Elliptical Gaussian solitons in synthetic nonlocal nonliear media*

Cheng Shuang(程 爽), Wang Qi(王 奇), Ge Li-Juan(葛丽娟),
Shi Jie-Long(施解龙), Ding Hai-Xia(丁海霞), and Shen Ming(申 明)†

Department of Physics, Shanghai University, Shanghai 200444, *China*

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This paper studies analytically and numerically the dynamics of two-dimensional elliptical Gaussian solitons in a "double-self-focusing" synthetic nonlocal media featuring elliptical and circular Gaussian response with different degrees of nonlocality. Based on the variational approach, it obtains the approximately analytical solution of such Gaussian elliptical solitons. It also computes the stability of the solitons by numerical simulations.

Keywords: elliptical Gaussian solitons, nonlocal nonlinearity, variational principle, beam propagation method

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1. Introduction

Nonlocal spatial solitons have attracted much attention in recent years because of their unique physical properties. Nonlocal nonlinearity which supports the formation of such solitons means that the response of a media at one point is determined by the wave intensity not only at that point but also in its vicinity. It is ubiquitous in many materials with different nonlocal mechanisms such as atomic vapours[1] due to the transport of excitation caused by atomic diffusion, thermal media[2] associated with transport processes in heat conduction and dipolar Bose–Einstein condensates (BEC)[3] and liquid crystals[4] because of their long-range interaction of constituent molecules or particles. It has been shown that the nonlocal nonlinearity can provide many novel nonlinear phenomena,[5] for instance, the attraction of dark solitons[6] and suppressing the modulational instability of plane wave.[7] It can also support many kinds of solitons including multipole solitons[8,9] and vortex solitons[10,11] which would be unstable in pure local media.

Recently, elliptical spatial optical solitons have been a subject of extensive research. It has been reported experimentally,[2,12,13] including the observation of elliptic incoherent solitons supported by the photorefractive nonlinearity,[12] the first realization of coherent elliptic solitons in nonlinear media with an infinite range nonlocality,[2] and the demonstration of elliptical discrete solitons in an optically induced two-dimensional photonic lattice.[13] Besides these extensive realization of such elliptical solitons, they have also fascinated a great interest in theory.[14−18] It has been discussed that the accessible white-light solitons with an elliptical Gaussian intensity profile can propagate in a strongly nonlocal medium,[14] and that elliptical photonic lattices can ensure the stable propagation of solitons in an elliptic motion due to the modulation of intensity.[15] The elliptical bright optical solitons have also been presented in strongly[16,17] and weakly[18] nonlocal nonlinear media.

After a natural physical setting provided by the pioneering work,[19] the competing nonlinearities drew considerable attention, such as the quadratic-cubic nonlinearities,[20−23] the cubic–quintic (C–Q) model,[24−27] and the synthetic of nonlocal nonlinearities proposed recently.[28,29] The quadratic-cubic(defocusing) nonlinearities can support the fundamental solitons,[20] three-dimensional spatiotemporal solitons,[21] vortex solitons[22] and spinning optical solitons,[23] which can not be achieved in both pure quadratic or de-focusing cubic nonlinearity. The C–Q nonlinearity can exist not only in the nonlinear medium[24] but also in the BEC due to its two- and three-body nonlinearity with negative cubic and

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†Corresponding author. E-mail: shenmingluck@shu.edu.cn

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positive quintic terms.[25] Subsequently, a drastic difference from the previously considered settings is "double-self-focusing" C–Q nonlinearity in BEC proposed by Khaykovich and Malomed,[26] which demonstrated how the residual three dimensionality affects the dynamics of matter-wave solitons. However, all the research above was limited to the local case. In 2006, Mihalache *et al.* had demonstrated the effect of competition between self-focusing cubic and self-defocusing quintic nonlocal nonlinearities.[27] Nonlocal competing nonlinearity can support the fundamental solitons in liquid crystals with the focusing reorientational nonlinearity and the de-focusing thermal contributions.[28] Recently, a new setting, synthetic material with competing nonlocal nonlinearities featuring both focusing and defocusing effects proposed by Kartashov *et al.*, was used to demonstrate the stabilization of higher-order vortices and one-dimensional multihump solitons which were highly unstable in natural nonlocal material with focusing nonlinearity.[29]

In this paper, we study the evolution of two-dimensional elliptical Gaussian solitons in a "double-self-focusing" synthetic nonlocal media analytically and numerically featuring elliptical and circular Gaussian response with different degrees of nonlocality. Based on the variational approach, we obtain the evolution dynamics of such Gaussian solitons. It is shown that the beam ellipticity η and the nonlocal response ellipticity β determine the propagation of the elliptical Gaussian solitons. We also demonstrate the accessible soliton in strongly-strongly synthetic nonlocal media. Our analytical results are compared with directly numerical simulations.

2. Synthetic nonlocal nonliear propagating model and the variational approach

We consider a two-dimensional Gaussian beam propagating in a "double-self-focusing" synthetic nonlocal nolinear medium featuring an elliptical and circular Gaussian response with different degrees of nonlocality. In this case, the nonlocal nonlinear equation is given by[29]

$$\mathrm{i}\frac{\partial u}{\partial z}+\frac{1}{2}\nabla_{\perp}^{2}u+u\int(\alpha_1 R_1(x-x',y-y')+\alpha_2 R_2(x-x',y-y'))|u(x',y'z)|^2\,\mathrm{d}x'\,\mathrm{d}y'=0, \quad (1)$$

where $u(x,y,z)$ is the paraxial beam amplitude, α_1 and α_2 are real coefficients and we assume $\alpha_1>0$ and $\alpha_2>0$ ($\alpha_1\neq\alpha_2$) in this paper, then the nonlocal nonlinearity is combined with two different nonlinear self-focusing parts. The $R_j(x-x',y-y')$ ($j=1,2$) are the normalized nonlocal response functions of media with $\int R_j(x-x',y-y')\mathrm{d}x'\,\mathrm{d}y'=1$. For convenience, we consider here the case of Gaussian nonlocal response function

$$R_1(x-x',y-y')=\frac{1}{\pi\sigma_1^2}\exp\left[-\frac{(x-x')^2+(y-y')^2}{\sigma_1^2}\right], \quad (2)$$

$$R_2(x-x',y-y')=\frac{1}{\pi\sigma_{2x}\sigma_{2y}}\exp\left[-\frac{(x-x')^2}{\sigma_{2x}^2}-\frac{(y-y')^2}{\sigma_{2y}^2}\right], \quad (3)$$

where R_1 is a circularly symmetric response function with a characteristic width σ_1, R_2 is elliptical Gaussian response function with characteristic widths σ_{2x} and σ_{2y} ($\sigma_{2x}\neq\sigma_{2y}$) in x and y directions, respectively. We also choose an elliptical Gaussian trial function for Eq. (1)

$$u(x,y,z)=A(z)\exp\left(-\frac{x^2}{2w_x^2(z)}-\frac{y^2}{2w_y^2(z)}+\mathrm{i}\theta(z)+\mathrm{i}(c_x(z)x^2+c_y(z)y^2)\right), \quad (4)$$

where $A(z)$ is the complex amplitude, $\theta(z)$ represents the phase, w_x, w_y, c_x and c_y are the beam widths and phase front curvatures of the beam in the x and y directions, respectively. The incident power of the beam is $P_0=\int\int|u(x,y,z)|^2\mathrm{d}x\mathrm{d}y=A^2\pi w_x w_y$. In this paper, we aim at achieving the soliton solution of Eq. (1) by using the variational approach which can be represented as $\delta\int L_z\mathrm{d}z=0$, where $L_z=\int\int L\mathrm{d}x\mathrm{d}y$ with

$$L=\frac{\mathrm{i}}{2}\left(u^*\frac{\partial u}{\partial z}-u\frac{\partial u^*}{\partial z}\right)-\frac{1}{2}\left(\left|\frac{\partial u}{\partial x}\right|^2+\left|\frac{\partial u}{\partial y}\right|^2\right)+\frac{1}{2}|u|^2\int(\alpha_1 R_1(x-x',y-y')+\alpha_2 R_2(x-x',y-y'))|u(x',y',z)|^2\mathrm{d}x'\,\mathrm{d}y'. \quad (5)$$

Substituting Eq. (4) into Eq. (5) and integrating once over the whole spatial coordinates, we can obtain the average Lagrangian density

$$L_z=-\pi A^2 w_x w_y\frac{\partial\theta}{\partial z}-\frac{\pi A^2 w_x^3 w_y}{2}\frac{\partial c_x}{\partial z}-\frac{\pi A^2 w_y^3 w_x}{2}\frac{\partial c_y}{\partial z}-\frac{\pi A^2 w_x}{4w_y}-\frac{\pi A^2 w_y}{4w_x}$$

$$
\begin{aligned}
&- \pi A^2 w_x^3 w_y c_x^2 - \pi A^2 w_y^3 w_x c_y^2 \\
&+ \frac{\pi A^4 \alpha_1 w_x^2 w_y^2}{2(2w_x^2+\sigma_1^2)^{1/2}(2w_y^2+\sigma_1^2)^{1/2}} \\
&+ \frac{\pi A^4 \alpha_2 w_x^2 w_y^2}{2(2w_x^2+\sigma_{2x}^2)^{1/2}(2w_y^2+\sigma_{2y}^2)^{1/2}}. \qquad (6)
\end{aligned}
$$

Based on the Euler–Lagrange equation

$$
\frac{\delta L_z}{\delta p_{\rm i}} = \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\partial L_z}{\partial p_{\rm i}'}\right) - \frac{\partial L_z}{\partial p_{\rm i}} = 0, \qquad (7)
$$

$$
p_{\rm i} = \{A, w_x, w_y, c_x, c_y, \theta\},
$$

$$
p_{\rm i}' = \left\{\frac{\mathrm{d}A}{\mathrm{d}z}, \frac{\mathrm{d}w_x}{\mathrm{d}z}, \frac{\mathrm{d}w_y}{\mathrm{d}z}, \frac{\mathrm{d}c_x}{\mathrm{d}z}, \frac{\mathrm{d}c_y}{\mathrm{d}z}, \frac{\mathrm{d}\theta}{\mathrm{d}z}\right\},
$$

we can obtain the evolution equations in x and y directions, respectively

$$
\begin{aligned}
&\frac{\mathrm{d}^2 w_x}{\mathrm{d}z^2} - \frac{1}{w_x^3} + \frac{2\alpha_1 p w_x}{\pi\sqrt{2w_x^2\eta^2+\sigma_1^2}\sqrt{(2w_x^2+\sigma_1^2)^3}} \\
&\quad + \frac{2\alpha_2 p w_x}{\pi\sqrt{2w_x^2\eta^2+\sigma_2^2\beta^2}\sqrt{(2w_x^2+\sigma_2^2)^3}} = 0, \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\mathrm{d}^2 w_y}{\mathrm{d}z^2} - \frac{1}{w_y^3} + \frac{2\alpha_1 p w_y}{\pi\sqrt{2\dfrac{w_y^2}{\eta^2}+\sigma_1^2}\sqrt{(2w_y^2+\sigma_1^2)^3}} \\
&\quad + \frac{2\alpha_2 p w_y}{\pi\sqrt{2\dfrac{w_y^2}{\eta^2}+\sigma_2^2}\sqrt{(2w_y^2+\sigma_2^2\beta^2)^3}} = 0, \qquad (9)
\end{aligned}
$$

where $\eta = w_y/w_x$ is the beam[30] and $\beta = \sigma_{2y}/\sigma_{2x}$ is the nonlocal response ellipticity (here we denote $\sigma_{2x} = \sigma_2$, so $\sigma_{2y} = \sigma_2\beta$). Equations (8) and (9) describe the dynamics of the beam widths and show a competition among self-trapping, diffraction and nonlocality. It is obvious that the beam ellipticity η and the nonlocal response ellipticity β determine the propagation of the Gaussian elliptical solitons. Letting $w_x(z) = w_{x0}$ and $w_y(z) = w_{y0}$, which means the beam keeps its initial widths during the propagation, we can obtain the critical powers for the stationary elliptical solitons

$$
p_{cx} = \frac{\pi}{2w_x^4}\frac{1}{\dfrac{\alpha_1}{(2w_x^2\eta^2+\sigma_1^2)^{1/2}(2w_x^2+\sigma_1^2)^{3/2}} + \dfrac{\alpha_2}{(2w_x^2\eta^2+\sigma_2^2\beta^2)^{1/2}(2w_x^2+\sigma_2^2)^{3/2}}}, \qquad (10)
$$

$$
p_{cy} = \frac{\pi}{2w_y^4}\frac{1}{\dfrac{\alpha_1}{\sqrt{2\dfrac{w_y^2}{\eta^2}+\sigma_1^2}\sqrt{(2w_y^2+\sigma_1^2)^3}} + \dfrac{\alpha_2}{\sqrt{2\dfrac{w_y^2}{\eta^2}+\sigma_2^2}\sqrt{(2w_y^2+\sigma_2^2\beta^2)^3}}}. \qquad (11)
$$

In Fig. 1, we plot the critical power as a function of beam width in x direction with different nonlinearity strength α_2 in figure (a), the nonlocal response ellipticity β in figure (b) and in x and y directions with different beam ellipticity η in figures (c) and (d), respectively. It is obvious that the critical power will decrease when the beam width increases. At a fixed beam width, the critical power will decrease with the increase of the nonlinearity extent α_2, On the contrary, it will increase when the nonlocal response ellipticity β increases. However, when beam ellipticity η increases, the critical power in x direction will increase whereas in y direction decrease.

These results can be explained as follows. As we know, the larger the power is, the larger the nonlinearity is.[31] When the nonlinearity strength α_2 increases, the nonlinearity will be strengthened, then the diffraction is weaker than the nonlinearity, so the powers must be weakened to balance this diffraction. We can also see that the synthetic media needs a much smaller critical power than its corresponding natural nonlocal media for $\alpha_2 = 0$ in Fig. 1(a). Critical power in x direction was related not only with response width in its direction but also with the one in y direction, if nonlocal response ellipticity β increases, the response width in y direction will increase, so the nonlinearity will be weakened and the power should be strengthened as shown in Fig. 1(b). The critical power in y direction has the same case. However, with the increase of beam ellipticity $\eta = w_y/w_x$, the width in x direction should decrease if we let the width in y direction be fixed, so the diffraction will increase and the power must be increased to balance the increasing diffraction. On the contrary, when width in x direction is fixed, the width in y direction should increase and then the diffraction decreases so the critical power must be decreased to balance the decreasing diffraction.

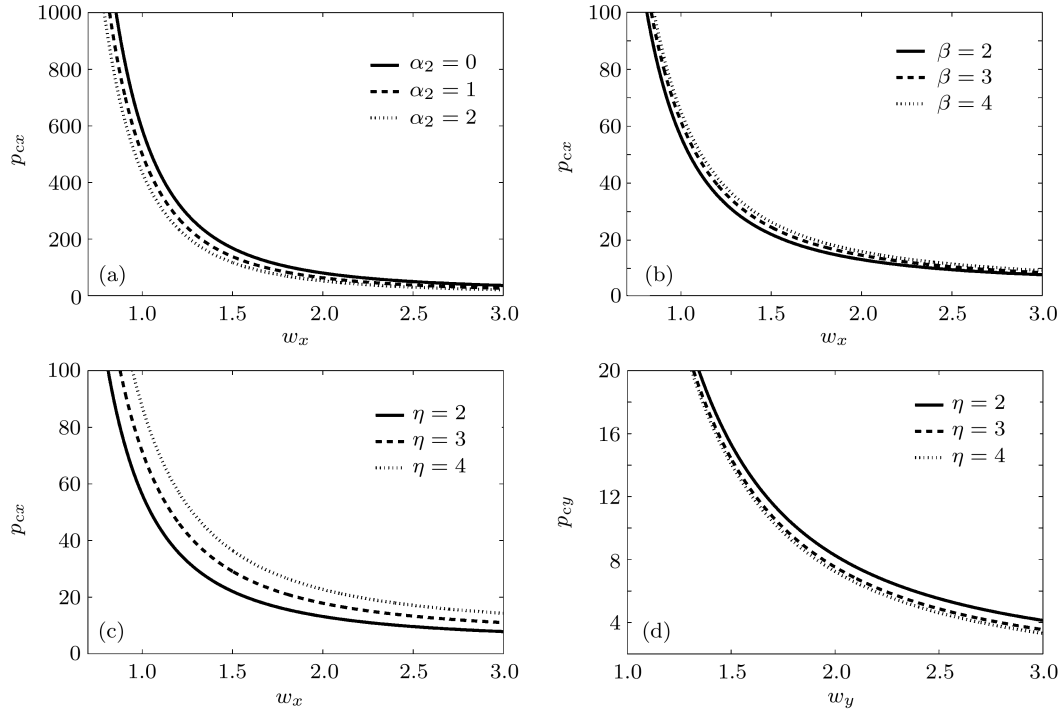


Fig. 1. Critical power of Gaussian solitons versus the beam width in x direction with different nonlinear strength α_2 in (a), nonlocal response ellipticity β in (b) and in x and y directions with different beam ellipticity η in (c) and (d), respectively. The initial parameters are $\alpha_1 = 1$, $\sigma_1 = 2$ and $\sigma_2 = 3$.

3. Strong-strong nonlocal nonlinear response

In the limit of strong-strong nonlocality, the characteristic widths of the nonlocal response function are much larger than the beam widths, i.e., $\sigma_1 \gg w_{i0}$ and $\sigma_{2i} \gg w_{i0}$ $(i = x, y)$, we can simplify Eqs. (8) and (9) as

$$\frac{\mathrm{d}^2 w_x}{\mathrm{d}z^2} - \frac{1}{w_x^3} + \frac{2\alpha_1 p w_x}{\pi\sigma_1^4} + \frac{2\alpha_2 p w_x}{\pi\sigma_2^4\beta} = 0, \tag{12}$$

$$\frac{\mathrm{d}^2 w_y}{\mathrm{d}z^2} - \frac{1}{w_y^3} + \frac{2\alpha_1 p w_y}{\pi\sigma_1^4} + \frac{2\alpha_2 p w_y}{\pi\sigma_2^4\beta^3} = 0. \tag{13}$$

Assuming the beam has $\mathrm{d}w_j/\mathrm{d}z|_{z=0} = 0$ $(j = x, y)$ and integrating Eqs. (12) and (13) once, we obtain the equations

$$\left[\frac{\mathrm{d}w_x}{\mathrm{d}z}\right]^2 + \frac{1}{w_{x0}^2}\left[\frac{w_{x0}^2}{w_x^2} - 1\right] + \frac{P_0}{\pi}\left[w_x^2 - w_{x0}^2\right]\left(\frac{\alpha_1}{\sigma_1^4} + \frac{\alpha_2}{\sigma_2^4\beta}\right) = 0, \tag{14}$$

$$\left[\frac{\mathrm{d}w_y}{\mathrm{d}z}\right]^2 + \frac{1}{w_{y0}^2}\left[\frac{w_{y0}^2}{w_y^2} - 1\right] + \frac{P_0}{\pi}\left[w_y^2 - w_{y0}^2\right]\left(\frac{\alpha_1}{\sigma_1^4} + \frac{\alpha_2}{\sigma_2^4\beta^3}\right) = 0. \tag{15}$$

If we denote a two-dimensional potential

$$\begin{aligned} V(\boldsymbol{w}) &= \frac{1}{w_{x0}^2}\left[\frac{w_{x0}^2}{w_x^2} - 1\right] + \frac{P_0}{\pi}\left[w_x^2 - w_{0x}^2\right]\left(\frac{\alpha_1}{\sigma_1^4} + \frac{\alpha_2}{\sigma_2^4\beta}\right) \\ &+ \frac{1}{w_{y0}^2}\left[\frac{w_{y0}^2}{w_y^2} - 1\right] + \frac{P_0}{\pi}\left[w_y^2 - w_{y0}^2\right]\left(\frac{\alpha_1}{\sigma_1^4} + \frac{\alpha_2}{\sigma_2^4\beta^3}\right), \end{aligned} \tag{16}$$

then Eqs. (14) and (15) can be casted in a single relation

$$\frac{\mathrm{d}^2\boldsymbol{w}}{\mathrm{d}z^2} = -\boldsymbol{\nabla}(\boldsymbol{w}), \tag{17}$$

where $\boldsymbol{w} = (w_x, w_y)$. Equation (17) can be treated as a classical Newtonian equation which represents as a two-dimensional dynamics of an effective particle with a kinetic energy $[\mathrm{d}w_x/\mathrm{d}z]^2 + [\mathrm{d}w_y/\mathrm{d}z]^2$ moving in the potential $V(\boldsymbol{w})$.

Stationary strongly-strongly nonlocal Gaussian elliptical solitons, called elliptical accessible soliton (the effective particles located at the bottom of the potential), can be obtained by $[\partial V(w_j)/\partial w_j]|_{w_j = w_{j0}} = 0$ $(j = x, y)$ and then the critical powers for the solitons can be represented as follows:

$$p_{cx} = \frac{\pi}{2w_x^4}\frac{1}{\dfrac{\alpha_1}{\sigma_1^4} + \dfrac{\alpha_2}{\sigma_2^4\beta}}, \tag{18}$$

$$p_{cy} = \frac{\pi}{2w_y^4}\frac{1}{\dfrac{\alpha_1}{\sigma_1^4} + \dfrac{\alpha_2}{\sigma_2^4\beta^3}}, \tag{19}$$

which imply the relation

$$\frac{p_{cy}}{p_{cx}} = \frac{\beta^2}{\eta^4}\frac{\alpha_1\sigma_2^4\beta + \alpha_2\sigma_1^4}{\alpha_1\sigma_2^4\beta^3 + \alpha_2\sigma_1^4}. \tag{20}$$

First, we briefly describe the case of general elliptical Gaussian soliton in the strongly nonlocal elliptical response media when $\alpha_1 = 0$. The Eq. (20) turns into $p_{cy}/p_{cx} = \beta^2/\eta^4$ and when $\eta = \sqrt{\beta}$ (dashed curve line in Fig. 2(a)), $p_{cx} = p_{cy} = p_c$. If the incident power p_0 equals p_c, the diffraction is strictly balanced by the nonlinearity, the beam keeps its initial widths in transverse directions simultaneously and a stationary elliptical Gaussian soliton is formed. However, in the case of general circularly symmetric media with $\beta = 1$, the ratio turns into $p_{cy}/p_{cx} = 1/\eta^4$ which means that only when $\eta = 1$, the soliton can be formed. That is to say, the circularly symmetric media can only support circular symmetric beam as demonstrated in Ref. [32]. However, it is a little different from the case of strongly-strongly nonlocal synthetic media in which the ratio of critical powers is determined not only by the ellipticity of beam and by nonlocal response but also by the ratio of nonlinearity strength α_2/α_1 and of nonlocality degree σ_2/σ_1 which could be achieved by the simplification of Eq. (20). When stationary elliptical soliton forms, η and β have to satisfy the following complicated relationship:

$$\eta^4 = \frac{\left(\frac{\sigma_2}{\sigma_1}\right)^4\beta^3 + \frac{\alpha_2}{\alpha_1}\beta^2}{\left(\frac{\sigma_2}{\sigma_1}\right)^4\beta^3 + \frac{\alpha_2}{\alpha_1}}, \tag{21}$$

as displayed in Fig. 2(a) by the solid line. With the increasing nonlocal response ellipticity β, the beam ellipticity η should increase first and then decrease to an asymptotic value when it reaches the maximum, rather than increase persistently as in the case of elliptical nonlocal media. We also depict the affections of the ratio of α_2/α_1 and σ_2/σ_1 on the relation between η and β in Figs. 2(b) and 2(c), respectively. In Fig. 2(b), we can see that η tends to a much larger asymptotic value with the increase of α_2/α_1. On the contrary, with the increase of σ_2/σ_1, beam ellipticity η tends to stabilization quickly but the asymptotic value becomes much smaller. Meanwhile, with further increase of σ_2/σ_1, η can keep a similar asymptotic value no matter how large the σ_2/σ_1 is, at the same time, the curve will no longer decline (Fig. 2(c)).

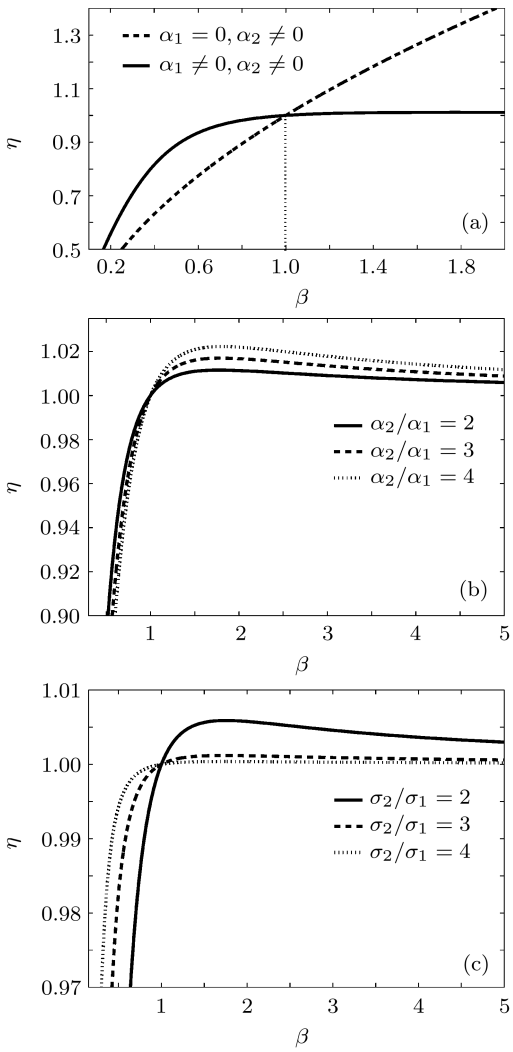


Fig. 2. Beam ellipticity η versus nonlocal response ellipticity β in the case of stationary elliptical Gaussian soliton. (a) represents the case in the natural elliptical (dashed line) and circular-elliptical synthetic nonlocal media (solid line), respectively. In circular-elliptical synthetic nonlocal media, the affections of α_2/α_1 and σ_2/σ_1 on the relation between η and β are shown in (b) and (c), respectively.

4. Oscillations of the solitons

The stationary solitons correspond to a specific relation among the soliton parameters such as widths, power and the properties of the nonlocality. For arbitrary choice of those parameters the elliptical Gaussian solitons experience dynamical evolution in the framework of the variational approach, this evolution is governed by the system of Eqs. (8) and (9). In Fig. 3, we plot the evolution of the beam widths by using directly numerical integration of Eqs. (8) and (9). Figure 3(a) represents the evolution of width with different initial powers (only x direction considered), from which we can see that the incident power can

solely determine the evolution trend. When the incident power is equal to the critical power, the beam keeps its initial width (solid line). When the incident power is smaller (larger) than the critical one, as shown by dot (dashed) line, the width expands (compresses) first and then oscillates periodically.

Figures 3(b)–3(d) illustrate the dynamics of beam widths in x and y directions with different incident powers. We let the critical power in x direction be smaller than that in the y direction. When the incident power is smaller (larger) than both of the critical powers, the widths in both directions will increase (decrease) first and then decrease (increase), oscillate with the propagation distance, as shown in Figs. 3(b) and 3(d). However, in Fig. 3(c), the incident power is larger than the critical power in x direction but smaller than the that in y direction, we can see that the dynamics of the beam widths has an opposite evolution behaviour.

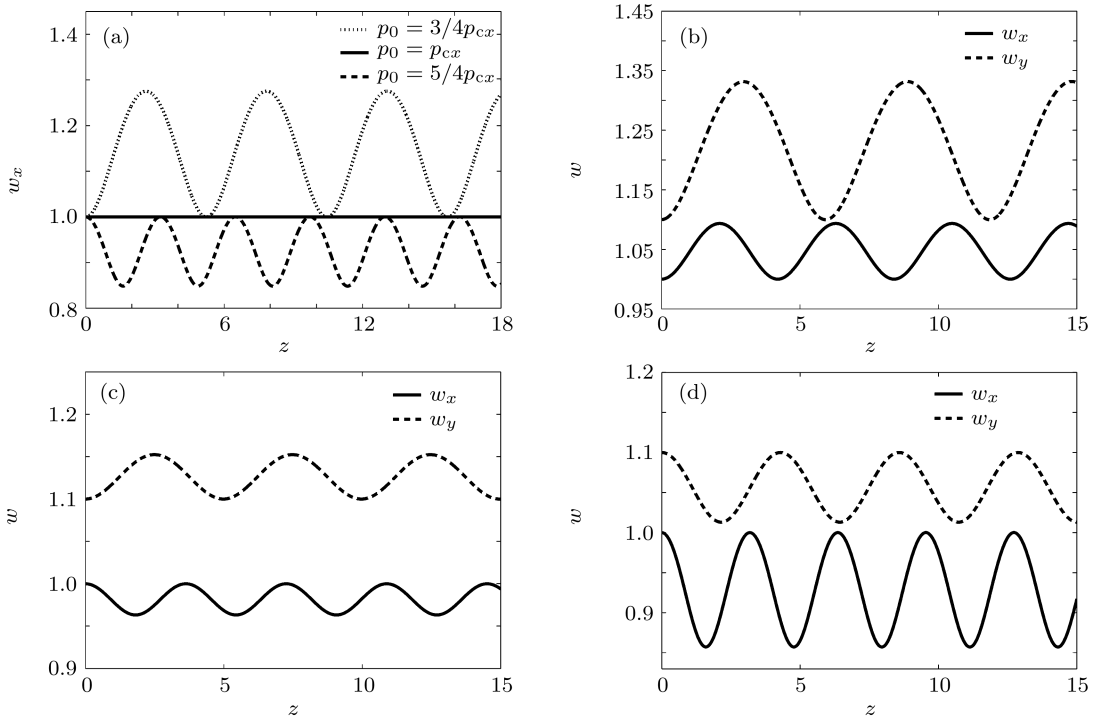


Fig. 3. Width of Gaussian solitons versus propagation distance z for different incident powers. The evolution of width in x direction with different initial powers is shown in (a), the dot line represents $p_0 < p_{cx}$, the solid line represents $p_0 = p_{cx}$, and the dashed line represents $p_0 > p_{cx}$. The widths in x and y directions with different incident powers: the cases of $p_0 < p_{cx} < p_{cy}$, $p_{cx} < p_0 < p_{cy}$ and $p_{cx} < p_{cy} < p_0$ were shown in (b), (c) and (d), respectively. The solid line represents width in x direction, while the dash one represents width in y direction. The initial parameters are $w_{0x} = 1$, $w_{0y} = 1.1$, $\alpha_1 = 1$, $\alpha_2 = 2$, $\eta = 1.1$, $\beta = 4$, $\sigma_1 = 2$ and $\sigma_2 = 2$.

5. Numerical result

To confirm the variational results, we numerically compute Eq. (1) by using the split step beam propagation method.[33] The analytical solutions are used as the initial conditions in our numerical code. We find that the numerical results are in good agreement with the analytical results.

As shown in Figs. 4–7, we show the evolutions of the elliptical solitons with different degrees of nonlocality. When the incident power equals the critical values, the elliptical solitons can propagate quasi-stationarily during propagation for both weak-weak (Fig. 4) and strong-strong (Fig. 6) types of degree of the nonlocality. Because of the collapsing tendency of high dimensional beam and the anisotropic diffraction of the elliptical beam, the elliptical solitons may slightly change its size and intensity distribution during the propagation. When the power does not satisfy the stationary conditions, the solitons will oscillate with propagation. It is obvious that the solitons will breakup into two parts at 20 propagation distance when the power is smaller (Fig. 5) or larger (Fig. 7) than the critical value in the strong-strong type of degree of nonlocality.

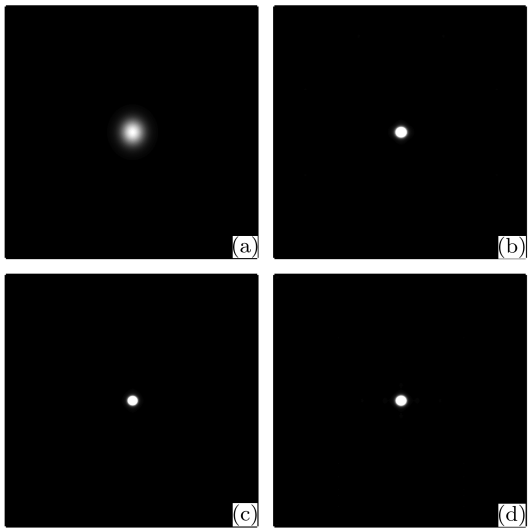


Fig. 4. The evolution of intensity of (2+1)-dimensional elliptical Gaussian beam in the synthetical weakly-weakly nonlocal media. The intensity of beam is shown in (a), (b), (c) and (d) corresponding to the propagation distance $z = 0$, $z = 10$, $z = 15$ and $z = 20$, respectively, where incident power $P_0 = 4.3$, $w_{0x} = 1$, $w_{0y} = 1.1$, $\alpha_1 = 1$, $\alpha_2 = 2$, $\eta = 1.1$, $\beta = 2$, $\sigma_1 = 0.1$ and $\sigma_2 = 0.2$.

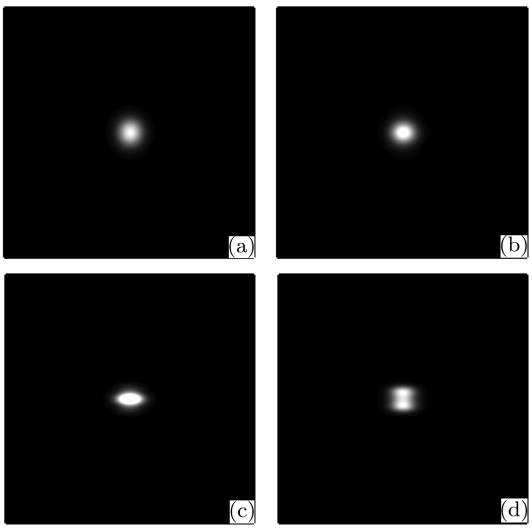
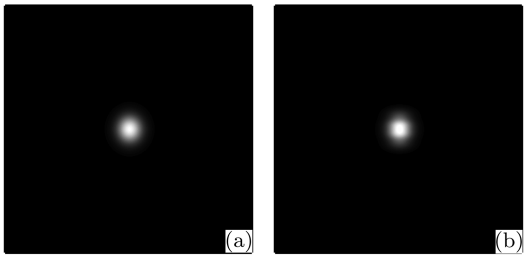


Fig. 5. The evolution of intensity of (2+1)-dimensional elliptical Gaussian beam in the synthetical strongly-strongly nonlocal media. The intensity of beam is shown in (a), (b), (c) and (d) corresponding to the propagation distance $z = 0$, $z = 6$, $z = 14$ and $z = 20$, respectively, where incident power $P_0 = 6500$ is smaller than the critical power, $w_{0x} = 1$, $w_{0y} = 1.1$, $\alpha_1 = 1$, $\alpha_2 = 2$, $\eta = 1.1$, $\beta = 2$, $\sigma_1 = 10$ and $\sigma_2 = 11$.



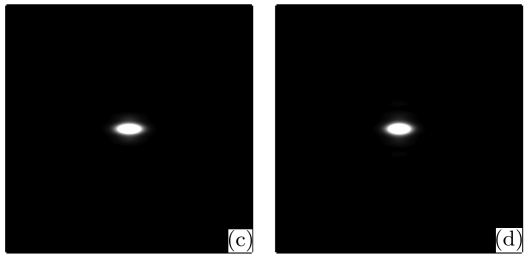


Fig. 6. The evolution of intensity of (2+1)-dimensional elliptical Gaussian beam in the synthetical strongly-strongly nonlocal media. The intensity of beam is shown in (a), (b), (c) and (d) corresponding to the propagation distance $z = 0$, $z = 6$, $z = 14$ and $z = 20$, respectively, where incident power $P_0 = 9328.5$ is equal to critical power, $w_{0x} = 1$, $w_{0y} = 1.1$, $\alpha_1 = 1$, $\alpha_2 = 2$, $\eta = 1.1$, $\beta = 2$, $\sigma_1 = 10$ and $\sigma_2 = 11$.

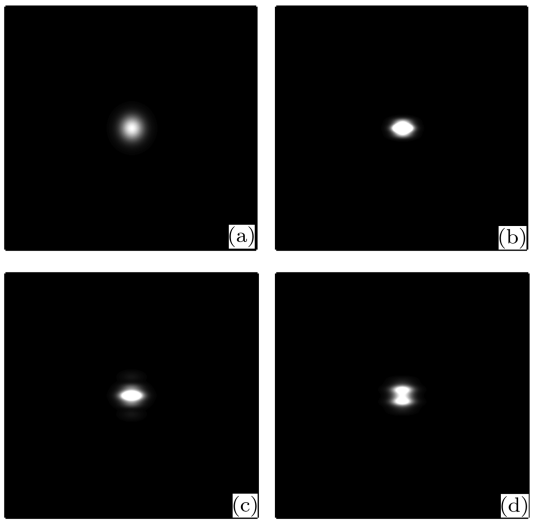


Fig. 7. The evolution of intensity of (2+1)-dimensional elliptical Gaussian beam in the synthetical strongly-strongly nonlocal media. The intensity of beam is shown in (a), (b), (c) and (d) corresponding to the propagation distance $z = 0$, $z = 6$, $z = 14$ and $z = 20$, respectively, where incident power $P_0 = 12000$ is larger than the critical power, $w_{0x} = 1$, $w_{0y} = 1.1$, $\alpha_1 = 1$, $\alpha_2 = 2$, $\eta = 1.1$, $\beta = 2$, $\sigma_1 = 10$ and $\sigma_2 = 11$.

6. Conclusion

In conclusion, we have studied the evolution of two-dimensional elliptical Gaussian solitons in a "double-self-focusing" synthetic nonlocal media featuring elliptical and circular Gaussian response with different degrees of nonlocality both analytically and numerically. Based on the variational approach, we obtain the general evolution dynamics of such Gaussian solitons. It is shown that the beam ellipticity η and the nonlocal response ellipticity β determine the propagation of the Gaussian elliptical solitons. We also demonstrate the propagation of the accessible soliton in strongly-strongly synthetic nonlocal media. At last, we investigate the stability of the solitons by using directly numerical simulations.

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