Rapid Communication

Magnetic quantum oscillations in a monolayer graphene under a perpendicular magnetic field*

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The de Haas–van Alphen (dHvA) oscillations of electronic magnetization in a monolayer graphene with structure-induced spin–orbit interaction (SOI) are studied. The results show that the dHvA oscillating centre in this system deviates from the well known (zero) value in a conventional two-dimensional electron gas. The inclusion of SOI will change the well-defined sawtooth pattern of magnetic quantum oscillations and result in a beating pattern. In addition, the SOI effects on Hall conductance and magnetic susceptibility are also discussed.

Keywords: graphene, spin–orbit interaction, magnetic quantum oscillations

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Magnetic oscillation, which was first predicted by Landau in 1930,[1] has been focus of research in condensed matter physics. One important reason is that the de Haas–van Alphen (dHvA) oscillations of magnetization provide a vigorous technique to study the properties of carriers around the Fermi surface. In the last decade, thanks to tremendous advances in microscopic semiconductor technology, the challenges encountered in measuring weak magnetization signals have been largely overcome. Magnetic dHvA oscillations have thus been extensively studied[2−11] in two-dimensional semiconductor systems, in which the magnetization oscillation mainly comes from the response of the electron k-quadratic kinetic energy to an external magnetic field.

Recently, graphene, as a perfect two-dimensional electron gas (2DEG) system, has been extensively studied both experimentally and theoretically.[12−14] Unlike the conventional semiconductor 2DEG, a peculiar characteristic of graphene is the presence of two atoms per unit cell, which results in a variety of unique properties. One intriguing fact is that graphene is described at low energies by a 2D massless Dirac wave equation with an additional pseudospin degree of freedom. Because of the Dirac spectrum and the pseudospin-induced fermionic chirality, the magnetic response properties in graphene sheets are naturally expected to display behaviour that should be absent in conventional semiconductor 2DEG systems. In addition, the Rashba-type spin–orbit interaction (SOI), which is as large as tens of meV in graphene,[15] is also expected to differ fundamentally from that in the conventional semiconductor 2DEG and plays a key role in determining the magnetic oscillation patterns in graphene. This is mainly due to the fact that unlike semiconductor heterostructures in which the SOI perturbs only Landau levels (LLs) in a minor way, both the kinetic term and the Rashba SOI term in graphene are k-linear near the two inequivalent Dirac points. Thus, the occurrence of SOI in graphene is expected to greatly change the LLs and the consequent magnetic responses.

Motivated by this observation, in the present paper we study the dHvA oscillations of the magnetization in a monolayer graphene with SOI in a perpendicular magnetic field. We find that the magnetization oscillation in the present system differs from that in traditional 2DEG by the calculated result that the dHvA oscillating centre deviates from the well known (zero) value. This deviation has an intimate relation

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with the Landau spectrum structure in the considered system. We also find that the SOI effect mixes the spin-up state and the spin-down state of neighbouring LLs and leads to a striking change in both the centre and the amplitude of the sawtooth-like magnetization oscillation.

The Hamiltonian of graphene with a modified Rashba SOI near the $\boldsymbol{K}$ point of the Brillouin zone is given by[16]

$$H_{\boldsymbol{K}} = \gamma\,(\boldsymbol{\sigma}\cdot\boldsymbol{k})\otimes\boldsymbol{I}_{2\times 2} + \frac{1}{2}\lambda\,(\boldsymbol{\sigma}\otimes\boldsymbol{s})\cdot\hat{z} = \begin{pmatrix} \boldsymbol{0} & h(\boldsymbol{k}) \\ h^{\dagger}(\boldsymbol{k}) & \boldsymbol{0} \end{pmatrix}, \tag{1}$$

with

$$h(\boldsymbol{k}) = \begin{pmatrix} \gamma\pi & -\mathrm{i}\,\lambda \\ \mathrm{i}\,\lambda & \gamma\pi \end{pmatrix},$$

where $\boldsymbol{\sigma}$ is the Pauli matrix for pseudospin describing the two sublattices in graphene, $\boldsymbol{s}$ is the Pauli matrix for electron spins, $\pi = k_x - \mathrm{i}\,k_y$, and $\gamma \equiv \hbar v_F = 639$ meV·nm (namely, the Fermi velocity $v_F = 10^6$ m/s). Following Ref. [15], in the present paper we choose the SOI parameter to be $\lambda = 13$ meV. The Hamiltonian near the $\boldsymbol{K'}$ point is the conjugate form of Eq. (1), $H_{\boldsymbol{K}'} = H^{\dagger}_{\boldsymbol{K}}$.

In the presence of a perpendicular magnetic field $\boldsymbol{B} = B\hat{z}$, the wave vector is transformed as $\boldsymbol{k}\rightarrow\boldsymbol{\Pi} = \boldsymbol{k} + e\boldsymbol{A}/\hbar c$ with $\boldsymbol{A} = (-By, 0, 0)$, which results in the commutation relation $\left[\pi, \pi^{\dagger}\right] = 2eB\hbar/c$. By introducing the harmonic oscillator operators a and $a^{\dagger}$ through the relation $\pi = \left(\sqrt{2}/l_B\right)a$, where $l_B = \sqrt{\hbar c/eB}$ is the magnetic length, the matrix $h(\boldsymbol{k})$ in Hamiltonian (1) is rewritten as

$$h(\boldsymbol{k}) = \begin{pmatrix} \dfrac{\sqrt{2}\gamma}{l_B}a & -\mathrm{i}\,\lambda \\ \mathrm{i}\,\lambda & \dfrac{\sqrt{2}\gamma}{l_B}a \end{pmatrix}. \tag{2}$$

In the case of $\lambda = 0$, the LLs are given by

$$\epsilon_n^{\mathrm{e/h}} = \pm\sqrt{2n}\gamma/l_B = \pm v_F\sqrt{2ne\hbar B/c}, \quad (n = 0, 1, 2, \ldots), \tag{3}$$

where the positive branch (ϵ_n^{e}) is for electrons and the negative branch (ϵ_n^{h}) is for holes. When the Rashba SOI is included, the LLs ($\epsilon_n^{\mathrm{e/h}}$) are then generally spin-split into the two branches $\epsilon_{n,+}^{\mathrm{e/h}}$ and $\epsilon_{n,-}^{\mathrm{e/h}}$ with their expressions given by

$$\begin{aligned} \epsilon_{n,s}^{\mathrm{e/h}} &= \pm\lambda\sqrt{\frac{1}{2}\left[1 + \Lambda_n + s\sqrt{1 + 2\Lambda_n + 4\Gamma^4}\right]},\ n \geq 2 \\ \epsilon_{1,s}^{\mathrm{e/h}} &= \begin{cases} \pm\lambda\sqrt{1 + 2\Gamma^2}, \\ 0, \end{cases} \\ \epsilon_{0,s}^{\mathrm{e/h}} &= 0, \end{aligned} \tag{4}$$

where $\Lambda_n = 2(2n-1)\Gamma^2$ and $\Gamma = \gamma/l_B\,|\lambda|$. Without loss of generality, in this paper we consider only the case of n-doping, for which only the electron branches of LLs are involved. Figure 1 shows the electron LLs each as a function of magnetic field.

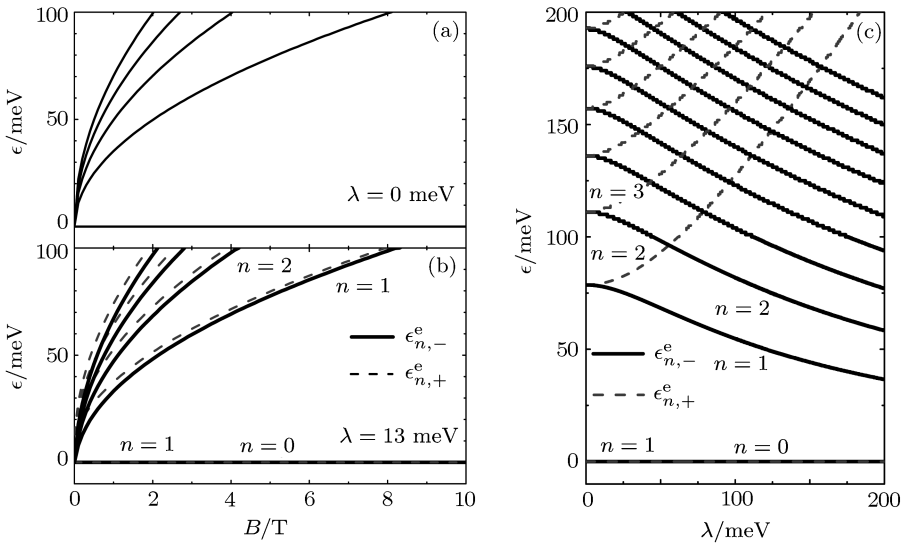


Fig. 1. Graphene Landau levels (a) without spin–orbital coupling, and (b) with spin–orbital coupling constant $\lambda = 13$ meV as functions of the external magnetic field B. And (c) the Landau levels as functions of the spin–orbital coupling parameter λ, with $B = 4.0$ T.

It is clear that for a monolayer graphene without SOI (see Eq. (3)), the energies of the electron LLs are proportional to the square root of the external magnetic field, which is shown in Fig. 1(a). However, when the SOI is introduced, it will mix the spin-up state and the spin-down state of neighbouring LLs (see Fig. 1(b)). Figure 1(c) plots the first ten electron LLs for a monolayer graphene as functions of the Rashba SOI parameter λ. With the SOI strength increasing, the spin degeneracy of the LLs is lifted and several crossings occur. It is important to note that the two *zero* modes, $\epsilon_{0,s}=0$ and $\epsilon_{1,s}^{(0)}=0$, which result in an additional fourfold degeneracy of the $\epsilon=0$ state, remain unaffected by changing the SOI strength. In the absence of Zeeman splitting, the Dirac–Landau energy spectra (3) and (4) are massless with a whole electron–hole symmetry and only half of these zero modes are occupied by electrons in the case of n-doping.

Now with the knowledge of LLs, we study the magnetic quantum oscillations in n-doped graphene. The electron magnetization density is the derivative of the Helmholtz free energy density with respect to B at fixed electron density $\mathcal{N}$ and temperature T, $M=-(\partial F/\partial B)|_{\mathcal{N},T}$. The Helmholtz free energy density of n-doped graphene is given by

$$F(B,T)=\mu\mathcal{N}-\frac{N_\nu}{\beta}\sum_{n=0,s}^{\infty}\ln[1+\mathrm{e}^{\beta(\mu-\epsilon_{n,s}^{\mathrm{e}})}], \quad (5)$$

where $\beta=1/k_BT$, $N_\nu=2/2\pi l_B^2$ is the degeneracy for a non-zero LL (namely, the number of electrons per unit area on an LL) with the factor 2 coming from the Dirac corner-degeneracy, n and s are respectively the orbital index and the spin index, and μ is the electron chemical potential. It should be noted that although the *zero* mode has fourfold degeneracy, the system has electron–hole symmetry and half of the particles in the zero mode are electrons. In this case, therefore, the electron filling factor for the zero mode is $N_0{=}\frac{1}{2}{\times}4/2\pi l_B^2=N_\nu$. The B-dependent chemical potential μ in Eq. (5) is related to the experimentally accessible electron density $\mathcal{N}$, which is given by

$$\mathcal{N}=N_\nu\sum_{n=0,s}^{\infty}f\left(\epsilon_{n,s}^{\mathrm{e}}\right) \quad (6)$$

with $f\left(\epsilon_{n,s}^{\mathrm{e}}\right)=1/[\mathrm{e}^{\beta\left(\mu-\epsilon_{n,s}^{\mathrm{e}}\right)}+1]$ being the Fermi–Dirac distribution function. From Eq. (5) the electron magnetization density becomes

$$\begin{aligned}M&=-\sum_{n=0,s}^{\infty}N_\nu f\left(\epsilon_{n,s}^{\mathrm{e}}\right)\frac{\partial\epsilon_{n,s}^{\mathrm{e}}}{\partial B}\\&\quad+\frac{2e}{h}\sum_{n=0,s}^{\infty}\frac{1}{\beta}\ln[1+\mathrm{e}^{\beta(\mu-\epsilon_{n,s}^{\mathrm{e}})}]\\&\equiv M^{(0)}+M^{(1)}. \quad (7)\end{aligned}$$

Here the first part $M^{(0)}$ on the right-hand side of Eq. (7) is the conventional contribution from the B dependence of the LLs and thus denotes a diamagnetic response; the second part $M^{(1)}$ comes from the B dependence of the level degeneracy factor N_ν, thus describing the effect of the variation in the density of states on magnetic field and denoting a paramagnetic contribution to the total magnetization. Obviously, $M^{(0)}$ is negative while $M^{(1)}$ is positive, thus the net result is an oscillation of the total magnetization M as a function of B.

We plot in Figs. 2(a) and 2(b) the sharp sawtooth-like oscillating behaviour of the chemical potential μ and the dHvA oscillating characteristic of magnetization M per electron, respectively, each as a function of the external magnetic field B without SOI at low temperature. The corresponding numerical results with SOI parameter $\lambda=13$ meV are shown in Figs. 2(c) and 2(d). It is clear that the magnetization in the presence of SOI displays a beating pattern compared with that without SOI since the SOI mixes the spin-up and spin-down states of neighbouring LLs. The weaker the magnetic field, the larger the change by SOI in the dHvA oscillations.

We find that there is a prominent difference in dHvA oscillating pattern between the monolayer graphene and the conventional semiconductor 2DEG. For the latter it is well known that the oscillating centre of the magnetization M remains exactly at zero. However, in the present monolayer graphene system the oscillating centre picks up a positive value, which implies the preference of a paramagnetic response. This deviation arises from the different LL spectrum structures in the two different systems considered. The LL in the conventional semiconductor 2DEG obeys a $B(n+\frac{1}{2})$ rule while that in the monolayer graphene system approximately obeys a $\sqrt{nB}$ rule. In particular, the diamagnetic term of the magnetization in the conventional semiconductor 2DEG turns to be $M^{(0)}{=}-\frac{e}{hc}\sum_{n=0,s}^{\mathrm{occu.}}\epsilon_{ns}^{\mathrm{e}}$ (at zero temperature) while the paramagnetic term is $M^{(1)}{=}\frac{e}{hc}\sum_{n=0,s}^{\mathrm{occu.}}(\mu_0-\epsilon_{ns}^{\mathrm{e}})$. Here μ_0 is the zero-temperature Fermi energy. The negative $M^{(0)}$ and the positive $M^{(1)}$ give that the net result is an oscillation of the total magnetization M as a function of B. The oscillating amplitude increases with B and the oscillating centre keeps at

zero. In addition, although the paramagnetic term in the monolayer graphene system also turns to be $M^{(1)}=\frac{2e}{hc}\sum_{n=0,s}^{\text{occu.}}(\mu_0-\epsilon_{ns}^{\text{e}})$ at zero temperature (The factor 2 comes from the Dirac corner-degeneracy), the diamagnetic contribution is nevertheless given by $M^{(0)}=-\frac{2e}{hc}\sum_{n=0,s}^{\text{occu.}}\frac{\epsilon_{ns}^{\text{e}}}{2}$. One can find that the diamagnetic contribution of $M^{(0)}$ in the monolayer garphene is half reduced compared with that in the conventional semiconductor 2DEG. The oscillating centre of the total magnetization now has a positive value. Although this simple comparison is not strict in mathematics, it affords an intuitive explanation of the difference in magnetization between the monolayer graphene and the conventional semiconductor 2DEG.

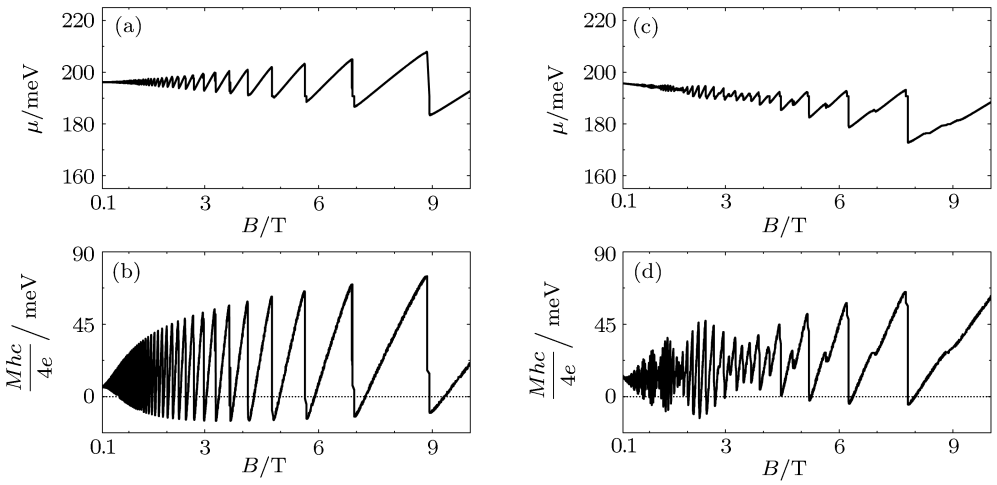


Fig. 2. Magnetic field dependences of chemical potential μ and magnetization M in an n-doped graphene system with electron density $\mathcal{N}=3.0\times10^{-2}$ nm^{-2}, and the temperature $T=4.0$ K. The left ((a), (b)) and right ((c), (d)) panels correspond to the cases of $\lambda=0$ and $\lambda=13$ meV, respectively.

Now we consider the magnetization properties in the case where the chemical potential is field independent. Figure 3(a) plots the magnetization in the monolayer graphene system as a function of the zero-temperature chemical potential (i.e., the Fermi energy) at the external magnetic field $B=2.0$ T. From Fig. 3(a) one can find that the dHvA oscillating patterns with and without SOI are prominently different (see the shaded regions) due to the fact that the inclusion of SOI will lift the spin-degenerate LLs and mix the LLs with different values of the integer n (see Fig. 1). We also calculate $\partial M/\partial\mu_0$, from which the Hall conductance σ_{H} is obtained by combining the thermodynamic relationship with Streda formula

$$\frac{\partial M}{\partial\mu_0}=\frac{\sigma_{\text{H}}}{e}. \tag{8}$$

The numerical result of the Hall conductance as a function of Fermi energy is plotted in Fig. 3(b), from which the Hall plateaus are clearly obtained. One can see that when the SOI is excluded (i.e., $\lambda=0$), due to the spin- and K-degeneracies of LLs the Hall conductance is quantized as $\frac{4e^2}{h}(n+1/2)$ as shown in Fig. 3(b) by the thick black lines. When the SOI is included, however, the platforms of the Hall conductance σ_{H} split and turn to be quantized as $\frac{2e^2}{h}(n+1)$ as shown in Fig. 3(b) by the thin gray lines.

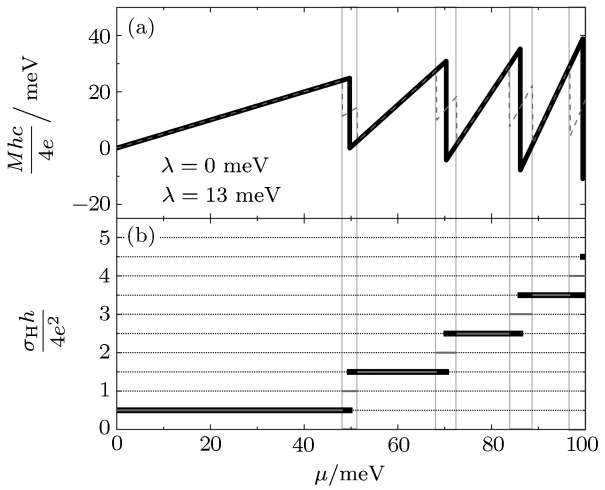


Fig. 3. (a) Calculated magnetization as a function of Fermi energy μ_0 with the external magnetic field chosen as $B=2$ T. (b) The derived Hall conductance from $\partial M/\partial\mu_0$. The black curves are for the case of $\lambda=0$ and the gray ones correspond to the case of $\lambda=13$ meV.

Before ending this paper, we investigate the mag-

netic susceptibility χ of the monolayer graphene system. The magnetic susceptibility χ is defined as the first derivative of the magnetization with respect to the external magnetic field, i.e., $\chi = \partial M/\partial B$. With the knowledge of the magnetization [Eq. (7)], one can easily obtain the final expression of magnetic susceptibility, which is given by

$$\chi = \frac{2e}{h}\sum_{n=0,s}^{\infty} f\left(\epsilon_{n,s}^{\mathrm{e}}\right)\left(\frac{\partial\mu}{\partial B} - 2\frac{\partial\epsilon_{n,s}^{\mathrm{e}}}{\partial B}\right) - \sum_{n=0,s}^{\infty} N_\nu\left[\frac{f\left(\epsilon_{n,s}^{\mathrm{e}}\right)}{\partial B}\frac{\partial\epsilon_{n,s}^{\mathrm{e}}}{\partial B} + f\left(\epsilon_{n,s}^{\mathrm{e}}\right)\frac{\partial^2\epsilon_{n,s}^{\mathrm{e}}}{\partial B^2}\right]. \quad (9)$$

Figure 4 shows the magnetic susceptibility of graphene as a function of $1/B$. As the resonant (for magnetic susceptibility) magnetic field only reflects the occupation of the LLs, which is the same as that in the conventional semiconductor 2DEG, it loses the information about the oscillating centre value of the magnetization. From Fig. 4 the effect of SOI on magnetic susceptibility is obvious. One can see that the distance between the neighbouring resonant peaks in the case with SOI included is one half of that in the case without SOI. With B decreasing, the effect of the SOI on magnetic susceptibility becomes more and more prominent.

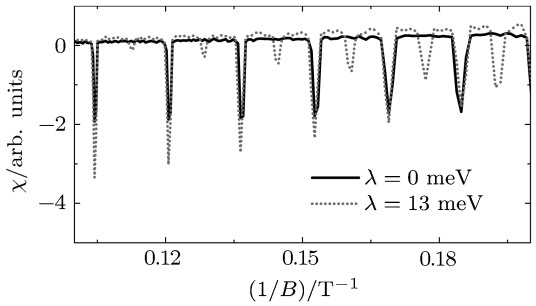


Fig. 4. Calculated magnetic susceptibility χ as a function of $1/B$. The parameters are same as those in Fig. 2. The black and gray curves correspond to the cases of $\lambda = 0$ and $\lambda = 13$ meV, respectively.

In summary, we have investigated the magnetic dHvA oscillations in a spin–orbit coupled graphene system. The results show that the oscillating centre of the magnetization maintains a positive value, which is fundamentally different from that found in conventional semiconductor 2DEG systems, in which the dHvA oscillating centre always remains at zero. This difference has an intimate relation with the difference in energy dispersions between these two kinds of systems. In addition to the SOI-induced beating pattern in dHvA oscillations, we have also studied the effect of SOI on the resonant behaviours of magnetic susceptibility and Hall conductance. These theoretical results are expected to be experimentally verified in the future.

References

[1] Landau L D 1930 *Z. Phys.* **64** 629

[2] Meinel I, Hengstmann T, Grundler D, Heitmann D, Wegscheider W and Bichler M 1999 *Phys. Rev. Lett.* **82** 819

[3] Meinel I, Grundler D, Heitmann D, Manolescu A, Gudmundsson V, Wegscheider W and Bichler M 2001 *Phys. Rev.* B **64** 121306(R)

[4] Meinel I, Grundler D, Bargstadt-Franke S, Heyn C and Heitmann D 1997 *Appl. Phys. Lett.* **70** 3305

[5] Wilde M A, Schwarz M P, Heyn C, Heitmann D, Grundler D, Reuter D and Wieck A D 2006 *Phys. Rev.* B **73** 125325

[6] Schwarz M P, Grundler D, Meinel I, Heyn C and Heitmann D 2000 *Appl. Phys. Lett.* **76** 3564

[7] Schwarz M P, Wilde M A, Groth S, Grundler D, Heyn C and Heitmann D 2002 *Phys. Rev.* B **65** 245315

[8] Wang Z and Zhang P 2009 *Phys. Rev.* B **79** 235327

[9] Wilde M A, Reuter D, Heyn C, Wieck A D and Grundler D 2009 *Phys. Rev.* B **79** 125330

[10] Fang C, Wang Z, Li S S and Zhang P 2009 *Chin. Phys.* B **18** 4430

[11] Fang C, Wang Z, Li S S and Zhang P 2009 *Chin. Phys.* B **18** 5431

[12] Castro Neto A H, Guinea F, Peres N M R, Novoselov K S and Geim A K 2009 *Rev. Mod. Phys.* **81** 109

[13] Sun J T, Du S X, Xiao W D, Hu H, Zhang Y Y, Li G and Gao H J 2009 *Chin. Phys.* B **18** 3008

[14] Li Q, Cheng Z G, Li Z J, Wang Z H and Fang Y 2010 *Chin. Phys.* B **19** 097307

[15] Varykhalov A, Sanchez-Barriga J, Shikin A M, Biswas C, Vescovo E, Rybkin A, Marchenko D and Rader O 2008 *Phys. Rev. Lett.* **101** 157601

[16] Rashba E I 2009 *Phys. Rev.* B **79** 161409(R)