Parametric analysis of the planar metamaterials based on complementary double-ring resonators*

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The planar metamaterials comprising complementary double-ring resonators (CDRRs) show its left handed behaviour. As a consequent work, this paper presents a detailed parametric study on the magnetically resonant transmission characteristics of the complementary double-ring metamaterials based on its structural parameters. This will be useful for the design of compact planar metamaterials based on the transmission lines loaded with CDRRs.

Keywords: metamaterial, complementary, left handed, transmission line

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1. Introduction

Recently, left-handed materials (LHM) have attracted growing attention which was inspired by the experimental demonstration of left handedness.[1] In such an artificial materials, the phase of the electromagnetic (EM) waves would propagate in a direction opposite to that of the EM energy flow, this fact exhibits many peculiar effects, including negative refraction, reversed Doppler shift and Cherenkov radiation. However, these effects could not be experimentally verified since, as Veselago pointed out, substances with negative permeability were not available at that time.[2] It was very recently that various metallic artificial structures, such as split ring resonators (SRRs) or double SRRs (DSRRs),[3,4] split ring chains,[5] cut-wire pairs,[6] and double-ring resonators (DRRs)[7,8] have been proposed in pursuing the magnetic response above gigahertz frequencies. Thanks to the magnetic response in artificial materials, significant progresses have been highlighted in the past decade such as the negative refraction,[9] superlens,[10] and cloaking effect.[11,12]

The original artificial LHMs was realized by the periodical arrangement of metal wires and SRRs.[3] The array of metal wires exhibits a resonant frequency response similar to that of a plasma medium which is responsible for the negative permittivity. The array of SRRs serves as magnetic dipoles, with a resonant response resulting from internal inductance and capacitance which are responsible for the negative permeability. Based on the Babinet principle and the duality concept, it was demonstrated that the negative permittivity can be achieved by the complementary screen of SRRs.[13] Complementary SRRs (CSRRs) behave as electrical dipoles that can be excited by axial electric field and exhibit negative permittivity upon their resonance. Further investigation reveals that by adding series capacitive gaps in the transmission line above the positions of CSRRs,[14] a negative effective permittivity and permeability simultaneously arise and the structure exhibits left-handed wave propagation behaviour. It should be noted that DRRs, which has no splits in both of the double rings and without the assistance of metallic wires, can generate a negative index of refraction (NIR).[7] These simple structures are very attractive not only for its convenience in fabrication but also for its isotropic electromagnetic response in the ring plane. In particular, the antisymmetric resonance modes associated with the antiparallel induced currents in the neighbouring edges of concentric double rings were suggested to contribute to the negative effective permeability.

In our recent work, complementary DRRs (CDRRs), as the dual part of DRRs, coupled with the planar transmission line, was investigated to show

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left-handed behaviour without any split introduced in either the etched rings or transmission lines.[15] The transmission lines loaded with CDRRs are very attractive for its simplicity in fabrication. It is the objective of this paper, as a consequent work, to explore the optimized response of the CDRR-loaded transmission line. The systematic numerical results on the magnetic resonance transmission of the planar metamatrials based on CDRRs is presented to study the geometric dependence, including the length, substrate and periodicity, etc. This will be very useful for optimization design of the CDRRs based planar metamaterials such as filters and other microwave devices.

2. Simulation details

There are many different parameters which affect the transmission property of the CDRRs-loaded transmission line, the parameters of the substrate and the structural parameters of the unit cell are most dominant. In the following sections, we show the effect of some of the parameters involved in the design which affect the magnetic resonance property and which are practically easy to vary. Figure 1 shows schematically the unit cell of the planar metamaterials comprising CDRRs. It contains a substrate with a copper strip on one side and a copper ground plane on the other side. Complementary double rings are etched in the ground plane. For the purpose of parametric optimization, all the geometric scales are represented by variables, i.e., dielectric constant ϵ of the substance, thickness of the substance h, width of the substance L_y, metallic thickness t, length of edge a, gap g, widths of the outer-ring edge w_1 and inner-ring edge w_2 and width of the transmission line w. In the parametric simulations, the initial values of various variables are as follows: $\epsilon = 10.2$, $h = 1$ mm, $t = 0.018$ mm.

This study is intended to aid us in the design of a performance enhanced microwave device based on planar metamaterials comprising CDRRs. All simulations presented in this paper are performed by using the commercial software CST Microwave Studio based on the finite integration technique. In our full-wave simulations, the simulation configuration has a dimension of $1 \times 1 \times 5$ units (i.e., one unit in the transverse yz-plane and five units in the electromagnetic propagation direction along the x-axis). The electromagnetic waves propagate along x direction and the polarized incident waves with electric field in the z direction and magnetic field in the y direction are satisfied by using transmission line structure.

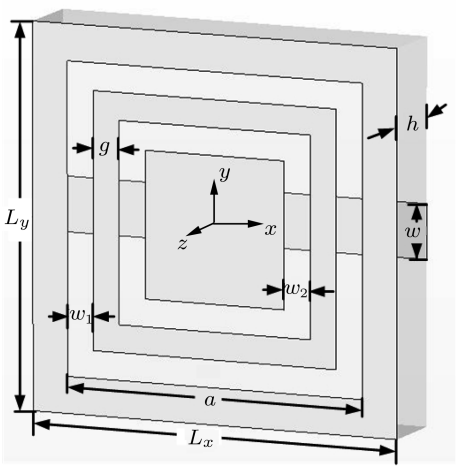


Fig. 1. Unit cell of the planar metamaterial comprising CDRRs.

3. Results and discussion

In order to obtain a better response of the magnetically resonant transmission such as its bandwidth, magnitude and quality factor Q, optimization of the complementary-double-ring metamaterial is needed. We study the dependence of the magnetically resonance frequency on the parameters of the CDDRs-based transmission line. In our detailed simulations, four sets of parameters are divided: the first set is the scales of the etched double rings (a, w_1, w_2, g), the second set is the unit cell sizes (L_x, L_y), the third set is the parameters of the substrate (ϵ, h) and the last set includes metallic thickness t and width of the transmission line w. From the detailed study of all these variables, we could understand the influence of geometrical factors on the magnetic resonance and thus obtain an optimization of the magnetic resonance of the planar metamaterials comprising CDRRs.

The relation between the resonant transmission and the geometric configuration of the complementary double rings could be interpreted by an equivalent LC circuit model. According to Ref. [8], L is proportional to the current circulating area, while C is proportional to the charge accumulation area. As we mentioned in Ref. [15], the antisymmetric current mode is concentrated in the top and bottom sides of the CDRRs, L is determined by the current circulating area, proportional to $2(a - 2w_1)$ and reversely proportional to g,t [$\sim (2(a - 2w_1)/gt)$], while C is determined by the charge accumulation area which is mainly concentrated in the left and right sides,[15] proportional to $[(a - 2w_1)t]$ and reversely proportional to w_1 [$\sim ((a - 2w_1)t/w_1)$]. Note that the total effective inductance L_{eff} equals $L/2$ because the top and

bottom inductors are in parallel, while the total effective capacitance $C_{\rm eff}$ equals $C/2$ because the left and right capacitors are in series. From the above analysis, we can estimate the inductance, capacitance and magnetic resonance frequency as

$$L \approx \mu_0\mu 2(a-2w_1)/gt, \tag{1}$$

and

$$C \approx \varepsilon_0\varepsilon(a-2w_1)t/w_1. \tag{2}$$

Consequently,

$$f_0 = \frac{1}{2\pi\sqrt{L_{\rm eff}C_{\rm eff}}} \approx \frac{\sqrt{gw_1}}{\pi(a-2w_1)\sqrt{2\mu_0\mu\varepsilon_0\varepsilon}}. \tag{3}$$

Figure 2 shows the transmission spectrum of the planar transmission line loaded with five CDRRs which has been confirmed in our previous work. It has been revealed that a left-handed transmission band occurred within an otherwise stop band if magnetic resonance is induced in this regime. Here, we mainly investigate the result of the resonance frequency as a function of the edge length a. As shown in Fig. 2, the magnetic resonance frequency decreases with the increase in the edge length a and a further analysis indicates that f_0 is roughly proportional to $1/a$, which is obviously shown in formula (3) (consider the approximation, $w_1 \ll a$).

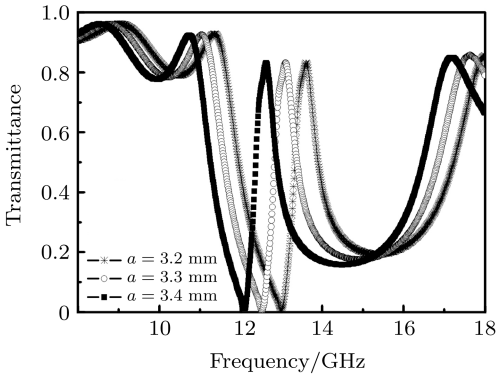


Fig. 2. Transmission spectra of the complementary-double-ring metamaterial with different lengths of edge a.

Next, we explore the relation between the resonance frequency and the edge width of the etched rings. As formula (3) indicates that f_0 is proportional to w_1 and does not show obvious dependence on w_2. The dependence of the resonance frequency on the width of the outer etched ring w_1 and the width of the inner etched ring w_2 are shown in Figs. 3 and 4, respectively. The results confirm that the resonance frequency is rather sensitive (proportional) to w_1 in the outside range than that of the inner one (w_2). Physically, the resonant electric field is concentrated mainly around w_1 but not the inner etched ring w_2.[15] Therefore, area w_1 contributes much more to the charge accumulation than that of w_2.

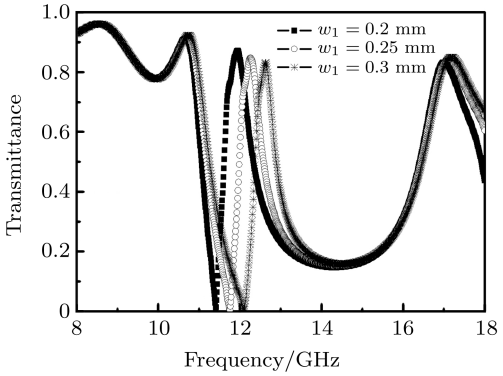


Fig. 3. Magnetic resonance frequency as a function of the width of outer etched ring w_1.

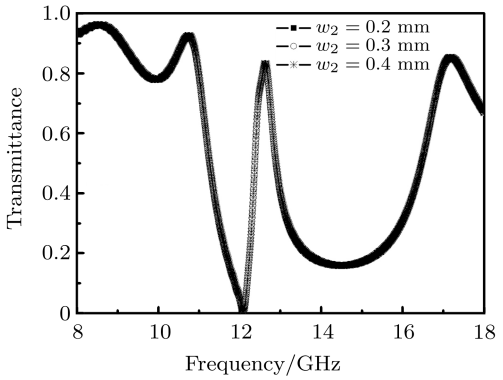


Fig. 4. Magnetic resonance frequency as a function of the width of inner etched ring w_2.

The gap between the outer and inner etched rings g is another important structural parameter. Figures 5 and 6 show the dependence of resonance frequency on gap g. It is worth mentioning that in Fig. 5, the changes in gap g are designed by altering the edge length of the inner etched ring without altering the outer etched ring. However, Figure 6 shows the alternative way to change gap g, which is altering the edge length of the outer ring while keeping the scale of the inner etched ring unchanged. From Fig. 5, it can be revealed that the transmission peak moves to a higher frequency when the gap g between the inner and outer rings increases. These increases can be explained by the effective inductance L, larger g leads to the decrease of L (see formula (1)) and thus a higher f_0. Figure 6 illustrates the different relation between the resonance frequency and gap g. The discrepant dependence of resonance frequency on gap g between these two schemes is due to the fact that the latter has an additional modification of the edge

length a, which is inversely proportional to the resonance frequency (see Fig. 2). Further investigation indicates that the magnitude of the resonant transmission increases with the increase of gap g in both of the two schemes. This implies that large gap should be preferable to reach sufficient magnitude of the resonant transmission peak. The simulation results are consistent with the estimation mentioned above.

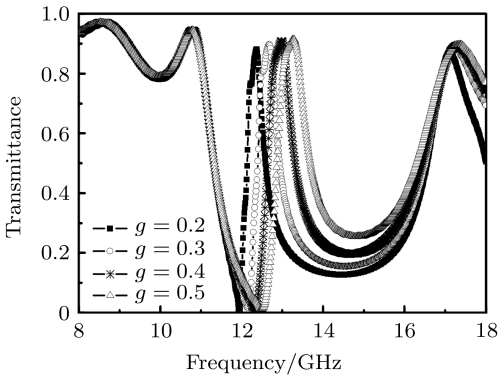


Fig. 5. Transmission spectra for different gap g by changing the inner etched ring.

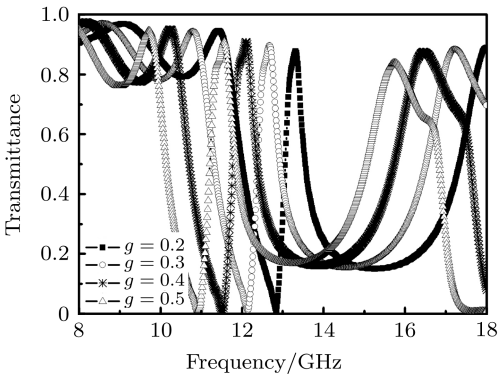


Fig. 6. Transmission spectra for different gap g by changing the outer etched ring.

In contrast to the parameters mentioned above, there is another set of parameters (L_x, L_y), which describe the size of the unit cell. Figure 7 shows smaller L_x results in higher resonance frequency, higher magnitude and wider bandwidth of the resonant transmission peak because smaller L_x cause stronger electric field coupling between CDRRs. Figure 8 reveals that L_y does not influence the transmission spectrum of the left handed transmission line which is reasonable that only the electromagnetic fields around the CDRRs mainly contribute the resonance. Hence, it can be concluded that only L_x can change the magnetic resonance frequency.

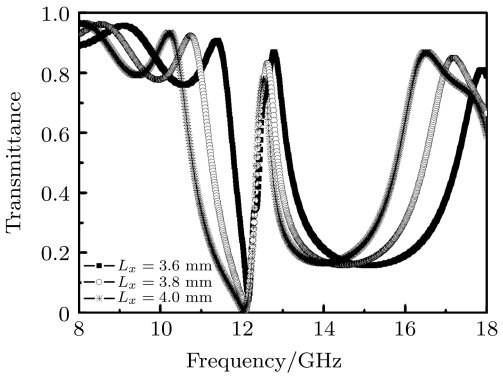


Fig. 7. Transmission spectra of the planar metamaterial comprising CDRRs with different lengthes of the unit cell L_x.

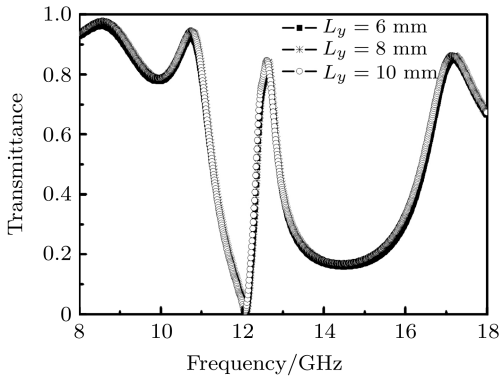


Fig. 8. Magnetic resonance frequency as a function of the width of the dielectric substrate.

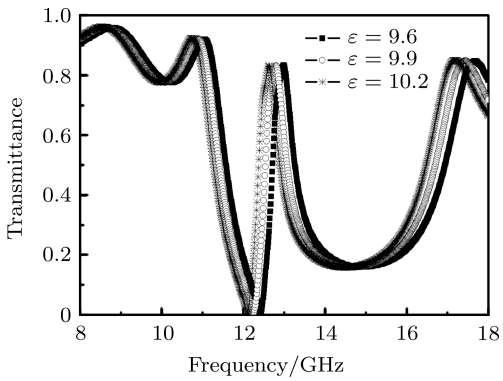


Fig. 9. Transmission spectra of the planar metamaterial comprising CDRRs with different dielectric substrate.

Another set contains the parameters of the substrate (ε, h). From Fig. 9, we can find that the magnetic resonance frequency decreases with the increase in the permittivity ε which can also be estimated in formulae (2) and (3), larger ε leads to larger C and thus lower f_0. Figure 10 tells us that the resonance frequency does not depend on h, however, both the magnitude and bandwidth of the resonant transmission peak increase with the increase in h.

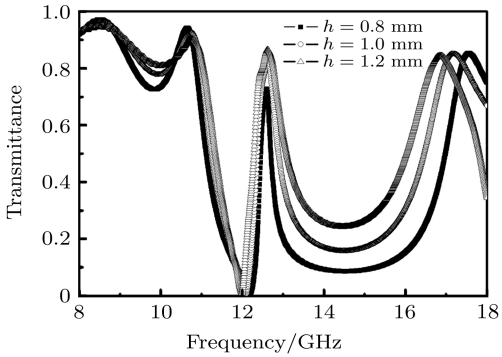


Fig. 10. Magnetic resonance frequency as a function of the thickness of the dielectric substrate.

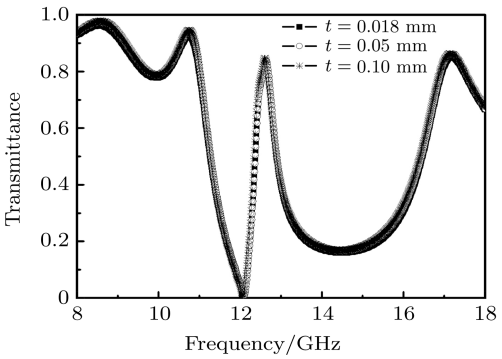


Fig. 11. Magnetic resonance frequency as a function of the metallic thickness t.

The last two parameters we considered are the thickness (t) of the etched ring and width (w) of the transmission line. The dependence of the resonance frequency on the two parameters are depicted in Figs. 11 and 12 respectively. In our simulation, the thickness in a larger range does not obviously change the magnetic resonance frequency (see formula (3)). However, a further investigation indicates that the magnitude increases slightly with the increase in the metallic thickness t. It implies that a bit of a thicker metallic layer is better for obtaining sufficient resonant transmission besides its influence on the resonance frequency. The transmission line here is to provide the transverse electromagnetic mode electromagnetic field to satisfy the polarization, so we can see from Fig. 12 that it does not show obvious dependence on w. However, the width w affects the impedance matching of the transmission line.

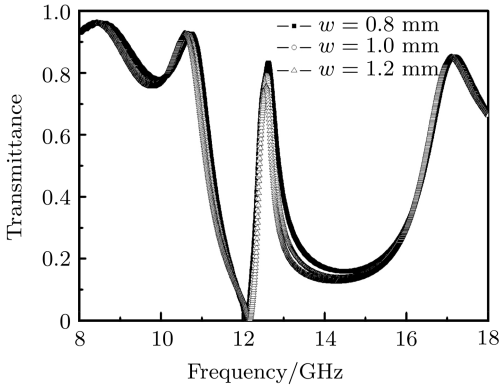


Fig. 12. Magnetic resonance frequency as a function of the width of the transmission line.

4. Conclusions

In conclusion, we have investigated the geometric dependence of the resonant transmission by systematic parametric simulations. The relationship between the parameters and the resonant response provides a guide to design the compact planar metamaterials based on the transmission lines loaded with CDRRs. Since the microstrip lines are widely used in microwave active circuit, this work can be of interest if this new kind transmission line is applied in the future.

References

[1] Shelby R A , Smith D R and Schultz S 2001 *Science* **292** 5514

[2] Veselago V G 1968 *Sov. Phys. Usp.* **10**

[3] Pendry J B, Holden A J, Robbins D J and Stewart W J 1999 *IEEE Trans. Microwave Theory Tech.* **47** 2075

[4] Yen T J, Padilla W J, Fang N, Vier D C, Smith D R, Pendry J B, Basov D N and Zhang X 2004 *Science* **303** 1494

[5] Liu H, Genov D A, Wu D M, Liu Y M, Steele J M, Sun C, Zhu S N and Zhang X 2006 *Phys. Rev. Lett.* **97** 243902

[6] Shalaev V M, Cai W, Chettiar U K, Yuan H K, Sarychev A K, Drachev V P and Kildishev A V 2005 *Opt. Lett.* **30** 3356

[7] Dong Z G, Xu M X, Lei S Y, Liu H, Li T, Wang F M and Zhu S N 2008 *Appl. Phys. Lett.* **92** 064101

[8] Dong Z G, Xu M X, Liu H, Li T and Zhu S N 2009 *J. Appl. Phys.* **105** 034907

[9] Zhang X and Liu Z 2008 *Nature Mater.* **7** 435

[10] Smith D R, Pendry J B and Wiltshire M C K 2004 *Science* **305** 788

[11] Pendry J B, Schurig D and Smith D R 2006 *Science* **312** 1780

[12] Liu R P, Ji C, Mock J J, Chin J Y, Cui T J and Smith D R 2009 *Science* **323** 5912

[13] Falcone F, Lopetegi T, Baena J B, Marqués R, Martín F and Sorolla M 2004 *IEEE Microw. Wirel. Compon. Lett.* **14** 280

[14] Falcone F, Lopetegi T, Laso M A G, Baena J D, Bonache J, Marqués R, Martín F and Sorolla M 2004 *Phys. Rev. Lett.* **93** 197401

[15] Lu W B, Ji Z F, Dong Z G, Ping X W and Cui T J 2010 *J. Appl. Phys.* **108** 033717