Composite-fermionization of the mixture composed of Tonks gas and Fermi gas*

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This paper investigates the ground-state properties of the mixture composed of the strongly interacting Tonks–Girardeau gas and spin polarized Fermi gas confined in one-dimensional harmonic traps, where the interaction between the Bose atoms and Fermi atoms is tunable. With a generalized Bose–Fermi transformation the mixture is mapped into a two-component Fermi gas. The homogeneous Fermi gas is exactly solvable by the Bethe-ansatz method and the ground state energy density can be obtained. Combining the ground-state energy function of the homogeneous system with local density approximation it obtains the ground-state density distributions of inhomogeneous mixture. It is shown that with the increase in boson–fermion interaction, the system exhibits composite-fermionization crossover.

Keywords: one-dimensional, Tonks–Fermi mixture, composite-fermionization

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1. Introduction

With the rapid experimental progress of the manipulation of quantum gases, we can realize the low-dimensional gas with strong anisotropic magnetic trap or two-dimensional optical lattice.[1−3] Theoretically this system corresponds to an effective one-dimensional (1D) model. In addition multi-components of atomic gas has also been investigated extensively and it turns out that they display rich exotic phenomena such as phase separation, novel quantum states and phase transitions, etc.,[4−15] which have been observed in the experiments.[16−22] It is the competition among the inter-species and intra-species interactions which induce richer phase structure in comparison with the single-component systems. Both the effective 1D intra- and inter-component interactions can be tuned via the magnetic Feshbach resonance or confinement induced resonance.[19,20]

For the strong quantum fluctuation in 1D the mean-field theory generally does not work well when the interaction between atoms becomes strong and we must turn to the non-perturbation methods. For example, Bethe ansatz,[23−28] Bosonization method,[7] exact diagonalization,[29−31] Bose–Fermi mapping (BFM) method,[32] multi-configuration Hartree theory[33] have exerted to investigate both the single component Bose gases and multi-component of quantum gases. In order to exactly study the large-size system we have developed density functional theory based on Bethe ansatz and applied it to two-components of Bose mixture and Bose–Fermi mixture.[34,35] The investigation shows that with the increase in repulsion strength the total density distribution continuously evolves from Gaussian-like distribution of bosons to Fermi-like and when the inter-component interaction is strong enough Bose–Fermi mixture displays phase separation.[12−15,21,22,34,35] It turns out that this theory can describe the binary Bose mixture very well in the full interacting regime from the weakly interacting regime to the strong interacting Tonks–Girardeau (TG) regime.

In this paper, we use the density-functional theory based on the Bethe ansatz to investigate the ground-state properties of the mixture composed of TG gases[36,37] and polarized Fermi gases trapped in harmonic potential with tunable inters-species interaction ranging from weak interaction to strong interaction. The present work will focus on the case that the Bose atoms and Fermi atoms are isotopes of the same species of atoms, where its corresponding model can be mapped into the Yang–Gaudin model with generalized Bose–Fermi transformation and the ground-state energy function can be obtained

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from the Bethe-ansatz solution of the Yang–Gaudin model.[38,39] Combining the ground state energy function with local density approximation (LDA),[23,40−44] we can obtain a set of coupled nonlinear Schrödinger equations (NLSEs) governing the system. By numerically solving NLSEs the ground state properties will be obtained. In this paper we shall investigate both the total density distributions and those of each component for different boson fraction. We also obtain the exact density profiles with Bose–Fermi mapping method in the limit of no inter-component interaction and in the limit of strong inter-component interaction. It turns out that the results from both methods match well.

The present paper is organized as follows. Section 2 will give the model for Tonks–Fermi mixture and the method. Section 3 is devoted to the density profiles of the mixture with tunable inter-species repulsion. A summary is given in the last section.

2. Model and method

We consider a 1D Bose–Fermi mixture composed of $N_{\rm b}$ hard core Bose atoms and $N_{\rm f}$ polarized Fermi atoms confined in the external potential $V_{\rm ext}(x)$, where the Bose atoms and Fermi atoms are isotopes of the same species of atoms such that the atomic mass $m_{\rm b} \approx m_{\rm f} = m$. The many-body Hamiltonian in second quantized form should take the form as

$$\begin{aligned}\mathscr{H} = & \int {\rm d}x \sum_{i=b,f} \left\{ \hat{\Psi}_i^\dagger(x) \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_{\rm ext}(x) \right] \hat{\Psi}_i(x) \right. \\ & \left. + \frac{g_{\rm bb}}{2} \hat{\Psi}_{\rm b}^\dagger(x) \hat{\Psi}_{\rm b}^\dagger(x) \hat{\Psi}_{\rm b}(x) \hat{\Psi}_{\rm b}(x) \right\} \\ & + g_{\rm bf} \int {\rm d}x \hat{\Psi}_{\rm b}^\dagger(x) \hat{\Psi}_{\rm f}^\dagger(x) \hat{\Psi}_{\rm f}(x) \hat{\Psi}_{\rm b}(x). \end{aligned} \tag{1}$$

Here the effective 1D boson–boson and boson–fermion interaction constants $g_{\rm bb}$ and $g_{\rm bf}$ can be controlled experimentally by tuning the corresponding scattering lengths[19,20] and transverse confinement. The field operators $\Psi_{\rm b(f)}^\dagger(x)$, $\Psi_{\rm b(f)}(x)$ create and annihilate a Bose (Fermi) atom at position x respectively. For the Tonks components of hard core interaction the boson–boson interaction will be infinitely strong ($g_{\rm bb} = \infty$). In this situation the original Hamiltonian (1) will be formulated as

$$\begin{aligned}\mathscr{H}^{\rm TGF} = & \int {\rm d}x \sum_{i={\rm b,f}} \left\{ \hat{\Psi}_i^\dagger(x) \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_{\rm ext}(x) \right] \hat{\Psi}_i(x) \right. \\ & \left. + g \hat{\Psi}_{\rm b}^\dagger(x) \hat{\Psi}_{\rm f}^\dagger(x) \hat{\Psi}_{\rm f}(x) \hat{\Psi}_{\rm b}(x) \right\}, \end{aligned} \tag{2}$$

with the hard-core constraints $\hat{\Psi}_{\rm b}^\dagger(x)\hat{\Psi}_{\rm b}^\dagger(x) = \hat{\Psi}_{\rm b}(x)\hat{\Psi}_{\rm b}(x) = 0$ and $\left\{ \hat{\Psi}_{\rm b}(x), \hat{\Psi}_{\rm b}^\dagger(x) \right\} = 1$. For the simplicity of expression the original inter-component interaction $g_{\rm bf}$ is replaced with g.

For the mixture of TG gases and spin-polarized Fermi gases, it is convenient to introduce the generalized Bose–Fermi transformations[45,46]

$$\begin{cases} \hat{\Psi}_{\rm b}(x) = \exp\left[{\rm i}\,\pi \int_{-\infty}^{x} \Psi_\uparrow^{F\dagger}(z)\,\Psi_\uparrow^{\rm F}(z)\,(z)\,{\rm d}z \right] \Psi_\uparrow^{\rm F}(x), \\ \hat{\Psi}_{\rm f}(x) = \exp\left[{\rm i}\,\pi \int_{-\infty}^{\infty} \Psi_\uparrow^{F\dagger}(z)\,\Psi_\uparrow^{\rm F}(z)\,(z)\,{\rm d}z \right] \Psi_\downarrow^{\rm F}(x), \end{cases} \tag{3}$$

where the field operators $\Psi_i^{\rm F\dagger}(x)$ and $\Psi_i^{\rm F}(x)$ create and annihilate an i-component ($i = \uparrow, \downarrow$) fermion at location x respectively. With the above transformations, the mixture of TG gases and Fermi gases (2) is mapped into two-component fermionic gas and the corresponding Hamiltonian is formulated as

$$\begin{aligned}\mathscr{H}^{\rm F} = & \int {\rm d}x \sum_{i=\uparrow,\downarrow} \left\{ \hat{\Psi}_i^{\rm F\dagger} \left[-\frac{\hbar^2}{2m_i}\frac{\partial^2}{\partial x^2} + V_{\rm ext}(x) \right] \hat{\Psi}_i^{\rm F} \right. \\ & \left. + g \hat{\Psi}_\uparrow^{\rm F\dagger}(x) \hat{\Psi}_\downarrow^{\rm F\dagger}(x) \hat{\Psi}_\downarrow^{\rm F}(x) \hat{\Psi}_\uparrow^{\rm F}(x) \right\}. \end{aligned} \tag{4}$$

This Fermi system has the exact same energy spectrum as the original mixture because they relate to each other by a unitary transformation (3). For the same reason the ground-state density distribution for the mixture of TG gases and Fermi gases can be obtained by consultation for the corresponding fermionic model. As $V_{\rm ext}(x) = 0$ the above Hamiltonian (4) is the exact solvable Yang–Gaudin model,[38,39] which can be diagonalized by Bethe ansatz method. The corresponding first quantized Hamiltonian is

$$H = \sum_{j=1}^{N} \left[-\frac{\partial^2}{\partial x_j^2} + V_{\rm ext}(x_j) \right] + 2c \sum_{j<l} \delta(x_j - x_l), \tag{5}$$

where $c = mg/\hbar^2$ and $N = N_{\rm b} + N_{\rm f} = N_\uparrow + N_\downarrow$ is the total particle number.

The ground state energy and wave function can be evaluated with quasi-momentum $\{k_j\}$ determined by the Bethe-ansatz equations

$$\begin{cases} k_j L = 2\pi I_j - 2 \sum_{\alpha=1}^{N_{\rm b}} \tan^{-1} \dfrac{k_j - \Lambda_\alpha}{c/2}, \\ \sum_{j=1}^{N} \tan^{-1} \dfrac{\Lambda_\alpha - k_j}{c/2} = \pi J_\alpha + \sum_{\beta=1}^{N_{\rm b}} \tan^{-1} \dfrac{\Lambda_\alpha - \Lambda_\beta}{c}, \end{cases} \tag{6}$$

with the quantum number I_j and J_α. Here $\{\Lambda_\alpha\}$ is rapidity and L is the length of system. In the thermodynamic limit ($N, L \to \infty$ but N/L is finite) equation

(6) reduces to two coupled integral equations

$$\begin{cases} \rho(k) = \dfrac{1}{\pi}\displaystyle\int_{-B}^{B} \mathrm{d}\Lambda \frac{c'\sigma(\Lambda)}{c'^2 + (k-\Lambda)^2} + \frac{1}{2\pi}, \\ \sigma(\Lambda) = -\dfrac{1}{\pi}\displaystyle\int_{-B}^{B} \mathrm{d}\Lambda' \frac{c\sigma(\Lambda')}{c^2 + (\Lambda-\Lambda')^2} \\ \qquad\quad + \dfrac{1}{\pi}\displaystyle\int_{-Q}^{Q} \mathrm{d}k \frac{c'\rho(k)}{c'^2 + (\Lambda-k)^2} \end{cases} \tag{7}$$

with $c' = c/2$, where Q and B are determined by the total particle density $\rho = N/L = \int_{-Q}^{Q} \rho(k)\mathrm{d}k$ and boson density $\rho\alpha = N_\mathrm{b}/L = \int_{-B}^{B} \sigma(\lambda)\mathrm{d}\lambda$ with α being the boson fraction N_b/N. By solving the integral equations we can evaluate the energy density of ground state by

$$\varepsilon_\mathrm{GS} = \frac{L}{N}\frac{\hbar^2}{2m}\int_{-Q}^{Q} k^2 \rho(k)\,\mathrm{d}k.$$

By numerically solving coupled integral equations, the ground state energy density can be obtained and parameterized as

$$\varepsilon_\mathrm{GS}(\rho,\alpha) = \frac{\pi^2\hbar^2\rho^2}{8m}\left[\left(1/3 + (2\alpha-1)^2\right) + f(\chi,\alpha)\right], \tag{8}$$

with

$$\begin{aligned} f(\chi,\alpha) = {} & [e(\chi) - 1/3]\{1 + \alpha(\chi)(2\alpha-1)^2 \\ & + \beta(\chi)(2\alpha-1)^4 \\ & - [1 + \alpha(\chi) + \beta(\chi)](2\alpha-1)^6\}, \end{aligned}$$

where $\chi = 2\gamma/\pi$ and $e(\chi)$, $\alpha(\chi)$ and $\beta(\chi)$ are given in Refs. [34] and [43].

In order to investigate the inhomogeneous system we use the density functional theory based on the above exact energy density evaluated by Bethe ansatz. With the LDA the energy functional of the inhomogeneous system in the external trap can be formulated as

$$\mathscr{E}(\rho) = \int \mathrm{d}x \left[\frac{\hbar^2}{2m}\left|\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{\rho}\right|^2 + V_\mathrm{ext}(x)\rho + \rho\epsilon(\rho)\right], \tag{9}$$

where $\epsilon(\rho)$ is local energy at point x with density $\rho(x) = \rho_\mathrm{b}(x) + \rho_\mathrm{f}(x)$ in the external trap. The ground state is determined by minimizing the energy functional Eq. (9) and $\sqrt{\rho}_i = \Phi_i$ satisfies the following coupled NLSEs:

$$\begin{cases} \left(-\dfrac{\hbar^2}{2m}\dfrac{\mathrm{d}^2}{\mathrm{d}x^2} + V_\mathrm{ext}\right)\Phi_\mathrm{b} + \tilde{F}_\mathrm{b}(\rho)\Phi_\mathrm{b} = \mu_\mathrm{b}\Phi_\mathrm{b}, \\ \left(-\dfrac{\hbar^2}{2m}\dfrac{\mathrm{d}^2}{\mathrm{d}x^2} + V_\mathrm{ext}\right)\Phi_\mathrm{f} + \tilde{F}_\mathrm{f}(\rho)\Phi_\mathrm{f} = \mu_\mathrm{f}\Phi_\mathrm{f}, \end{cases} \tag{10}$$

with the normalization condition $\int \mathrm{d}x|\Phi_i(x)|^2 = N_i$, where $\tilde{F}_i(\rho,\alpha) = \partial\rho\varepsilon_\mathrm{GS}/\partial\rho_i$ and the Lagrange multiplier $\mu_{\mathrm{b(f)}}$ is introduced to conserve $N_{\mathrm{b(f)}}$. By numerically solving coupled NLSEs, we can obtain the ground-state density distributions. In the present work we shall consider the TG–Fermi mixture trapped in a weak harmonic trap $V_\mathrm{ext}(x) = \frac{1}{2}m\omega^2x^2$.

3. Composite ferminization

In Fig. 1 we display the total ground state density distributions from the weakly Bose–Fermi interacting regime to the strongly Bose–Fermi interacting regime for different boson fractions. Here the length x is in unit of $l = \sqrt{\hbar/m\omega}$ and $U = g/l\hbar\omega$ denotes the dimensionless interacting constant. In the case of $U = 0$, TG gas does not interact with Fermi atom so the ferminized TG gas displays the same distribution as spinless Fermi atoms, which can be obtained exactly by the BFM method, while the distribution of polarized Fermi atoms is easy to evaluate. The total distribution is plotted in Fig. 1 (short dotted lines). It is shown that the TG–Fermi mixture with weak Bose–Fermi interaction can be approximatively described by the mixture without inter-component interaction. The weak interaction will not sharply change the density distribution of ground state of TG–Fermi mixture. For the balance case (Fig. 1(a)), i.e., there are equal Bose atom number and Fermi atom number in the harmonic trap (here $N_\mathrm{b} = N_\mathrm{f} = 20$), the total density profile exhibits the similar structure of single peak in the full interacting regime. In the weak inter-components interacting regime atoms will prefer to distribute in the centre of the trap. While with the increase in interaction more and more atoms are repelled to the location far away from the centre such that the half-width of the density profile becomes wider. In the strong inter-component interacting regime the total density profiles of TG–Fermi mixture match well with polarized fermi gases or single component TG gases with total atom number $N = 40$ (short dash-dot lines). This means that the ferminized TG gases ferminize again with the increase in the Bose–Fermi interaction, i.e., composite-ferminization happens.[47] For the imbalance case (Fig. 1(b)), i.e., the Bose atom number is not equal to the Fermi atom number (here $N_\mathrm{b} = 35$ and $N_\mathrm{f} = 5$), the total density distribution behaves in different way in different interacting regime. For the weak Bose–Fermi interaction the distribution displays tongue-shape profiles, in which there are two saddles

at $x \sim \pm 3$. Comparing with the balance case, we find that there are more atoms far away from the centre of the trap for the strong Bose–Bose interaction in the TG gases. But when the inter-component interaction becomes strong, the density profiles for imbalance case are same with those for balance case. The mixture also exhibits the behaviour of composite-ferminization.

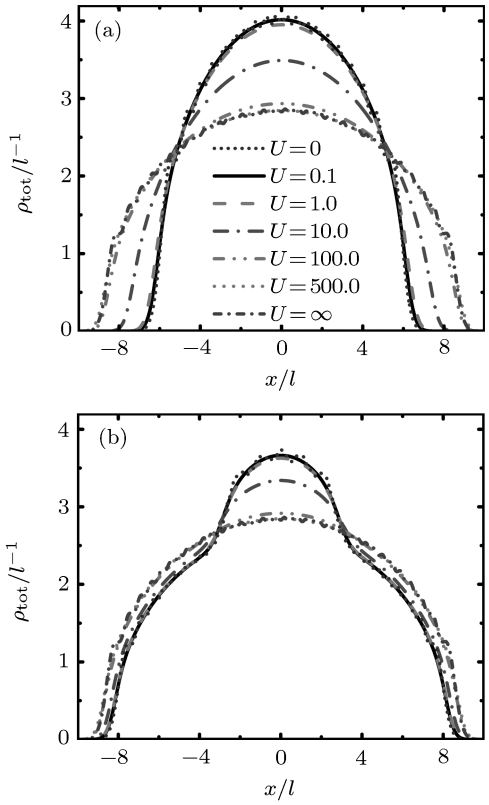


Fig. 1. Total ground state density distribution for the mixture composed of TG gases and Fermi gases with total atom number $N_{\rm total} = 40$: (a) $N_1 = N_2 = 20$; (b) $N_1 = 35$, $N_2 = 5$. Unit of $\rho_1(x)$: $1/l$; Unit of x: l.

The ground state density distributions for each component are displayed in Fig. 2 (balance case) and Fig. 3 (imbalance case) for the total atom number $N = 40$. It is shown that for the equal mixture of TG gases and polarized Fermi gases ($N_{\rm b} = N_{\rm f} = 20$) the Bose atoms exhibit exactly the same density profiles of those of the Fermi atoms in the full interacting regime. This again proves that the equivalence of properties is independent of the statistics between the strongly interacting Bose gases and the free Fermi gases. With the increase in Bose–Fermi interaction both Bose atoms and Fermi atoms are repelled far away from the centre of the trap. We also display the exact density profiles for the case of $U = 0$ (short dashed lines) and $U = \infty$ (short dash-dot lines) obtained by the Bose–Fermi method. In the former case the TG gases of $N_{\rm b} = 20$ show the same density distribution with Fermi gases of $N_{\rm f} = 20$. While in the latter case both TG gases and Fermi gases ($N_{\rm b} = N_{\rm f} = 20$) show the same density distribution as those of the polarized Fermi gases of $N = 40$, i.e., $\rho_{\rm b}(x) = \rho_{\rm f}(x) = \frac{1}{2}\rho_{\rm PF}^{N=40}(x)$. It is shown that the results obtained by BFM match well with the numerical results.

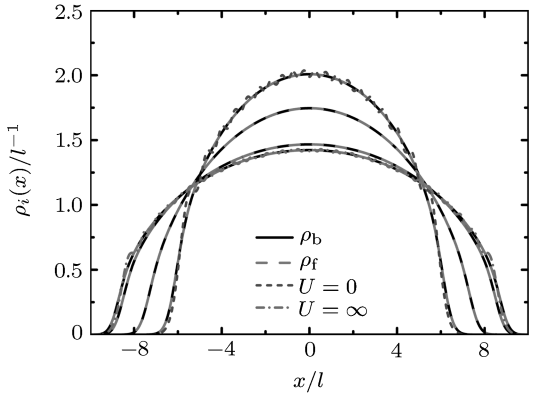


Fig. 2. Ground state density distribution for the mixture composed of TG gases and Fermi gases with $N_1 = N_2 = 20$ for different Bose–Fermi interactions. From the top to the bottom (solid lines and dashed lines): $U = 0.1$, 10, 100, 500. Short dash lines: $U = 0$;. Short dash-dot lines: $U = \infty$. Unit of $\rho_i(x)$: $1/l$. Unit of x: l.

In Fig. 3 we show the ground state density profiles of each component for the imbalance case. Here we study the system with different boson fractions but there are the same total atom numbers in the harmonic trap ($N = 40$ and $N_{\rm b} > N_{\rm f}$). In this situation Bose atoms display different density profiles from Fermi atoms. The Fermi gases always behave as a single peak located at the centre of harmonic trap and with the increase in interaction they distribute in a wider regime. However, the TG gases exhibit rich structure with the increase in inter-component interaction and boson fraction. Similar to the balance case, in the weakly interacting regime, TG gases show ferminized density distribution. As the inter-component interaction is strong enough the Bose atoms populate as the maximum probability at the position away from the trap centre, which is more obvious for the larger boson fraction. But when the Bose–Fermi interaction approaches infinity both TG gases and Fermi gases display the same structure of single peak as spinless Fermi gases of $N = N_{\rm b} + N_{\rm f}$ ($\rho_{\rm b}(x) = (N_{\rm b}/N)\rho_{\rm PF}^{N}(x)$ and $\rho_{\rm f}(x) = (N_{\rm f}/N)\rho_{\rm PF}^{N}(x)$). This means that composite-ferminization happens. We also obtain the exact profiles for the case of $U = 0$ and $U = \infty$ with BFM method. It turns out that they match well with the numerical result for the weak interaction and for the much strong interaction, respectively. According to the symmetry of Hamiltonian Eq. (4), as $N_{\rm b} < N_{\rm f}$ the TG gases will exhibit the same behaviour as the above Fermi gases and the Fermi gases will exhibit the behaviour as the above Bose atoms.

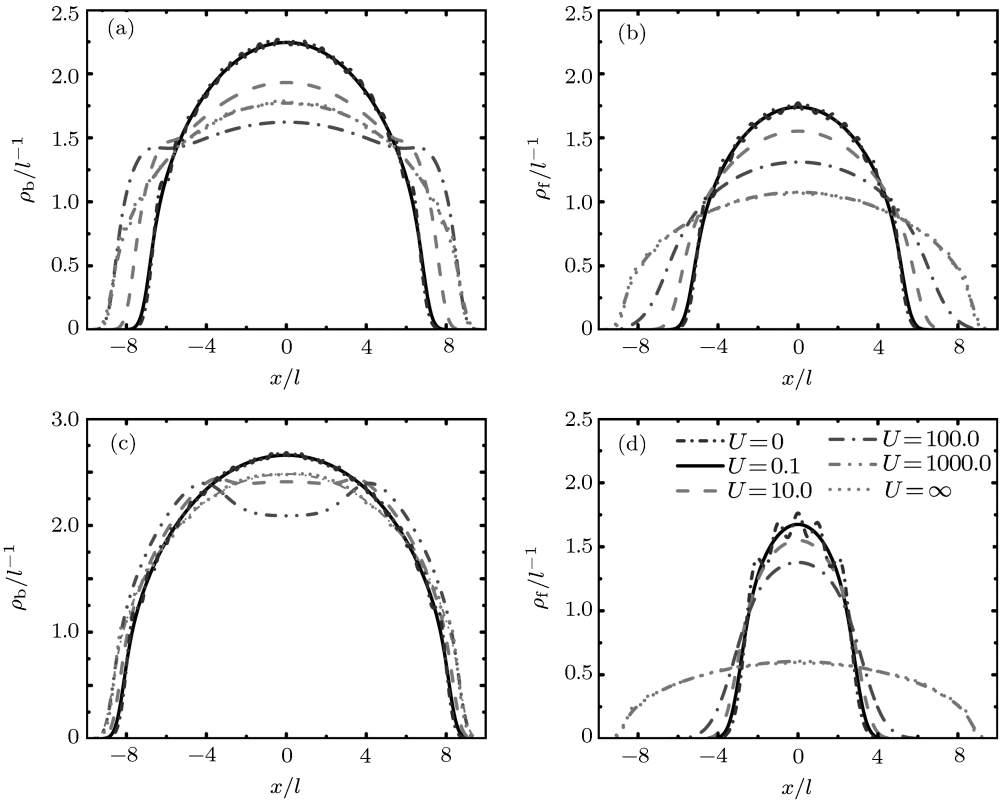


Fig. 3. Ground state density distributions for the mixture composed of TG gases and Fermi gases with total atom number $N_{\text{total}} = 40$: (a) and (b) $N_{\text{b}} = 25$, $N_{\text{f}} = 15$; (c) and (d) $N_{\text{b}} = 35$, $N_{\text{f}} = 5$. Unit of $\rho_i(x)$: $1/l$. Unit of x: l.

4. Summary

In summary, we have investigated the ground-state density distribution of TG–Fermi mixture trapped in 1D harmonic traps by combining the LDA and the exact ground state energy density of homogeneous system obtained by the Bethe-ansatz method. It is shown that the weak Bose–Fermi interaction will not sharply change the density profiles and with the increase in interaction the profiles of ferminized TG gases and Fermi gases ferminized again. In the strongly interacting limit their distributions exhibit the same profiles as those of polarized Fermi gases or TG gases with the total atom number $N = N_{\text{b}} + N_{\text{f}}$, i.e., $\rho_{\text{b}}(x) = (N_{\text{b}}/N)\rho_{\text{PF}}^{N}(x)$ and $\rho_{\text{f}}(x) = (N_{\text{f}}/N)\rho_{\text{PF}}^{N}(x)$. In both noninteracting limit and infinite strong interacting limit we have obtained the exact density profiles with BFM method. All of them match well with the numerical results. While in the middle interacting regime only the numerical method is available. It is shown that with the change of boson fraction each component behaves in different ways. For the balance case TG gases display the same density profiles as Fermi gases in the full interacting regime. But for the imbalance case each component exhibits a different behaviour. The components with less atom number always display the structure of single peak located at the centre of trap, while the components with more atom number will populate as the maximum probability at the position away from the centre of trap.

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