Tunneling of Bose–Einstein condensate and interference effect in a harmonic trap with a Gaussian energy barrier*

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The tunneling effect of Bose–Einstein condensate (BEC) in a harmonic trap with a Gaussian energy barrier is studied in this paper. The initial condensate evolves into two separate moving condensates after the tunneling time under certain conditions. The interference pattern between the two moving condensates is given as a comparison and as a further demonstration of the existence of the global phase.

Keywords: Bose–Einstein condensate, tunneling effect, interference effect

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Bose–Einstein condensation (BEC) has aroused varying interest since its realization and there have been many studies on the physical properties of BEC systems.[1−7] Among them, considerable investigations have arisen both experimentally and theoretically on the phase of condensates and the interference pattern.[8−13] Recently, interference of an array of independent condensates was studied and illustrative examples of the interference fringes of the phase-correlated and the phase uncorrelated condensates were compared.[14] Phase defects were revealed by the observation of topologically different patterns and dislocation lines were found in the interference pattern.[15] The interference of initially independent condensates was studied and it was shown that there will exist interference term when interaction was included.[16] Interference of a condensate between the halves of a double-well trap was studied.[17,18]

Since the condensate can be well described by the macroscopic wavefunction, the study of its feature is of much importance and the investigation of interference phenomena is naturally suited for this purpose. There are many approaches suggested theoretically to detect the interference effect. For example, it can be tested by means of different hyperfine atomic states in a single trap,[10] or be tested by the propagation of two condensates released from two wells,[11] and by condensates released from the wells.[12,13]

In Ref. [7] we have studied the tunneling effect of a condensate on a barrier by initiating the condensate and removing the harmonic trapping potential. It is shown that two moving condensates can be obtained after the tunneling time. However, without the trapping potential the two moving condensates could hardly overlap and it is not convenient to study the interference effect. In this paper, the harmonic trapping potential is retained throughout the tunneling process. We propose an approach to study the interference phenomena by means of two moving condensates obtained by the tunneling of an initial condensate on a Gaussian energy barrier, which is ramped up in the middle of the harmonic trap after the condensate has been started. When the two desired moving condensates are obtained, the trapping potential and the barrier are removed and the interference pattern is studied. The tunneling effect and the interference pattern are studied closely. It shows that the interference between the reflected wave and the incident wave becomes stronger with the increase of the energy barrier, and the tunneling effect can nearly be suppressed when the barrier is sufficiently high. The interference pattern between the two moving condensates is obvious, which gives a further demonstration of the existence of the global phase and the intensive interference will also make it easier for experimental observation.

In the laboratory, when the lateral motion of the condensate is confined or negligible, the condensate is essentially 1D and can be described by the 1D time-dependent Gross–Pitaevskii (GP) equation[11,12]

$$\left[-\frac{\partial^2}{\partial\xi^2}+\frac{\xi^2}{4}+\alpha\left|\Phi(\xi,\tau)\right|^2\right]\Phi\left(\xi,\tau\right)=\mathrm{i}\frac{\partial\Phi\left(\xi,\tau\right)}{\partial\tau}. \tag{1}$$

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It is rescaled by $\xi = x/a$, $\tau = \omega t$ and $\Phi(\xi,\tau) = \psi(x,t)\sqrt{a}$. Here ω is the oscillation frequency of the trap, $a \equiv \sqrt{\hbar/2m\omega}$ is the size of the harmonic trapping potential and m is the mass of a single atom. This GP equation has a form similar to the nonlinear Schrödinger equation and the nonlinear coefficient $\alpha = 8\pi N a_{\rm s}/a$ denotes the strength of the interatomic interaction, with $a_{\rm s}$ denoting the s-wave scattering length of the atom and N denoting the number of atoms in the condensate. The wavefunction of the condensate is normalized to unity. We consider the case of $\alpha > 0$ which implies the interatomic interaction is repulsive. Our computation is performed for ^{133}Cs with $m = 133$ amu (amu is atomic mass unit), $a_{\rm s} = 3.45$ nm, $\omega = 2\pi \times 10$ Hz,[9] and a small condensate of $N = 1000$ atoms is considered. Equation (1) is solved by symplectic method[19,20] and the fourth-order symplectic RK method is used. Zero boundary condition is adopted at sufficient large distance. By using the symplectic method, all iterations conserve normalization of the wave function and the method preserves the symplectic structure of the Hamiltonian formulation. The wavefunction and the corresponding eigenvalue are calculated. The chemical potential is $6.56(\hbar\omega)$.

First we start the moving of the condensate by displacing it from the centre of the harmonic trapping potential in the ξ direction by a distance of $\Delta\xi = -15a$ at time $\tau = 0$.[21] The result is shown in Fig. 1. Since the condensate is away from the centre of the trap, it performs periodic oscillation about the minimum of the harmonic trap with an amplitude 15 $a = 29$ μm. The period is $T = 6.284(\omega^{-1})$. When $\alpha > 0$, the interaction between atoms is repulsive, then this period is larger than 2π. The force on the condensate is $F = -m\omega^2\xi$, the condensate gains the maximum momentum at $\xi = 0$ which can be seen from Fig. 1.

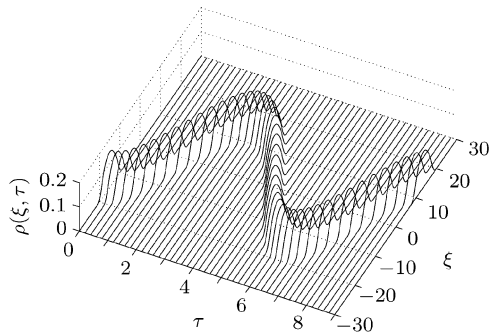


Fig. 1. Evolution of the probability density of the condensate with time. When the condensate is formed at time $\tau = 0$, displace it from the centre of the trap in the ξ direction by a distance of $\Delta\xi = -15a$; length in unit of a, time in unit of ω^{-1} and density in unit of a^{-1}.

If we split the harmonic trap by erecting a barrier in the middle, it is expected that tunneling effect will happen between the condensate and the barrier.

Secondly we investigate the case of a Gaussian potential barrier that is ramped up rapidly just after the condensate has been displaced at $\tau = 0$ and simulate the dynamical evolution process. The form of the barrier is[22]

$$V(\xi) = b\,{\rm e}^{-c\xi^2}. \tag{2}$$

We take $c = 1(a^{-2})$, which controls the width of the barrier and is not disturbed by the initial condensate at time $\tau = 0$. Here $b(\hbar\omega)$ controls the height of the barrier that can be modulated by tuning the laser placed at the bottom of the harmonic potential. The barrier splits the harmonic potential into two wells and it is expected that the initial condensate will behave differently in the tunneling process with the increase of the height of the barrier. We study the evolution of the probability density with the increase of the parameter b and the result is presented in Fig. 2. In Fig. 2(a) $b = 30$, the Gaussian barrier is low compared with the kinetic energy of the condensate, therefore the latter just performs oscillation in the harmonic potential with a little deformation in the barrier region. In Fig. 2(b) $b = 40$, a large fraction of the condensate tunnels through the barrier. We can also see the interference between the reflected wave function and the incident wave function and the interference dies down with time and a little condensate forms in the left well. Due to the confinement of the magnetic harmonic trap, the two condensates in the left well and right well evolve independently until they meet again in the barrier region. Because of the interaction between the two condensates and between the condensate and the barrier, this time the interference is intensive. In Fig. 2(c) $b = 50$ and in Fig. 2(d) $b = 60$, these cases are alike. Part of the initial condensate tunnels through the barrier and the interference between the reflected wave and the incident wave is stronger than that of Fig. 2(b). After this tunneling process there are two condensates of similar size that evolve in the two wells. In Fig. 2(e) $b = 70$, a small fraction of the initial condensate tunnels through the barrier and the rest is reflected back. In Fig. 2(f) $b = 80$, almost all the condensate is reflected back and its motion is confined in the left well.

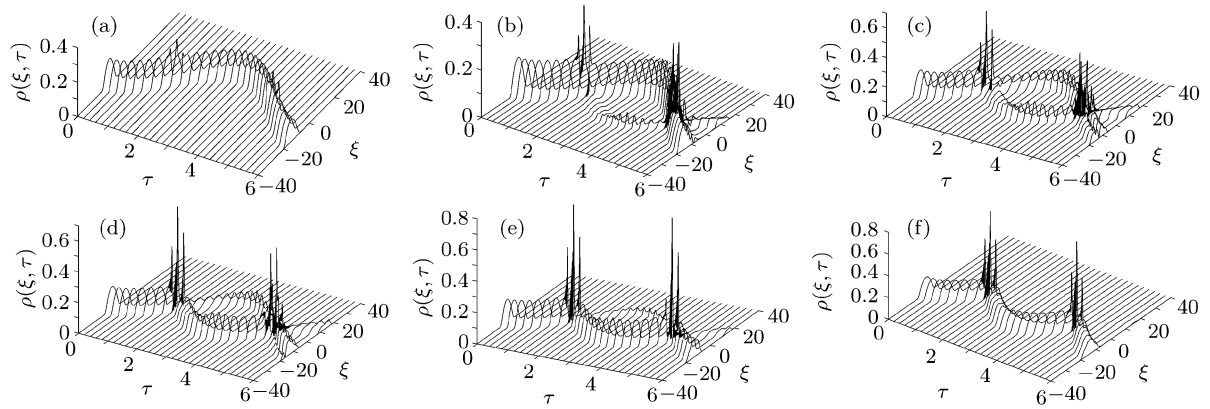


Fig. 2. Evolution of the probability density of the condensate with time. When the condensate is formed at time $\tau = 0$, it is displaced from the centre of the trap in ξ direction by a distance of $\Delta\xi = -15a$ and subsequently a Gaussian energy barrier $V(\xi) = b\,\mathrm{e}^{-\xi^2}$ centred at $\xi = 0$ is rapidly erected. The parameter b in Figs. 2(a)–2(f) is taken as: (a) $b = 30$, (b) $b = 40$, (c) $b = 50$, (d) $b = 60$, (e) $b = 70$, and (f) $b = 80$. Length is given in unit of a, time in unit of ω^{-1}, and density in unit of a^{-1}.

From Fig. 2 we can draw the conclusion that the interference between the reflected wave and the incident wave becomes stronger with the increase of the height of the barrier. Due to the combination of the harmonic potential and the barrier, the interference fringes are more intensive than those without trapping potential in Refs. [7] and [22] and are more suitable for detection. Tunneling effect can be nearly suppressed when the energy barrier is increased to a certain degree. Noticing that there is a moving condensate in each well after the tunneling process in Fig. 2(c), we developed a scheme to study the global phase of the two condensates by switching off the traps and the barrier at a proper time, then the two condensates are expected to expand ballistically and move along the axial direction at the same time and interference fringes can be observed when they overlap. The probability density is assumed to be governed by Eq. (1).

Finally we investigate the interference effect by switching off the harmonic trap and the Gaussian energy barrier; $\Delta\xi = -15a$, $b = 50$ is considered. However, the case of two condensates moving away from each other is not expected because they will not overlap after being released, so we need to switch off the traps at a time $T/2 < \tau < 3T/4$ and we choose (a) $\tau = 3.5$, (b) $\tau = 4$. The probability density at different instants during evolution is shown in Fig. 3. In Fig. 3(a), the two condensates released at $\tau = 3.5$ can be seen clearly. The number of atoms contained in each condensate is not equal, then the interference fringe is not symmetric about $\xi = 0$, however it does not affect the essence of the interference phenomenon. At $\tau = 30$, the fringe spacing is 28 μm. In Fig. 3(b), the condensates gain more momentum along the ξ axis when being released at time $\tau = 4$. They almost overlap fully at $\tau = 5$ and the interference is much stronger and thicker than that of Fig. 3(a).

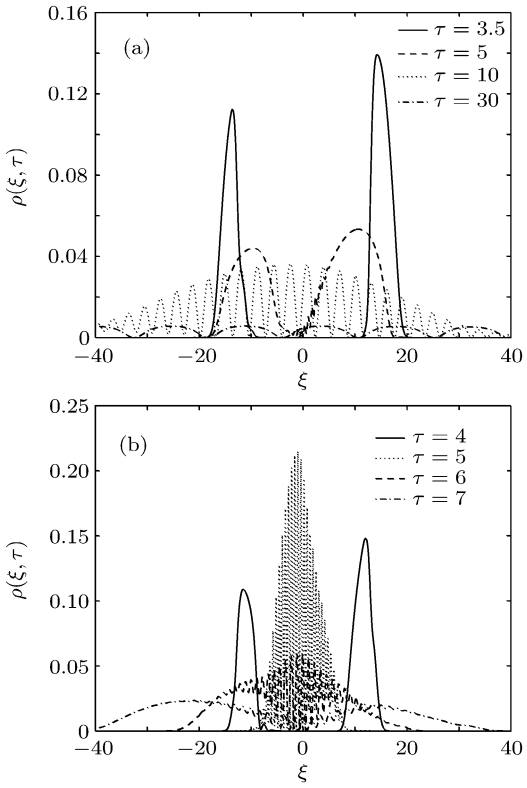


Fig. 3. Probability density of the condensate at different time during evolution, $\Delta\xi = -15a$, $b = 50$, $c = 1$. In Fig. 3(a) $\tau = 3.5$ and 3(b) $\tau = 4$, the trapping potential and barrier are suddenly removed and the condensates expand freely in the ξ direction. Length is given in unit of a, time in unit of ω^{-1}, density in unit of a^{-1}.

They move apart from each other, which can be identified at $\tau = 6$ and this phenomenon is more evident at $\tau = 7$. In Fig. 3 we can see the interference fringes clearly and thus give a further demonstration of the global phase even though the two condensates are obtained in tunneling process. Another feature is that

the interference effect is much stronger if the two condensates have larger momentum and this will facilitate detection due to better visibility of the fringes.

In conclusion, we have developed a scheme to study interference. Two moving condensates have been obtained in the tunneling process with a harmonic potential and a Gaussian energy barrier. By shutting off the combined potential the two condensates expand ballistically and intense interference fringes can be observed as a further demonstration of the existence of the global phase. The momentum of the particles in the condensate has a great impact on the intensity of the interference effect.

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