Perturbation to Mei symmetry and Mei adiabatic invariants for discrete generalized Birkhoffian system*

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Based on the concept of discrete adiabatic invariant, this paper studies the perturbation to Mei symmetry and Mei adiabatic invariants of the discrete generalized Birkhoffian system. The discrete Mei exact invariant induced from the Mei symmetry of the system without perturbation is given. The criterion of the perturbation to Mei symmetry is established and the discrete Mei adiabatic invariant induced from the perturbation to Mei symmetry is obtained. Meanwhile, an example is discussed to illustrate the application of the results.

Keywords: discrete generalized Birkhoffian system, Mei symmetry, perturbation, Mei adiabatic invariant

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1. Introduction

We know that even a small change, which we can call a perturbation, may influence the original symmetries and exact invariants of mechanical systems. Perturbation to symmetries and adiabatic invariants plays a very important role in mechanical systems. An adiabatic invariant is a certain physical quantity which changes more slowly than a slowly-varying parameter of the system.[1] So the study on perturbation to symmetries and adiabatic invariants of the Birkhoffian system has great significance.

The Birkhoffian dynamics are more general than the Hamiltonian mechanics and are extensively applied to the fields of modern physics. In 1927, Birkhoff made primary studies on Birkhoffian dynamics.[2] Then the researches on Birkhoffian dynamics, such as the symmetry theory of Birkhoffian system, become more and more active.[3−14] Since 2000, some studies have been carried out to discuss the perturbation to symmetries and adiabatic invariants for the continuous Birkhoffian system.[15−21] However, the perturbation to symmetry of the discrete Birkhoffian system has never been studied so far.

In 2002, Guo *et al.*[22−26] presented a difference discrete variational principle and introduced the concept of Euler–Lagrange cohomology. Using the difference discrete variational approach, we study the perturbation to Mei symmetry and Mei adiabatic invariants of the discrete generalized Birkhoffian system, give the discrete Mei exact invariant induced from the Mei symmetry without perturbation and obtain the discrete Mei adiabatic invariant induced from the perturbation to Mei symmetry.

2. Mei symmetry and exact invariants of discrete generalized Birkhoffian system

The generalized Birkhoffian equation can be written as

$$\left(\frac{\partial R_\nu}{\partial a^\mu}-\frac{\partial R_\mu}{\partial a^\nu}\right)\dot{a}^\nu-\frac{\partial B}{\partial a^\mu}-\frac{\partial R_\mu}{\partial t}=-\Lambda_\mu,\quad(\mu,\nu=1,\ldots,2n),\tag{1}$$

where $B=B(t,\boldsymbol{a})$ is called a Birkhoffian, $R_\mu=R_\mu(t,\boldsymbol{a})$ are called Birkhoffian functions, a^μ are variables, Λ_μ are additional items.

In discrete mechanics, time interval (t_1,t_2) is discretized into a point sequence (t_k), $k=0,1,\ldots,N$, the discrete versions of a^μ, $\Lambda_\mu(t,\boldsymbol{a})$, $B(t,\boldsymbol{a})$, $R_\mu(t,\boldsymbol{a})$ are $a^{\mu,k}=a^\mu(t_k)$, $\Lambda_\mu^k=\Lambda_\mu(t_k,\boldsymbol{a}^k)$, $B_{\rm D}^k=B_{\rm D}(t_k,\boldsymbol{a}^k)$ and $R_\mu^k=R_\mu(t_k,\boldsymbol{a}^k)$, respectively.

The discrete equations of motion of the system are[27]

$$\frac{\partial R_\nu^k}{\partial a^{\mu,k}}\Delta a^{\nu,k}-\Delta R_\mu^{k-1}-\frac{\partial B_{\rm D}^k}{\partial a^{\mu,k}}+\Lambda_\mu^k=0,\tag{2}$$

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$$\Delta B_{\rm D}^{k-1} - \frac{\partial B_{\rm D}^k}{\partial t_k} + \frac{\partial R_\mu^k}{\partial t_k}\Delta a^{\mu,k} - \Lambda_\mu^k \Delta a^{\mu,k} = 0. \quad (3)$$

Choose the infinitesimal transformations of discrete time and discrete coordinates as

$$\begin{aligned} t_k^* &= t_k + \delta_t t_k = t_k + \varepsilon \xi_{0,k}^0(t_k, \boldsymbol{a}^k), \\ a^{\mu,k*}(t_k^*) &= a^{\mu,k}(t_k) + \delta_t a^{\mu,k}(t_k) \\ &= a^{\mu,k}(t_k) + \varepsilon \xi_{\mu,k}^0(t_k, \boldsymbol{a}^k), \end{aligned} \quad (4)$$

where ε is an infinitesimal parameter, while $\xi_{0,k}^0$, $\xi_{\mu,k}^0$ are discrete infinitesimal generators.

We introduce the recursive operators of discrete system for any discrete variable or function as

$$R_\pm f(z_k) = f(z_{k\pm 1}), \quad (5)$$

the vector field of generators is

$$X_{0,\rm D}^{(0)} = \xi_{0,k}^0 \frac{\partial}{\partial t_k} + \xi_{\mu,k}^0 \frac{\partial}{\partial a^{u,k}}. \quad (6)$$

According to Mei symmetry theory, the criteria of Mei symmetry for the discrete generalized Birkhoffian system can be written as

$$\begin{aligned} &\frac{\partial X_{0,\rm D}^{(0)}(R_\nu^k)}{\partial a^{\mu,k}}\Delta a^{\nu,k} - \Delta R_- X_{0,\rm D}^{(0)}(R_\mu^k) \\ &\quad - \frac{\partial X_{0,\rm D}^{(0)}(B_{\rm D}^k)}{\partial a^{\mu,k}} + X_{0,\rm D}^{(0)}(\Lambda_\mu^k) = 0, \end{aligned} \quad (7)$$

$$\begin{aligned} &\Delta R_- X_{0,\rm D}^{(0)}(B_{\rm D}^k) - \frac{\partial X_{0,\rm D}^{(0)}(B_{\rm D}^k)}{\partial t_k} \\ &\quad + \frac{\partial X_{0,\rm D}^{(0)}(R_\mu^k)}{\partial t_k}\Delta a^{\mu,k} - X_{0,\rm D}^{(0)}(\Lambda_\mu^k)\Delta a^{\mu,k} = 0. \end{aligned} \quad (8)$$

For the discrete generalized Birkhoffian system Eqs. (2) and (3), if the discrete infinitesimal generators $\xi_{0,k}^0$, $\xi_{\mu,k}^0$ satisfy Eqs. (7) and (8), the symmetry is Mei symmetry of the system.

Proposition 1[27] If a discrete gauge function $G_{{\rm M}k}^0(t_k, \boldsymbol{a}^k)$ exists such that the infinitesimal generators $\xi_{0,k}^0$, $\xi_{\mu,k}^0$ of Mei symmetry satisfy the following structure equation:

$$\begin{aligned} &X_{0,\rm D}^{(0)}\{X_{0,\rm D}^{(0)}(R_\mu^k)\}\Delta a^{\mu,k} - X_{0,\rm D}^{(0)}\{X_{0,\rm D}^{(0)}(B_{\rm D}^k)\} \\ &\quad - X_{0,\rm D}^{(0)}(B_{\rm D}^k)\Delta\xi_{0,k}^0 + X_{0,\rm D}^{(0)}(R_\mu^k)\Delta\xi_{\mu,k}^0 + X_{0,\rm D}^{(0)}(\Lambda_\mu^k) \\ &\quad \times (\xi_{\mu,k}^0 - \xi_{0,k}^0\Delta a^{\mu,k}) + \Delta G_{{\rm M}k}^0 = 0, \end{aligned} \quad (9)$$

the Mei symmetry of discrete generalized Birkhoffian system can induce the discrete Mei exact invariant as

$$\begin{aligned} I_{\rm M0,D} &= R_- X_{0,\rm D}^{(0)}(R_\mu^k)\xi_{\mu,k}^0 - R_- X_{\rm D}^{(0)}(B_{\rm D}^k)\xi_{0,k}^0 + G_{{\rm M}k}^0 \\ &= \text{const.} \end{aligned} \quad (10)$$

3. Perturbation to Mei symmetry and adiabatic invariants of system

According to the concept of adiabatic invariant, for the discrete generalized Birkhoffian system we have the following definition and proposition:

Definition If $I_{z,\rm D}(t_k, \boldsymbol{a}^k, \varepsilon)$ is a physical quantity of a discrete generalized Birkhoffian system including ε whose highest power is z and its first order difference with respect to discrete time t_k is directly proportional to ε^{z+1}, then $I_{z,\rm D}$ is called a zth order adiabatic invariant of the discrete generalized Birkhoffian systems.

Suppose the discrete generalized Birkhoffian system Eqs. (2) and (3) are perturbed by small quantities $\varepsilon W_{s,k}$, then the equations of motion for the systems become

$$\begin{aligned} &\frac{\partial R_\nu^k}{\partial a^{\mu,k}}\Delta a^{\nu,k} - \Delta R_\mu^{k-1} - \frac{\partial B_{\rm D}^k}{\partial a^{\mu,k}} \\ &\quad + \Lambda_\mu^k + \varepsilon W_{s,k} = 0, \end{aligned} \quad (11)$$

$$\begin{aligned} &\Delta B_{\rm D}^{k-1} - \frac{\partial B_{\rm D}^k}{\partial t_k} + \frac{\partial R_\mu^k}{\partial t_k}\Delta a^{\mu,k} \\ &\quad - \Delta a^{\mu,k}(\Lambda_\mu^k + \varepsilon W_{s,k}) = 0. \end{aligned} \quad (12)$$

Because of the action of $\varepsilon W_{s,k}$, the original symmetries and invariants of the system may vary. Assume that the variation is a small perturbation based on the symmetrical transformations of the system without perturbation and that $\xi_{0,k}$, $\xi_{\mu,k}$ express the generators of infinitesimal transformations after being perturbed, then

$$\begin{aligned} \xi_{0,k} &= \xi_{0,k}^0 + \varepsilon\xi_{0,k}^1 + \varepsilon^2\xi_{0,k}^2 + \cdots, \\ \xi_{\mu,k} &= \xi_{\mu,k}^0 + \varepsilon\xi_{\mu,k}^1 + \varepsilon^2\xi_{\mu,k}^2 + \cdots. \end{aligned} \quad (13)$$

The infinitesimal transformations become

$$\begin{aligned} t_k^* &= t_k + \delta_t t_k = t_k + \varepsilon\xi_{0,k}(t_k, \boldsymbol{a}^k), \\ a^{\mu,k*}(t_k^*) &= a^{\mu,k}(t_k) + \delta_t a^{\mu,k}(t_k) \\ &= a^{\mu,k}(t_k) + \varepsilon\xi_{\mu,k}(t_k, \boldsymbol{a}^k). \end{aligned} \quad (14)$$

According to the Mei symmetry theory, the determining equations of Mei symmetry of the discrete generalized Birkhoffian system perturbed by small quantity $\varepsilon W_{s,k}$ become

$$\begin{aligned} &\frac{\partial X_{\rm D}^{(0)}(R_\nu^k)}{\partial a^{\mu,k}}\Delta a^{\nu,k} \\ &\quad - \Delta R_- X_{\rm D}^{(0)}(R_\mu^k) - \frac{\partial X_{\rm D}^{(0)}(B_{\rm D}^k)}{\partial a^{\mu,k}} + X_{\rm D}^{(0)}(\Lambda_\mu^k) \end{aligned}$$

$$+\varepsilon X_{\rm D}^{(0)}(W_{s,k})=0, \tag{15}$$

$$\Delta R_{-}X_{\rm D}^{(0)}(B_{\rm D}^{k})-\frac{\partial X_{\rm D}^{(0)}(B_{\rm D}^{k})}{\partial t_k}+\frac{\partial X_{\rm D}^{(0)}(R_{\mu}^{k})}{\partial t_k}\Delta a^{\mu,k}-\Delta a^{\mu,k}[X_{\rm D}^{(0)}(\Lambda_{\mu}^{k})+\varepsilon X_{\rm D}^{(0)}(W_{s,k})]=0, \tag{16}$$

where

$$X_{\rm D}^{(0)}=\xi_{0,k}\frac{\partial}{\partial t_k}+\xi_{\mu,k}\frac{\partial}{\partial a^{u,k}}. \tag{17}$$

Substituting Eq. (13) into Eq. (17), we have

$$X_{\rm D}^{(0)}=\varepsilon^{m}X_{m,\rm D}^{(0)},\quad m=0,1,\ldots,z, \tag{18}$$

$$X_{m,\rm D}^{(0)}=\xi_{0,k}^{m}\frac{\partial}{\partial t_k}+\xi_{\mu,k}^{m}\frac{\partial}{\partial a^{u,k}}. \tag{19}$$

Substituting Eq. (13) into Eqs. (15) and (16), noticing Eqs. (17)–(19) and making the coefficients of ε^m equal, we obtain

$$\frac{\partial X_{m,\rm D}^{(0)}(R_{\nu}^{k})}{\partial a^{\mu,k}}\Delta a^{\nu,k}-\Delta R_{-}X_{m,\rm D}^{(0)}(R_{\mu}^{k})-\frac{\partial X_{m,\rm D}^{(0)}(B_{\rm D}^{k})}{\partial a^{\mu,k}}+X_{m,\rm D}^{(0)}(\Lambda_{\mu}^{k})+X_{m-1,D}^{(0)}(W_{s,k})=0, \tag{20}$$

$$\Delta R_{-}X_{m,\rm D}^{(0)}(B_{\rm D}^{k})-\frac{\partial X_{m,\rm D}^{(0)}(B_{\rm D}^{k})}{\partial t_k}+\frac{\partial X_{m,\rm D}^{(0)}(R_{\mu}^{k})}{\partial t_k}\Delta a^{\mu,k}-\Delta a^{\mu,k}[X_{m,\rm D}^{(0)}(\Lambda_{\mu}^{k})+X_{m-1,D}^{(0)}(W_{s,k})]=0. \tag{21}$$

When $m=0$, $\varepsilon W_{s,k}$ vanishes, we appoint that $\xi_{0,k}^{m-1}=\xi_{\mu,k}^{m-1}=0$ holds, then Eqs. (20) and (21) turn into Eqs. (7) and (8) respectively.

For the discrete generalized Birkhoffian system, when it is perturbed by small quantities $\varepsilon W_{s,k}$, if the discrete infinitesimal generators $\xi_{0,k}^{m}$, $\xi_{\mu,k}^{m}$ satisfy Eqs. (20) and (21), the change of the symmetry is called the perturbation to Mei symmetry of the system.

Proposition 2 For the discrete generalized Birkhoffian system perturbed by small quantity $\varepsilon W_{s,k}$, if the infinitesimal generators $\xi_{0,k}^{m}$, $\xi_{\mu,k}^{m}$ satisfy Eqs. (20) and (21), and there exists a discrete gauge function $G_{{\rm M}k}^{m}(t_k,\boldsymbol{a}^{k})$ satisfying the following structure equation:

$$X_{0,\rm D}^{(0)}\{X_{m,\rm D}^{(0)}(R_{\mu}^{k})\}\Delta a^{\mu,k}-X_{0,\rm D}^{(0)}\{X_{m,\rm D}^{(0)}(B_{\rm D}^{k})\}-X_{m,\rm D}^{(0)}(B_{\rm D}^{k})\Delta\xi_{0,k}^{0}+X_{m,\rm D}^{(0)}(R_{\mu}^{k})\Delta\xi_{\mu,k}^{0}+X_{m,\rm D}^{(0)}(\Lambda_{\mu}^{k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})+\varepsilon X_{m,\rm D}^{(0)}(W_{s,k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})+\Delta G_{{\rm M}k}^{m}=0, \tag{22}$$

then the perturbation to Mei symmetry of the system can induce a zth order discrete adiabatic invariant as

$$I_{{\rm M}z,\rm D}=\varepsilon^{m}\{R_{-}X_{m,\rm D}^{(0)}(R_{\mu}^{k})\xi_{\mu,k}^{0}-R_{-}X_{m,\rm D}^{(0)}(B_{\rm D}^{k})\xi_{0,k}^{0}+G_{{\rm M}k}^{m}\},\quad m=0,1,\ldots,z. \tag{23}$$

Proof By using the Leibniz law

$$\Delta(f_kg_k)=\Delta f_k\cdot g_k+f_{k+1}\cdot\Delta g_k, \tag{24}$$

the expansion of Eq. (22) gives

$$\begin{aligned}
&X_{0,\rm D}^{(0)}\{X_{m,\rm D}^{(0)}(R_{\mu}^{k})\}\Delta a^{\mu,k}-X_{0,\rm D}^{(0)}\{X_{m,\rm D}^{(0)}(B_{\rm D}^{k})\}\\
&\quad-X_{m,\rm D}^{(0)}(B_{\rm D}^{k})\Delta\xi_{0,k}^{0}+X_{m,\rm D}^{(0)}(R_{\mu}^{k})\Delta\xi_{\mu,k}^{0}\\
&\quad+X_{m,\rm D}^{(0)}(\Lambda_{\mu}^{k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})\\
&\quad+\varepsilon X_{m,\rm D}^{(0)}(W_{s,k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})+\Delta G_{{\rm M}k}^{m}\\
&=\frac{\partial X_{m,\rm D}^{(0)}(R_{\nu}^{k})}{\partial a^{\mu,k}}\Delta a^{\nu,k}\xi_{\mu,k}^{0}+\frac{\partial X_{m,\rm D}^{(0)}(R_{\mu}^{k})}{\partial t_k}\Delta a^{\mu,k}\xi_{0,k}^{0}\\
&\quad-\frac{\partial X_{m,\rm D}^{(0)}(B_{\rm D}^{k})}{\partial a^{\mu,k}}\xi_{\mu,k}^{0}-\frac{\partial X_{m,\rm D}^{(0)}(B_{\rm D}^{k})}{\partial t_k}\xi_{0,k}^{0}\\
&\quad-\Delta[R_{-}X_{m,\rm D}^{(0)}(B_{\rm D}^{k})\xi_{0,k}^{0}]+\Delta[R_{-}X_{m,\rm D}^{(0)}(B_{\rm D}^{k})]\xi_{0,k}^{0}\\
&\quad+\Delta[R_{-}X_{m,\rm D}^{(0)}(R_{\mu}^{k})\xi_{\mu,k}^{0}]-\Delta[R_{-}X_{m,\rm D}^{(0)}(R_{\mu}^{k})]\xi_{\mu,k}^{0}\\
&\quad+X_{m,\rm D}^{(0)}(\Lambda_{\mu}^{k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})\\
&\quad+\varepsilon X_{m,\rm D}^{(0)}(W_{s,k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})+\Delta G_{{\rm M}k}^{m}\\
&=\left\{\frac{\partial X_{m,\rm D}^{(0)}(R_{\nu}^{k})}{\partial a^{\mu,k}}\Delta a^{\nu,k}-\Delta R_{-}X_{m,\rm D}^{(0)}(R_{\mu}^{k})-\frac{\partial X_{m,\rm D}^{(0)}(B_{\rm D}^{k})}{\partial a^{\mu,k}}+X_{m,\rm D}^{(0)}(\Lambda_{\mu}^{k})\right\}\xi_{\mu,k}^{0}\\
&\quad+\left\{\Delta R_{-}X_{m,\rm D}^{(0)}(B_{\rm D}^{k})-\frac{\partial X_{m,\rm D}^{(0)}(B_{\rm D}^{k})}{\partial t_k}+\frac{\partial X_{m,\rm D}^{(0)}(R_{\mu}^{k})}{\partial t_k}\Delta a^{\mu,k}-X_{m,\rm D}^{(0)}(\Lambda_{\mu}^{k})\Delta a^{\mu,k}\right\}\xi_{0,k}^{0}\\
&\quad+\Delta\{R_{-}X_{m,\rm D}^{(0)}(R_{\mu}^{k})\xi_{\mu,k}^{0}-R_{-}X_{m,\rm D}^{(0)}(B_{\rm D}^{k})\xi_{0,k}^{0}+G_{{\rm M}k}^{m}\}\\
&\quad+\varepsilon X_{m,\rm D}^{(0)}(W_{s,k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})=0.
\end{aligned} \tag{25}$$

Using Eqs. (20) and (21), we have

$$\begin{aligned}
&\Delta\{\varepsilon^{m}\{R_{-}X_{m,\rm D}^{(0)}(R_{\mu}^{k})\xi_{\mu,k}^{0}-R_{-}X_{m,\rm D}^{(0)}(B_{\rm D}^{k})\xi_{0,k}^{0}+G_{{\rm M}k}^{m}\}\}=\Delta I_{{\rm M}z,\rm D}\\
&=\varepsilon^{m}\{-X_{m-1,\rm D}^{(0)}(W_{s,k})\xi_{0,k}^{0}\Delta a^{\mu,k}+X_{m-1,\rm D}^{(0)}(W_{s,k})\xi_{\mu,k}^{0}-\varepsilon X_{m,\rm D}^{(0)}(W_{s,k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})\}\\
&=\varepsilon^{m}\{X_{m-1,\rm D}^{(0)}(W_{s,k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})-\varepsilon X_{m,\rm D}^{(0)}(W_{s,k})(\xi_{\mu,k}^{0}-\xi_{0,k}^{0}\Delta a^{\mu,k})\}
\end{aligned}$$

$$= -\varepsilon^{z+1} X^{(0)}_{z,\mathrm{D}}(\xi^0_{\mu,k} - \xi^0_{0,k}\Delta a^{\mu,k}). \quad (26)$$

It shows that $\Delta I_{z,\mathrm{D}}$ is in direct proportion to ε^{z+1}.

4. An example

The discrete Birkhoffian system is

$$R^k_1 = a^{3,k}, \quad R^k_2 = a^{4,k}, \quad R^k_3 = R^k_4 = 0,$$
$$B^k_{\mathrm{D}} = a^{2,k} + \frac{1}{2}\{(a^{3,k})^2 + (a^{4,k})^2\}, \quad (27)$$

the additional items are $\Lambda^k_1 = a^{1,k} + t_k$, $\Lambda^k_2 = 0$, $\Lambda^k_3 = a^{3,k}$, $\Lambda^k_4 = 0$, and the system is perturbed by the following small quantities:

$$\varepsilon W^k_1 = 0, \quad \varepsilon W^k_2 = \varepsilon a^{2,k},$$
$$\varepsilon W^k_3 = 0, \quad \varepsilon W^k_4 = \varepsilon a^{4,k}. \quad (28)$$

Let us study the perturbation to Mei symmetry and Mei adiabatic invariants of the discrete generalized Birkhoffian system. The equations of motion for the system without perturbation are

$$-\Delta a^{3,k-1} + a^{1,k} + t_k = 0, \quad -\Delta a^{4,k-1} - 1 = 0,$$
$$\Delta a^{1,k} = 0, \quad \Delta a^{2,k} - a^{4,k} = 0,$$
$$\Delta B^{k-1}_{\mathrm{D}} - a^{3,k}\Delta a^{3,k} = 0. \quad (29)$$

First, we seek the Mei exact invariants. Choose infinitesimal generators as

$$\xi^0_{0,k} = -a^{4,k}, \quad \xi^0_{1,k} = a^{4,k} - 1,$$
$$\xi^0_{2,k} = a^{4,k}, \quad \xi^0_{3,k} = 0, \quad \xi^0_{4,k} = -1. \quad (30)$$

It is easy to verify that they satisfy Eqs. (7) and (8), so the infinitesimal generators Eq. (30) are Mei symmetrical for the system Eq. (27). Also, they satisfy Eq. (9).

From Eq. (9), we obtain the gauge function

$$G^0_{\mathrm{M}k} = a^{4,k} + a^{2,k} - t_k. \quad (31)$$

According to Proposition 1, Mei symmetry of the system Eq. (27) can lead to an exact invariant

$$I_{\mathrm{M0,D}} = (a^{4,k})^2 + a^{4,k} + a^{2,k} - t_k = \mathrm{const}. \quad (32)$$

Then we seek the first order Mei adiabatic invariants. Choose infinitesimal generators as

$$\xi^1_{0,k} = -a^{4,k}, \quad \xi^1_{1,k} = a^{4,k} - 1,$$
$$\xi^1_{2,k} = \xi^1_{3,k} = \xi^1_{4,k} = 0. \quad (33)$$

First of all, the infinitesimal generators Eq. (33) should satisfy the perturbed criteria of Mei symmetry, Eqs. (20) and (21), then it should also satisfy Eq. (22). From Eq. (22), we obtain

$$G^1_{\mathrm{M}k} = a^{2,k} - t_k. \quad (34)$$

According to Proposition 2, we can obtain the first order Mei adiabatic invariant induced from the perturbation to Mei symmetry of the system as

$$I_{\mathrm{M1,D}} = (\varepsilon + 1)[(a^{4,k})^2 + a^{2,k} - t_k)] + a^{4,k}. \quad (35)$$

Further we can obtain more discrete high-order adiabatic invariants.

5. Conclusion

In this paper, we study the perturbation to Mei symmetry and Mei adiabatic invariants for the discrete generalized Birkhoffian system. By the perturbation to Mei symmetry, we obtain the condition of existence of the discrete Mei adiabatic invariant. The discrete Mei adiabatic invariants shown in this paper are a new type of adiabatic invariant for the system. Using the difference discrete variational approach, we disperse the original continuous system and retain the structure and nature of the original system as much as possible. Many geometric properties of continuous mechanical systems are also the corresponding existence in discrete systems. When $z = 0$, the Mei adiabatic invariants obtained will turn into the corresponding exact invariants naturally.

References

[1] Zhao Y Y and Mei F X 1999 *Symmetries and Invariants of Mechanical Systems* (Beijing: Science Press) (in Chinese)

[2] Birkhoff G D 1927 *Dynamical Systems* (Providence RI: AMS College Publications)

[3] Santilli R M 1978 *Foundations of Theoretical Mechanics* (New York: Springer)

[4] Mei F X 1993 *Sci. China Ser.* A **36** 1456

[5] Chen X W, Luo S K and Mei F X 2000 *Acta Mech. Sol. Sin.* **21** 251 (in Chinese)

[6] Guo H Y, Luo S K and Mei F X 2001 *Rep. Math. Phys.* **47** 33

[7] Fu J L, Chen L Q, Luo Y and Luo S K 2003 *Chin. Phys.* **12** 351
[8] Luo S K and Cai J L 2003 *Chin. Phys.* **12** 357
[9] Zhang Y 2004 *Commun. Theor. Phys.* **42** 669
[10] Gu S L and Zhang H B 2004 *Chin. Phys.* **13** 979
[11] Xu X J, Mei F X and Qin M C 2004 *Chin. Phys.* **13** 1999
[12] Xu Z X 2005 *Acta Phys. Sin.* **54** 4971 (in Chinese)
[13] Qiao Y F, Zhao S H and Li R J 2006 *Chin. Phys.* **15** 2777
[14] Zhang P Y and Fang J H 2006 *Acta Phys. Sin.* **55** 3813 (in Chinese)
[15] Chen X W, Zhang R C and Mei F X 2000 *Acta Mech. Sin.* **16** 282
[16] Zhang Y 2002 *Acta Phys. Sin.* **51** 1666 (in Chinese)
[17] Fu J L, Chen L Q and Xie F P 2003 *Acta Phys. Sin.* **52** 2664 (in Chinese)
[18] Fu J L and Chen L Q 2004 *Phys. Lett.* A **324** 95
[19] Luo S K and Guo Y X 2007 *Commun. Theor. Phys.* **47** 25
[20] Ding N and Fang J H 2008 *Commun. Theor. Phys.* **49** 1410
[21] Ding N, Fang J H and Chen X X 2008 *Chin. Phys.* B **17** 1967
[22] Guo H Y, Li Y Q, Wu K and Wang S K 2002 *Commun. Theor. Phys.* **37** 1
[23] Guo H Y, Li Y Q, Wu K and Wang S K 2002 *Commun. Theor. Phys.* **37** 129
[24] Guo H Y, Li Y Q, Wu K and Wang S K 2002 *Commun. Theor. Phys.* **37** 257
[25] Guo H Y and Wu K 2003 *J. Math. Phys.* **44** 5978
[26] Luo X D, Guo H Y, Li Y Q and Wu K 2004 *Commun. Theor. Phys.* **42** 443
[27] Zhang K J, Fang J H and Li Y 2010 *Chin. Phys.* B **19** 124601