Persistent excitation in adaptive parameter identification of uncertain chaotic system*

Zhao Jun-Chan(赵军产)[a)†], Zhang Qun-Jiao(张群娇)[a)], and Lu Jun-An(陆君安)[b)]

[a)] College of Science, Wuhan Textile University, Wuhan 430073, China

[b)] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China

(Received 28 September 2010; revised manuscript received 13 December 2010)

This paper studies the parameter identification problem of chaotic systems. Adaptive identification laws are proposed to estimate the parameters of uncertain chaotic systems. It proves that the asymptotical identification is ensured by a persistently exciting condition. Additionally, the method can be applied to identify the uncertain parameters with any number. Numerical simulations are given to validate the theoretical analysis.

Keywords: parameter identification, adaptive control, persistent excitation, chaotic system

PACS: 05.45.Gg **DOI:** 10.1088/1674-1056/20/5/050507

1. Introduction

Over the last few decades, chaotic systems have arisen worldwide concerns. They have been widely explored in many areas such as secure communication, information processing and neural network. So far, a large variety of approaches have been proposed for chaotic control and synchronization, such as the OGY method, Pecora and Carrol method, adaptive method, time delay feedback approach and backstepping method.[1−10]

Recently, a great many results have been reported about parameter identification for chaotic systems. Parlitz[11] and Yu *et al.*[12] estimated the model parameters from time series. Lü and Zhang,[13] and Hu *et al.*[14] identified unknown parameters by backstepping method. By using adaptive control, Li and Han,[15] and Huang *et al.*[16] proposed a novel method to track the uncertain parameters.[15−18] In this paper, we study the parameter identification of uncertain chaotic systems by adaptive scheme. Particularly, we derive a critical persistent excitation condition. It is a very important condition in the process of parameter identification. It guarantees that the parameters will be estimated successfully under this condition. Additionally, there is no limitation for the number of uncertain parameters.

2. Parameters identification scheme

In this section, we will discuss the parameter identification of the uncertain chaotic system. A persistently exciting condition will be pointed out, which is essential for ensuring parameter identification of general dynamical systems.

Consider an uncertain chaotic system with state equation in the form

$$\dot{x} = F(x) = f(x) + g(x)\alpha, \tag{1}$$

where $x \in R^n$, $F(x) \in R^n$ and α is an unknown parameter vector, rewrite Eq. (1) as

$$\begin{cases} \dot{x}_i = f_i(x) + g_i(x)\alpha_i, \ i \in D, \\ \dot{x}_j = f_j(x), \qquad\qquad j \in \{1, 2, \ldots, n\} - D, \end{cases} \tag{2}$$

where $x_i \in R$ is the component of state vector, $D \subseteq N$ and the corresponding equation includes uncertain parameter, $f_i \in R$, $g_i^{\mathrm{T}} \in R^q$ and $\alpha_i \in R^q$ is an unknown parameter vector.

Assume that the function $F(x)$ satisfies the uniform Lipschitz condition, i.e., there exists a nonnegative function $\varphi(x)$ such that

$$(x - y)^{\mathrm{T}}(F(x) - F(y)) \leq \varphi(x)\|x - y\|^2 \tag{3}$$

for all $x, y \in R^n$ and $t \in R$.

*Project supported in part by National Natural Science Foundation of China (Grant Nos. 11047114 and 60974081) and in part by the Key Project of Chinese Ministry of Education (Grant No. 210141).

†Corresponding author. E-mail: junchanzhao@163.com

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Note that inequality (3) is very mild. For example, all linear and piecewise linear functions satisfy this condition. In addition, if the solution of system is bounded for any initial condition, the above condition is satisfied. So, it includes many well-known chaotic systems, such as the Lorenz system,[19] Chen system,[20] Lü system,[21] Chua's circuit,[22] and so on. However, if the system contains a nonlinear term, it does not satisfy the Lipschitz condition. The reason is that the bound of $\varphi(x)$ is related to the initial condition, so we could not find a constant L to replace $\varphi(x)$.

In order to state our main results, the following definition and lemma are needed:

Persistently exciting condition[23,24] (PE condition) A function $w : R^+ \to R^n$ is persistently exciting if there exist $T, \delta_1, \delta_2 > 0$ such that

$$\delta_1 \boldsymbol{I}_n \leq \int_t^{t+T} w(\tau) w^{\mathrm{T}}(\tau) \mathrm{d}\tau \leq \delta_2 \boldsymbol{I}_n \tag{4}$$

holds for all $t \geq 0$, where $\boldsymbol{I}_n$ is the identity matrix of order n.

Lemma 1[23,24] Given a system of the following form:

$$\begin{cases} \dot{e}_1 = g(t)e_2 + f_1(t), \quad e_1 \in R^p, \\ \dot{e}_2 = f_2(t), \end{cases} \tag{5}$$

such that

(i) $\lim_{t\to\infty} \|e_1(t)\| = 0$, $\lim_{t\to\infty} \|f_1(t)\| = 0$, $\lim_{t\to\infty} \|f_2(t)\| = 0$,

(ii) $g(t), \dot{g}(t)$ are bounded and $g^{\mathrm{T}}(t)$ is persistently exciting;

then $\lim_{t\to\infty} \|e_2(t)\| = 0$.

To estimate the unknown parameters in α, we consider system (2) as a drive system. A response system is given by

$$\begin{cases} \dot{y}_i = f_i(y) + g_i(y)\tilde{\alpha}_i + u_i, \quad i \in D, \\ \dot{y}_j = f_j(y) + u_j, \quad j \in \{1, 2, \ldots, n\} - D, \end{cases} \tag{6}$$

where $\tilde{\alpha}_i \in R^q$ represents the estimate of unknown parameter vector $\boldsymbol{\alpha}_i$ and $u_i \in R$ is a controller function.

Define synchronization error as

$$e_i(t) = y_i(t) - x_i(t), \quad i = 1, 2, \ldots, n. \tag{7}$$

The parameter estimation updated algorithm and adaptive controller are chosen as

$$\begin{cases} \dot{\tilde{\alpha}}_i = -g_i(y)^{\mathrm{T}} e_i, \ i \in D, \\ u_i = -k_i e_i, \qquad \dot{k}_i = c_i e_i^2, \ i = 1, 2, \ldots, n. \end{cases} \tag{8}$$

Then the error equation is

$$\begin{cases} \dot{e}_i = f_i(y) + g_i(y)\tilde{\alpha}_i - f_i(x) - g_i(x)\alpha_i - k_i e_i, \\ \qquad i \in D \\ \dot{e}_j = f_j(y) - f_j(x) - k_j e_j, \\ \qquad j \in \{1, 2, \ldots, n\} - D. \end{cases} \tag{9}$$

In what follows, it will be shown that if (i) $f(x)$ satisfies Assumption (3) and $\varphi(x)$ is bounded for any initial condition and (ii) f_i, g_i are bounded for i, then the respond system (6) can synchronize the drive system (2) with updated laws Eq. (8). Additionally, if $g_i(y)$ is persistently exciting, and $\dot{g}_i(y)$ is bounded, then $\lim_{t\to\infty}(\tilde{\alpha}_i - \alpha_i) = 0$ for $i \in D$.

Consider the following Lyapunov functional candidate:

$$V = \frac{1}{2}\sum_{i=1}^{n}\left(e_i^2 + \frac{(k_i - k_i^*)^2}{c_i}\right) + \frac{1}{2}\sum_{i\in D}(\tilde{\alpha}_i - \alpha_i)^{\mathrm{T}}(\tilde{\alpha}_i - \alpha_i),$$

where k_i^* is a positive constant to be determined. Calculating the derivative of V along the trajectory of Eq. (9) and with updated laws Eq. (8), we obtain

$$\begin{aligned} \dot{V}|_{(9)} &= \sum_{i=1}^{n} e_i(f_i(y) - f_i(x) - k_i e_i) \\ &\quad + \sum_{i=1}^{n}(k_i - k_i^*)e_i^2 + \sum_{i\in D}(g_i(y)\tilde{\alpha}_i - g_i(x)\alpha_i \\ &\quad - (\tilde{\alpha}_i - \alpha_i)^{\mathrm{T}} g_i(y)^{\mathrm{T}})e_i \\ &= e^{\mathrm{T}}(f(y) + g(y)\alpha - f(x) - g(x)\alpha - k^* e) \\ &= e^{\mathrm{T}}(F(y) - F(x) - k^* e) \leq (\varphi(x) - k^*)\|e\|^2 \\ &\leq (L - k^*)\|e\|^2, \end{aligned}$$

where $e = (e_1, \ldots, e_n)^{\mathrm{T}}$, $|\varphi(x)| \leq L$. And $k^* = \min_{1\leq i\leq n}\{k_i^*\}$, choose $k^* = L+1$, then $\dot{V}|_{(9)} \leq -\|e\|^2$, which means that $\|e\|, \|\tilde{\alpha}_i - \alpha_i\|$ are bounded. For any t, we have

$$\begin{aligned} \int_0^t \|e\|^2 \mathrm{d}\tau &\leq -\int_0^t \dot{V}(\tau)\mathrm{d}\tau = V(0) - V(t) \\ &\leq V(0), \end{aligned} \tag{10}$$

then $e(t) \in L^2$.

Since $\dot{e}_i = f_i(y) + g_i(y)\alpha - f_i(x) - g_i(x)\alpha + g_i^{\mathrm{T}}(y)(\tilde{\alpha}_i - \alpha_i) - k_i e_i$, $\dot{e}_i$ is also bounded and thus, according to Barbalat's lemma, we have $\lim_{t\to\infty} |e_i(t)| = 0$.

Rewrite the equations for $e_i(t)$ and $(\tilde{\alpha}_i - \alpha_i)$ as

$$\begin{cases} \dot{e}_i = g(t)e_\alpha + f_1(t), \\ \dot{e}_\alpha = -g_i(y)^{\mathrm{T}} e_i, \end{cases} \tag{11}$$

where $f_1(t) = f_i(y) + g_i(y)\alpha_i - f_i(x) - g_i(x)\alpha_i - k_i e_i$, $e_\alpha = \tilde{\alpha}_i - \alpha_i$ and $\lim_{t\to\infty} f_1(t) = 0$. Since $g(y)$ is persistently exciting and $\dot{g}$ is bounded, by *lemma 1*, we can conclude that $\lim_{t\to\infty}(\tilde{\alpha}_i - \alpha_i) = 0$.

Note that $g(y)$ satisfies PE condition, because the respond system is still chaotic when it synchronizes to the drive system. Therefore, $y = 0$ is not a stable equilibrium point. It implies that $g(y)$ is persistently exciting. If $y = 0$ is a stable equilibrium point, parameter identification can not be identified successfully. We will give a detailed explanation via numerical results in the next section.

3. Numerical simulations

In this section, we consider the Lorenz system as an example to illustrate the effectiveness of our method, the system is described by

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1), \\ \dot{x}_2 = \rho x_1 - x_1 x_3 - x_2, \\ \dot{x}_3 = x_1 x_2 - \beta x_3, \end{cases} \tag{12}$$

where the parameters $\sigma = 10$, $\beta = 8/3$, $\rho = 28$. It is a 3-dimensional structure corresponding to the long-term behaviour of a chaotic flow, noted for its butterfly shape.[19] Suppose that only two parameters σ, ρ are unknown. Our purpose is to estimate these unknown parameters.

First, we prove that the system satisfies inequality (3),

$$\begin{aligned} &(y-x)^{\mathrm{T}}(F(y)-F(x)) \\ &= (e_1,e_2,e_3)^{\mathrm{T}}(\sigma(e_2-e_1), \rho e_1 - y_3 e_1 \\ &\quad - x_1 e_3 - e_2, y_2 e_1 + x_1 e_2 - \beta e_3) \\ &= -\sigma e_1^2 - e_2^2 - \beta e_3^2 + (\sigma+\rho) e_1 e_2 \\ &\quad - (e_3 + x_3) e_1 e_2 + (e_2 + x_2) e_1 e_3 \\ &= -\sigma e_1^2 - e_2^2 - \beta e_3^2 + (\sigma+\rho) e_1 e_2 - x_3 e_1 e_2 + x_2 e_1 e_3 \\ &\le \left(\frac{\sigma+\rho+|x_2|+|x_3|}{2} - \sigma\right) e_1^2 \\ &\quad + \left(\frac{\sigma+\rho+|x_3|}{2} - 1\right) e_2^2 + \left(\frac{|x_2|}{2} - \beta\right) e_3^2 \\ &\le \left\| \left[\left(\frac{\sigma+\rho+|x_2|+|x_3|}{2} - \sigma\right), \left(\frac{\sigma+\rho+|x_3|}{2} - 1\right), \right.\right. \\ &\quad \left.\left. \left(\frac{|x_2|}{2} - \beta\right) \right] \right\| \|[e_1, e_2, e_3]^{\mathrm{T}}\|, \end{aligned}$$

because x_1, x_2, x_3 are bounded, there exists such a nonnegative function $\varphi(x)$ that inequality (3) holds.

Set system (12) as a drive system and rewrite it as

$$\begin{cases} \dot{x}_1 = f_1(x) + g_1(x)\alpha_1, \\ \dot{x}_2 = f_2(x) + g_2\alpha_2, \\ \dot{x}_3 = f_3(x), \end{cases} \tag{13}$$

where

$$\begin{aligned} &f_1(x) = 0, \quad g_1(x) = x_2 - x_1; \\ &f_2(x) = -x_1 x_3 - x_2, \quad g_2(x) = x_1; \\ &f_3(x) = x_1 x_2 - \beta x_3. \end{aligned}$$

The controlled response system is as follows:

$$\begin{cases} \dot{y}_1 = f_1(y) + g_1(y)\tilde{\alpha}_1 + u_1, \\ \dot{y}_2 = f_2(y) + g_2\tilde{\alpha}_2 + u_2, \\ \dot{y}_3 = f_3(y) + u_3, \end{cases} \tag{14}$$

where

$$\begin{aligned} &f_1(y) = 0, \quad g_1(y) = y_2 - y_1; \\ &f_2(y) = -y_1 y_3 - y_2, \quad g_2(y) = y_1; \\ &f_3(y) = y_1 y_2 - \beta y_3, \end{aligned}$$

and adaptive laws are introduced as

$$\begin{cases} \dot{\tilde{\alpha}}_i = -g_i(y)^{\mathrm{T}} e_i, & i = 1,2, \\ u_i = -k_i e_i, \qquad \dot{k}_i = c_i e_i^2, & i = 1,2,3. \end{cases} \tag{15}$$

Then, the respond system (14) can synchronize the drive system (13), as shown in Fig. 1(a). Meanwhile, Fig. 1(b) shows that the evolutions of the variables $g_1 = y_2 - y_1$, $g_2 = y_1$. Define $W_i(t) = \int_t^{t+T_0} g_i(\tau) g_i^{\mathrm{T}}(\tau) \mathrm{d}\tau$. Figure 1(c) shows that $\lambda_{\min}(W_i(t)) > 0$, for all i, t and $T_0 = 1.5$, which indicates that g_i satisfies the PE condition. As a result, the parameters adapt themselves to the true value, as shown in Fig. 1(d).

Next, we will discuss the importance of the PE condition. If the parameters $\sigma = 5$, $\beta = 0$, $\rho = 1$, the scheme does not work. Because the system is stable at equilibrium $(0,0,0)$ under this parameters, when the response system is synchronous with the drive system, the response system will be stable at equilibrium $(0,0,0)$. Figure 2(c) shows that $\lambda_{\min}(W_i(t)) = 0$, which indicates that g_i does not satisfy the PE condition. Therefore, parameter identification can not be realized in this case. It can be easily seen in the numerical result in Fig. 2.

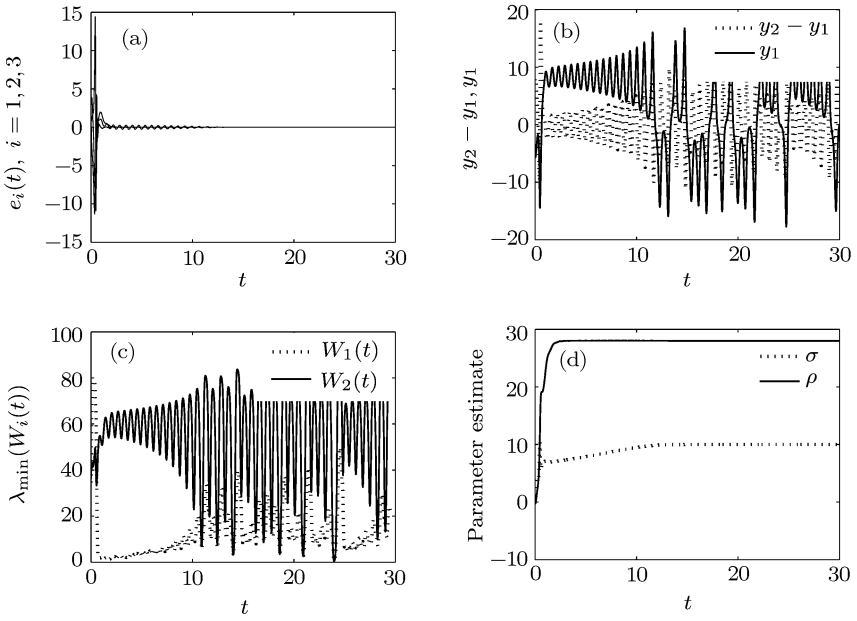


Fig. 1. (a) The synchronization errors between drive system and respond system. (b) The evolutions of the variables $g_1 = y_2 - y_1$, $g_2 = y_1$. (c) The minimum eigenvalue of the matrix $W_i(t)$. (d) Time evolutions of adaptive parameters.

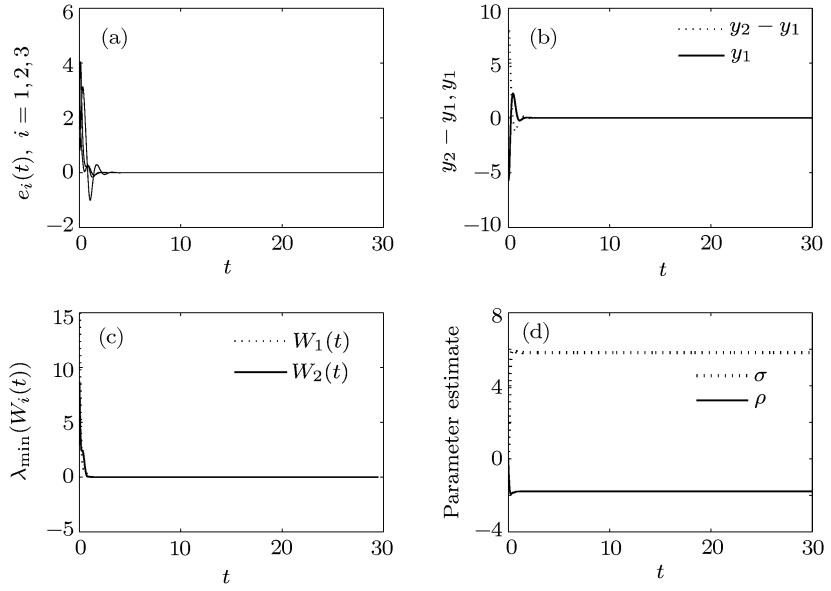


Fig. 2. (a) The synchronization errors between drive system and respond system. (b) The evolutions of the variables $g_1 = y_2 - y_1$, $g_2 = y_1$. (c) The minimum eigenvalue of the matrix $W_i(t)$. (d) Time evolutions of adaptive parameters.

4. Conclusion

In this paper, we have proposed an adaptive feedback controlling method to identify the uncertain parameters of chaotic systems. By using the Lyapunov direct method, we derive sufficient conditions for the uncertain chaotic systems to achieve parameter identification. In comparison with the resulted methods, we point out an important condition to track the estimated parameters successfully. Meanwhile, there is no limitation for the number of uncertain parameters. Finally, the illustrative simulations have been provided to verify the effectiveness of the proposed scheme. In the future, we will consider the parameter identification with noise.

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