Rapid Communication

Decoherence of elliptical states in phase space*

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The effects of decoherence on elliptical states which concern the quantum superposition of N coherent states on an ellipse in the α plane are studied. The characteristic decoherence times are determined. The evolutions of the Wigner functions associated with these states are investigated theoretically and the losses of nonclassicality as a result of decoherence are discussed. The result shows that the decoherence of elliptical states is slower than circular states relying on the number of coherent states and the amplitude, and the constructed states have a higher resilience to losses.

Keywords: elliptical states, Wigner distribution function, decoherence

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1. Introduction

Among the different representations of quantum states, the Wigner quasiprobability representation has been widely exploited as evidence of nonclassical properties.[1] In particular, the presence of negative quasiprobability regions has been considered to be a consequence of the quantum superposition of distinct physical states.[2−6] These superposition states play a key role in entanglement distillation,[7−9] loophole-free tests of Bell's inequality,[10,11] quantum computing[12] and quantum information.[13] Their nonclassicality is a necessary condition for generating an inseparable entanglement state via a beam splitter.[14,15] However, nonclassical optical fields inevitably interact with their surrounding environment, which leads to the extremely short persistence of quantum coherence, i.e., extremely rapid decoherence.[12,16−18] Quantum decoherence is responsible for washing out the quantum interference effects desirable to be seen as signals in experiments and is also the main problem found in quantum optics and physics of quantum information and computation. In many cases, one is interested in understanding decoherence because one wants to prevent decoherence from damaging quantum states and to protect information stored in quantum states from degradation during interaction with the environment.[17−20]

Based on circular states, which are very instructive examples in which many aspects of the nonclassical properties have been studied,[21−26] and which concern a sum of N coherent states having the same amplitude and distributed with equal probability on a circle, we have constructed the elliptical states described in Ref. [27], namely the amplitude of the superposed coherent state changes with ellipse. Various nonclassical properties of elliptical states have been investigated. It was shown that the constructed new states exhibited infinite quadrature squeezing. The interference fringes between the coherent states form the elliptic annuli of Fock states in the Wigner-function picture. The phase distribution was no longer uniform like in circular states. Following Ref. [27], in this paper, we further study the problem of the influence of an environment on the stability of the superposition of quantum states. The difference of nonclassicality between elliptical states and circular states is discussed in the presence of decoherence.

The paper is organized as follows. In Section 2, we determine the decoherence times of the system when the initial states are circular states and ellipti-

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cal states, respectively, namely the characteristic time when the system becomes essentially classical. An explicit analytic expression of the Wigner functions for elliptical states is derived in presence of decoherence in Section 3. The negative value of the Wigner representation in the phase space for such states is evidence of the persistence of quantum properties. This aspect is then compared with the case of circular states. A conclusion is given in Section 4.

2. The characteristic time of decoherence

The elliptical states in Ref. [27] are constructed as

$$|\psi_N\rangle = \lambda_N \sum_{k=1}^{N} C_k |\alpha_k\rangle, \tag{1}$$

where

$$\alpha_k = r_k \exp(\mathrm{i}\phi_k), \tag{2}$$

$$\phi_k = \phi_0 + \frac{2\pi k}{N}, \tag{3}$$

$$r_k = \left(\frac{\cos^2\phi_k}{a^2} + \frac{\sin^2\phi_k}{b^2}\right)^{-1/2}, \tag{4}$$

where a, b are real numbers. The normalization constant λ_N is

$$|\lambda_N|^{-2} = \sum_{j,k=1}^{N} C_j^* C_k \exp\left[-\frac{1}{2}(r_j^2 + r_k^2)\right] \exp(\alpha_j^* \alpha_k). \tag{5}$$

Obviously, when $a = b$, i.e., $r_k = r$, r is the modulus of circular states, the elliptical states back to circular states.[25] For N is even and $C_{k+N/2} = \pm C_k$, the parity of the states may be even or odd,

$$|\psi_N(-a,-b)\rangle = \pm |\psi_N(a,b)\rangle . \tag{6}$$

Here we consider, in particular, the case $C_k = 1$, the even elliptical state (EES). For $N > 1$, this state is non-classical according to Ref. [27].

Decoherence implies a degradation of quantum interference effects (manifesting themselves in the existence of quantum superpositions) due to interactions with the 'external world'. Here we consider coupling with the reservoir in which the amplitude of the harmonic oscillator is damped. In the Born and Markov approximations, the reduced density operator for the harmonic oscillator in the interaction picture obeys

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = \frac{\gamma}{2}\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right), \tag{7}$$

where γ is the damping constant. The rotating-wave approximation has been taken and the temperature of the reservoir was set to be zero ($T = 0$). In this case, the reservoir is the quantum vacuum field, in which the mechanical energy of the oscillating particle is dissipated. Much work has been done on quantum radiation.[28,29]

The normally ordered characteristic function is defined as

$$X_N(\eta, t) = \mathrm{Tr}[\hat{\rho}(t) \exp(\eta\hat{a}^\dagger) \exp(-\eta^*\hat{a})]. \tag{8}$$

We can write

$$\begin{aligned}\frac{\partial X_N(\eta,t)}{\partial t} &= \mathrm{Tr}\left[\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t}\exp(\eta\hat{a}^\dagger)\exp(-\eta^*\hat{a})\right] \\ &= -\frac{\gamma}{2}\left[\eta\frac{\partial X_N(\eta,t)}{\partial\eta} + \eta^*\frac{\partial X_N(\eta,t)}{\partial\eta^*}\right].\end{aligned} \tag{9}$$

One can show that the solution of Eq. (9) is

$$X_N(\eta, t) = X_N[\eta\exp(\gamma t/2), 0] = X_N(\eta(t), 0). \tag{10}$$

For elliptical states, the initial condition is

$$\begin{aligned}&X_N(\eta, 0) \\ &= \mathrm{Tr}\left[\hat{\rho}(0)\exp(\eta\hat{a}^\dagger)\exp(-\eta^*\hat{a})\right] \\ &= |\lambda_N|^{-2}\sum_{j,k=1}^{N} C_j^* C_k \langle\alpha_j|\alpha_k\rangle \exp(\eta\alpha_j^* - \eta^*\alpha_k).\end{aligned} \tag{11}$$

Thus the normally ordered characteristic function of the damped elliptical states is

$$\begin{aligned}&X_N(\eta, t) \\ &= |\lambda_N|^{-2}\sum_{j,k=1}^{N} C_j^* C_k \langle\alpha_j|\alpha_k\rangle \\ &\quad\times \exp[(\eta\alpha_j^* - \eta^*\alpha_k)\exp(-\gamma t/2)].\end{aligned} \tag{12}$$

It is not difficult to show that the corresponding density matrix ρ is given by

$$\hat{\rho} = |\lambda_N|^{-2}\sum_{j,k=1}^{N} C_j^* C_k \langle\alpha_j|\alpha_k\rangle^{(1-\exp(-\gamma t))} |\alpha_k\exp(-\gamma t/2)\rangle\langle\alpha_j\exp(-\gamma t/2)|$$

$$= |\lambda_N|^{-2} \sum_{k=1}^{N} |C_k|^2 \, |\alpha_k \exp(-\gamma t/2)\rangle \, \langle \alpha_k \exp(-\gamma t/2)|$$

$$+ |\lambda_N|^{-2} \sum_{j,k=1,j\neq k}^{N} C_j^* C_k \, \langle \alpha_j | \alpha_k \rangle^{(1-\exp(-\gamma t))} \, |\alpha_k \exp(-\gamma t/2)\rangle \, \langle \alpha_j \exp(-\gamma t/2)| \, . \quad (13)$$

The energy decays at the same rate as that for a coherence state, but there is another quantity now undergoing a decay, namely the quantity that multiplies the "coherences" or the "off-diagonal" terms. If $\gamma \ll 1$, then the relevant exponential factor multiplying the crossed terms becomes

$$\sum_{j,k=1,j\neq k}^{N} C_j^* C_k \exp\left[\left(-\frac{1}{2}r_j^2 - \frac{1}{2}r_k^2 + \alpha_j^* \alpha_k\right)\gamma t\right] \equiv \exp(-t/t_{\rm c}). \quad (14)$$

From which we can obtain the decoherence time

$$t_{\rm c}^{-1} = \sum_{j,k=1,j\neq k}^{N} \left[\gamma\left(\frac{1}{2}r_j^2 + \frac{1}{2}r_k^2 - \alpha_j^* \alpha_k\right) - \frac{\ln(C_j^* C_k)}{t}\right].$$

This is the characteristic time over which the coherences tend to vanish.

When the initial states are circular, the decoherence time versus the amplitude value r for different N is plotted in Fig. 1. It is easy to see that the characteristic time decreases with the increase of r. The larger the numbers of superposed coherent states are, the faster the curves decline when r takes a small value. The decoherence times are independent of the initial phase ϕ_0.

In order to see the effect of the variations of a and b on the decoherence time, we plot the decoherence times of elliptical states versus a when N, b and ϕ_0 are fixed in Fig. 2. It is obvious that the curves decline as a increases. When b becomes large, the positions of the curves along the vertical axis move down and this decreasing of the curves becomes more gentle with the increasing a. This indicates that the process of decoherence slows down. For different ϕ_0, the decoherence times are distinct. In particular, initial phase ϕ_0 has a significant effect on $N = 4$ and $N = 8$. So we can say that elliptical states show very different characteristics compared with circular states in the presence of decoherence.

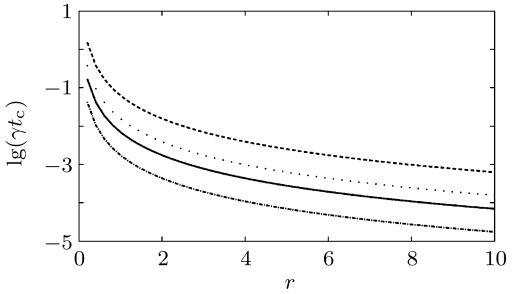


Fig. 1. The decoherence times of circular states versus the amplitude r. $N = 4$ (dashed line), $N = 8$ (dotted line), $N = 12$ (solid line) and $N = 24$ (dot-dashed line).

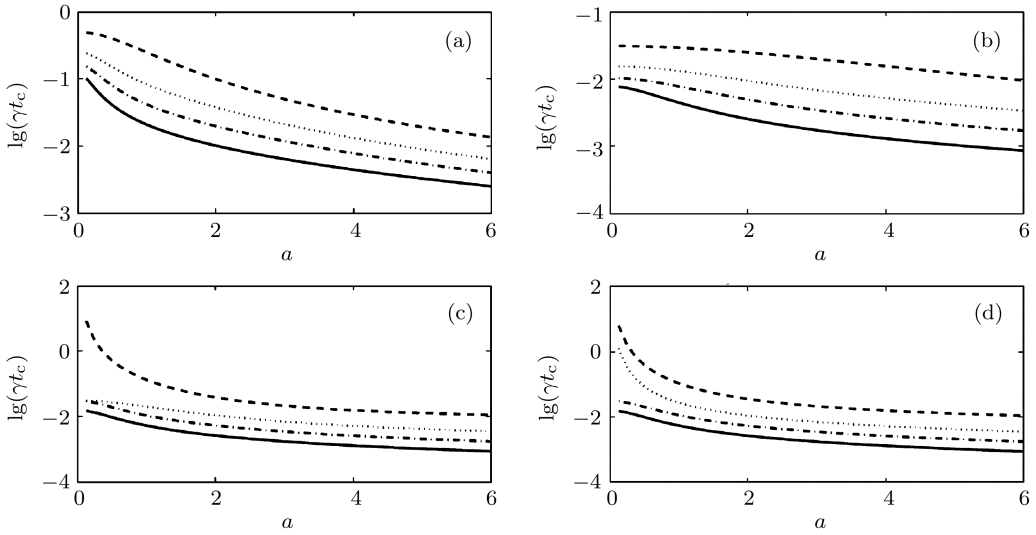


Fig. 2. The decoherence times of elliptical states: $N = 4$ (dashed line), $N = 8$ (dotted line), $N = 12$ (dot-dashed line) and $N = 24$ (solid line). (a) $\phi_0 = 0.5\pi, b = 1$; (b) $\phi_0 = 0.5\pi, b = 4$; (c) $\phi_0 = 0.25\pi, b = 4$; (d) $\phi_0 = \pi/3, b = 4$.

3. The evolution of the Wigner function

A well-known feature of "nonclassical" states is the existence of negative values of the Wigner quasidistribution functions. To obtain the conditions for the disappearance of negative values for circular states and elliptical states as the initial state, we can find the time-dependence of the Wigner function. Considering the density matrix of the elliptical states Eq. (13) after losses, we obtain

$$\begin{aligned} & W(\beta,t) \\ &= \frac{2}{\pi}|\lambda_N|^2 \sum_{j,k=1}^{N} C_j^* C_k \exp\left(-\frac{1}{2}r_j^2 - \frac{1}{2}r_k^2 + \alpha_j^*\alpha_k\right) \\ &\times \exp\{-2[\beta - \alpha_k \exp(-\gamma t/2)] \\ &\times [\beta^* - \alpha_j^* \exp(-\gamma t/2)]\}. \end{aligned} \tag{15}$$

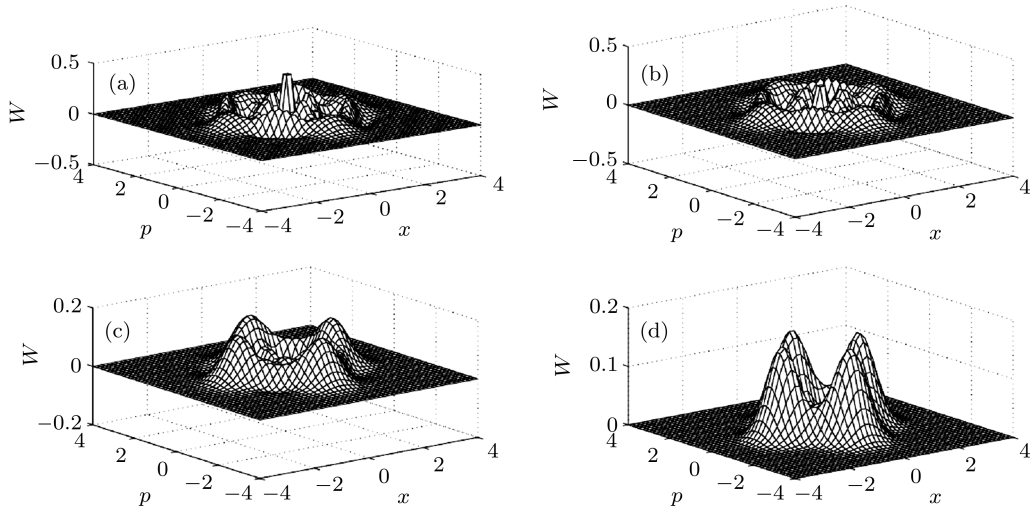


Fig. 3. Wigner function of the circular states for $N = 4$, $r = 2$, $\phi_0 = 0.5\pi$ and for different times. (a) $\gamma t = 0.01$, (b) $\gamma t = 0.1$, (c) $\gamma t = 0.4$, (d) $\gamma t = 0.8$.

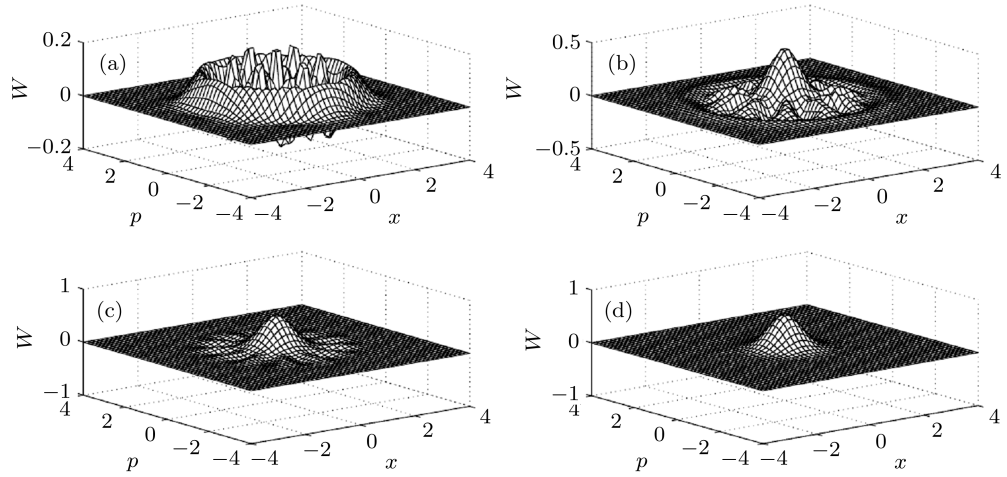


Fig. 4. The evolution of the Wigner function of the circular states for $\gamma t = 0.1$, $r = 2$, $\phi_0 = 0.5\pi$. (a) $N = 6$, (b) $N = 10$, (c) $N = 12$, (d) $N = 16$.

In Figs. 3–6 we show the evolution of the Wigner function of elliptical states and circular states for different time scales and different numbers of superposed coherent states, where x, p are the real and imaginary parts of β. One can easily see how the negative region of the Wigner function gradually diminishes with the increasing time. The more coherent superposition states there are, the faster the evolution is. For $N = 4$ of the circular states, the interference fringes almost disappear when $\gamma t = 0.8$ in Fig. 3(d). With increasing

N and $\gamma t = 0.1$, the effects of interference of the superposed states become increasingly less obvious. When $N = 16$, the Winger function changes into a Gaussian wave packet similar to that of the vacuum. However, when we change the amplitude into elliptical states, we can see that there are three peaks when $N = 4$ and $\gamma t = 0.8$ in Fig. 5 (d), unlike the four distinct peaks of the circular states. In particular, for elliptical states, the Wigner function when $N = 24$ is not like that of circular states evolving into the vacuum state when $N = 16$ but has the slowest decay along the direction of the semi-major axis. Therefore, quantum interference disappears more slowly than in circular states. From this discussion we can conclude that elliptical states are more resilient to losses and are more easily observed than circular states.

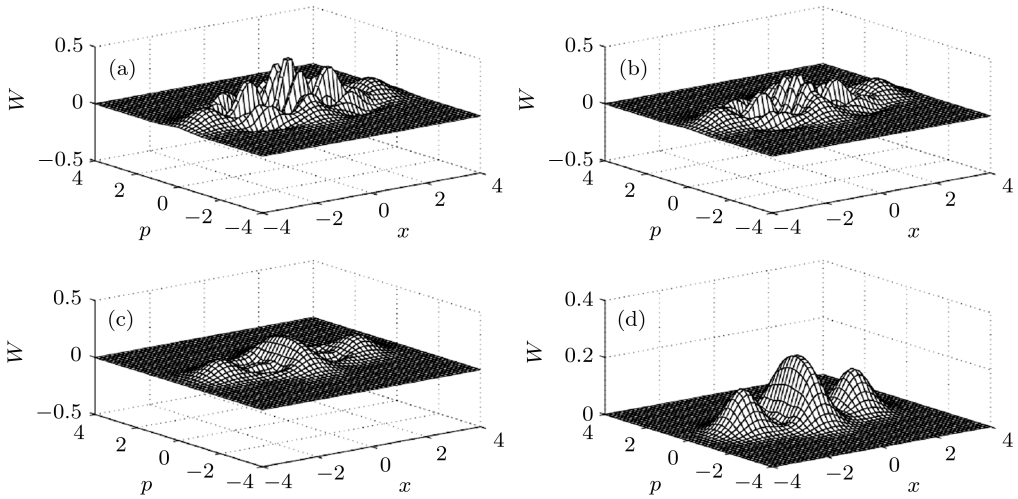


Fig. 5. Wigner function of the EESs for $N = 4$, $a = 3$, $b = 1$, $\phi_0 = 0.5\pi$ and at different times. (a) $\gamma t = 0.01$, (b) $\gamma t = 0.1$, (c) $\gamma t = 0.4$, (d) $\gamma t = 0.8$.

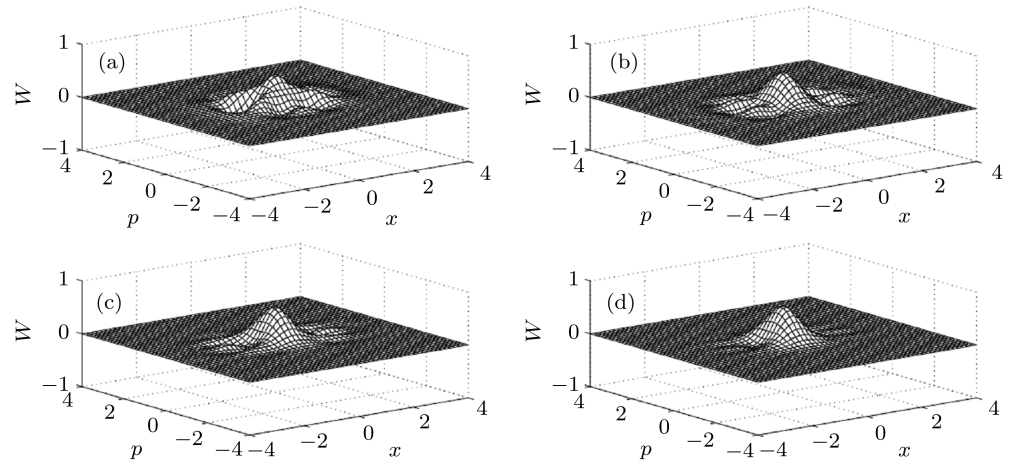


Fig. 6. The evolution of Wigner function of the EESs for $\gamma t = 0.1$, $a = 3$, $b = 1$, $\phi_0 = 0.5\pi$. (a) $N = 6$, (b) $N = 10$, (c) $N = 16$, (d) $N = 24$.

After a sufficiently long time, the interference fringes disappear while the Gaussians associated with each of the coherent state components of the original superposition remain distinct, although after a very long time these superposed coherent states merge to form the Wigner function of the vacuum.

4. Conclusion

In summary, we have investigated how elliptical and circular states evolve in the presence of decoherence. By solving the master equation of this system embedded in an external environment, the characteristic times of decoherence are determined. In the case

of amplitude decay, the numerical computation results show that elliptical states clearly exhibit the different features of quantum interference and are distinct from circular states. The nonclassical indicator, i.e., the negativity of the Wigner function of the superposition state, shows unique quality in their dependence on the amplitude of α, ϕ_0 and the number of the microscopic elements the states. The Wigner function loses its non-Gaussian nature and becomes Gaussian, corresponding to vacuum, for a very long time. A close comparison with circular states allows us to identify differences and analogies. Our results show that the decoherence of elliptical states is slower than of circular states and those elliptical states along the direction of the semi-major axis are the most easy to observe. Making these states is potentially significant in several areas of quantum physics, such as experimental studies of decoherence, quantum computing and cryptography.

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