will keep zero when h increases from zero and will jump to a large nonzero value when h exceeds some threshold value. $\langle A^2 \rangle$ will jump from zero to 18.48 when h exceeds the threshold value 0.53 in the case $\beta = 0.1$, and will jump to 19.32 when h exceeds the threshold value 0.93 in the case $\beta = 0.2$. One may call such a phenomenon stochastic jump, which leads to an interesting phenomenon of the random impact system.

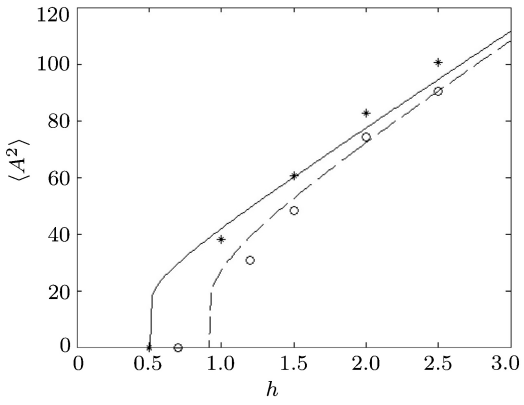


Fig. 3. Frequency response of system (1) for $\alpha = 0.02$, $\sigma = 0.1$, $e = 0.9$, and $\Omega_1 = 2.1$. The solid and dash lines are the theoretical solutions for $\beta = 0.1$ and 0.2 respectively, while the symbols $*$ and $\circ$ represent the numerical solutions for $\beta = 0.1$ and 0.2 respectively.

Next we consider the effect of e on the mean-square amplitude $\langle A^2 \rangle$ of the system. Variations of the steady-state response $\langle A^2 \rangle$ with e are shown in Fig. 4 for the case where $\alpha = 0.02$, $\Omega_1 = 2.0$, $\beta = 0.1$, $h = 2.0$, and $\sigma = 0.1$. For comparison, the theoretical results given by Eq. (23) are also shown in Fig. 4. It can be seen from Fig. 4 that the response amplitude will increase as e increases. Some impact losses are observed in the case $e < 1$, which is in accordance with the foregoing theoretical analysis.

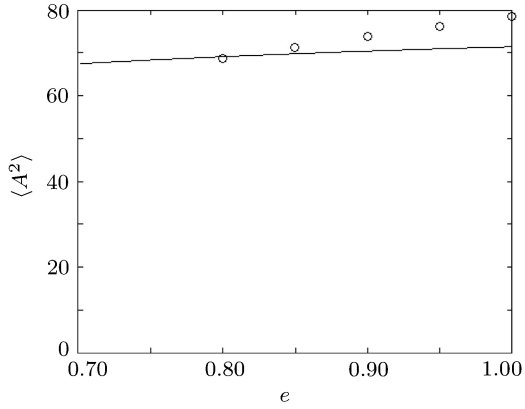


Fig. 4. Frequency response of system (1) for $\alpha = 0.02$, $\Omega_1 = 2.0$, $\beta = 0.1$, $h = 2.0$, and $\sigma = 0.1$. The solid line is the theoretical solution and the circle is the numerical solution.

Finally, we consider the effect of σ on the response of the system. The phase trajectories of system (1) are shown in Fig. 5 in the case where $\alpha = 0.02$, $\Omega_1 = 2.0$, $\beta = 0.1$, $h = 3.0$, and $e = 0.9$ for different $\sigma = 0.0,\ 0.05,\ 0.1,\ 30.0$, where $z(t) = \dot{y}(t)$ denotes the velocity of the mass. The response of system (1) shown in Fig. 5(a) is a periodic one while the phase trajectory is a limit cycle in the case $\sigma = 0.0$. The phase trajectories shown in Figs. 5(b) and 5(c) show that the responses are similar to periodic solutions and have some pervasions in the case $\sigma = 0.05$ and 0.1, which may be called quasi-periodic solutions. The pervasion of the phase trajectory will strengthen as σ increases. Such a pervasion will even destroy the topological property of the phase trajectory as shown in Fig. 5(d) in the case $\sigma = 30$. In fact, the steady-state response shown in Fig. 5(d) is a trivial one. Once again, the phenomenon of stochastic jump can be observed from Fig. 5, that is, the steady-state response of system will jump from a large non-trivial solution to a trivial one as σ increases and exceeds the threshold value $\sigma = 26.08$. The frequency region of the random excitation $\xi(t)$ will become wide and depart from the resonance frequency $\Omega_1 = 2\Omega = 2$ when σ increases from Eq. (3), whence the nonlinear effect will become weak.

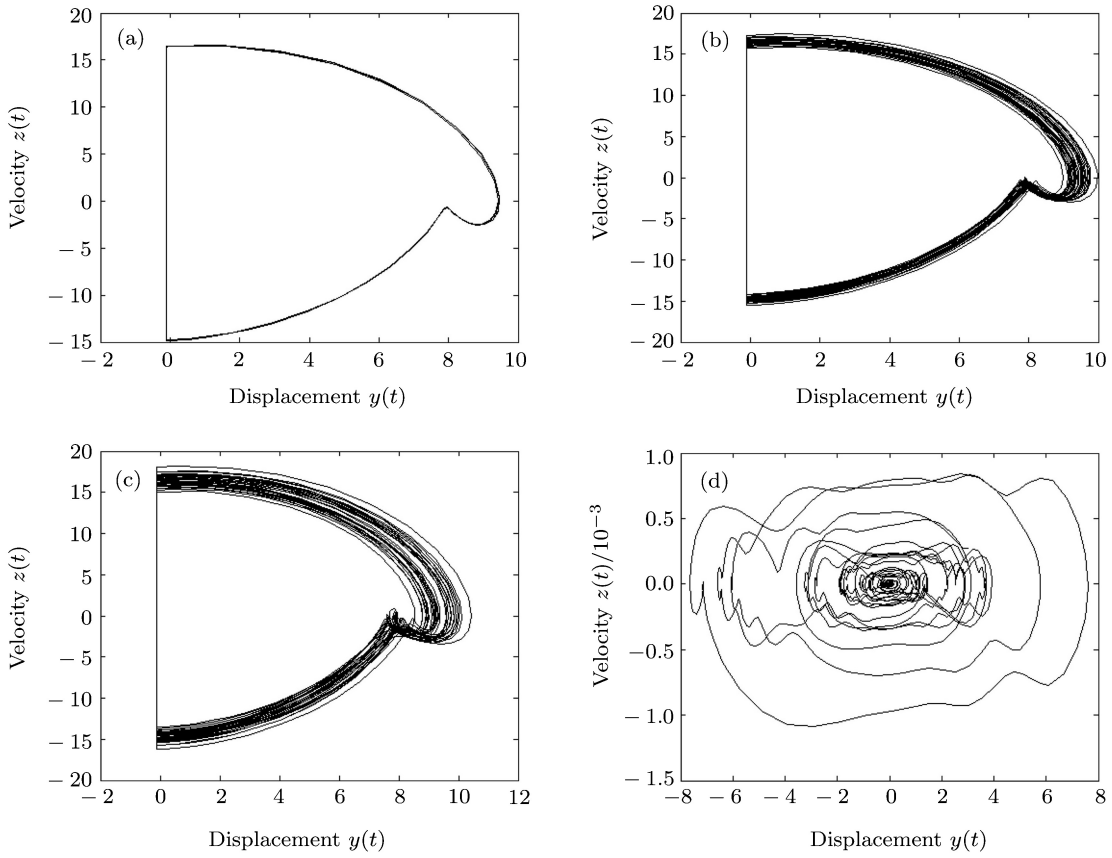


Fig. 5. Phase plots of system (1) for $\alpha = 0.02$, $\Omega_1 = 2.0$, $\beta = 0.1$, $h = 2.0$, and $e = 0.9$: (a) $\sigma = 0.0$, (b) $\sigma = 0.05$, (c) $\sigma = 0.10$, and (d) $\sigma = 30.0$.

4. Conclusions and discussion

In this paper, the resonance response of a nonlinear impact system under a special kind of narrow-band random parametric excitation—bounded noise, is discussed. The steps of the approach presented in the paper may be summarized as follows. First, use the method of Zhuravlev transformation to convert the original impact system to a common vibration system without impact, which can be reduced into shortened equations by averaging the fast state variables. The averaged equations are solved exactly and an algebraic equation of the amplitude of the response is obtained in the case of the perfect periodic excitation without random disorder. The methods of linearization and moment are used to obtain the formula of the

mean-square amplitude approximately for the case of random noise excitation with random disorder. The obtained analytical results are compared with Monte–Carlo simulation results, showing that the analytical solution has a high accuracy. From the analysis may be drawn a basic conclusion that the peak response amplitude will become small at large damping or large nonlinear intensity, but will become large at large amplitude or frequency of the random excitation.

A phenomenon of stochastic jump is observed, i.e., the steady response of the system will jump from a trivial solution to a large non-trivial one when the amplitude of the random excitation increases from zero and exceeds some threshold value, or will jump from a large non-trivial solution to a trivial one when the intensity of the random disorder of the random excitation increases and exceeds some threshold value. This is an interesting phenomenon of random impact system. Further analysis of the stochastic jump from theory and physical nature will be a continuation of this research.

The bounded noise model in Eq. (2) may be less accurate in some applications. Therefore other models such as the filtered Gaussian white noise will be more suited for the basic narrow-band excitation, which will also be a continuation of this research.

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