Effects of quantization on detrended fluctuation analysis*

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Detrended fluctuation analysis (DFA) is a method foro estimating the long-range power-law correlation exponent in noisy signals. It has been used successfully in many different fields, especially in the research of physiological signals. As an inherent part of these studies, quantization of continuous signals is inevitable. In addition, coarse-graining, to transfer original signals into symbol series in symbolic dynamic analysis, can also be considered as a quantization-like operation. Therefore, it is worth considering whether the quantization of signal has any effect on the result of DFA and if so, how large the effect will be. In this paper we study how the quantized degrees for three types of noise series (anti-correlated, uncorrelated and long-range power-law correlated signals) affect the results of DFA and find that their effects are completely different. The conclusion has an essential value in choosing the resolution of data acquisition instrument and in the processing of coarse-graining of signals.

Keywords: detrended fluctuation analysis, quantization

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1. Introduction

In recent years, the detrended fluctuation analysis (DFA) proposed by Peng *et al.*[1] has been widely used in research into the detection of long-range correlation and monofractal of nonstationarity time series. This method is much better than the normal spectral analysis and time or frequency domain analysis in that it can be used to detect the long-range correlations embedded in the seemingly nonstationary time series and also avoid the spurious detection of apparent long-range correlations that are an artifact of nonstationarity.[2] Thus, it has been successfully applied to many different fields, especially to the fields of the climate, econophysics and physiology, such as determining extreme events of climate,[3] cardiac dynamics,[4] analysis of neural signals,[5] research of DNA[1,6] and human gait,[7] heart rate variability and neuron spiking.[8]

Data acquisition is a crucial step in DFA study. In the acquiring data process, continuous analog-signals need to be quantized to discrete data on the amplitude. This can be seen as a continuous observation value range being divided into many intervals and the part of signals in each interval will be given an identical number. In addition, in the symbolic dynamic analysis,[9,10] all the data in a series are usually divided into groups according to their amplitudes in the same group represented by an identical symbol. In this way, the original series could be converted into a series consisting of a finite number of symbols; a process called coarse graining. No matter how nice the quantization or a quantization-like operation of coarse graining for signals is, some details in signals may be lost, or in other words, some noise will be produced at the same time.[11,12]

To correctly analyse a given system by DFA, it is important to know what effects on the DFA can be induced by the quantization of signals. In this paper, we investigate how differently the quantized levels for three types of noise signals (anti-correlated, uncorrelated and long-range powerlaw correlated signals) affect the results of DFA. We find that the scaling behaviours of quantized long-range power-law correlated signals are robust even in the case of large partition size, but the scaling behaviours of quantized anti-correlated signals are susceptible.

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2. Methods

2.1. DFA method

To illustrate the DFA method, we consider the series $x(i)$ $(i = 1, 2, \ldots, N_{\max})$.

(i) We integrate the series,

$$y(j) = \sum_{i=1}^{j} (x(i) - \langle x \rangle), \quad j = 1, 2, \ldots, N_{\max}, \tag{1}$$

where $\langle x \rangle$ is the mean of $x(i)$.

(ii) We detrend the integrated time series $y(i)$ by subtracting the local trend $y_{\text{fit}}(i)$ in each box and we calculate the detrended fluctuation function

$$Y(j) = y(i) - y_{\text{fit}}(i). \tag{2}$$

(iii) For a given box size n, we calculate the root mean square function of all the trended fluctuation function

$$F(n) = \sqrt{\frac{1}{N_{\max}} \sum_{i=1}^{N_{\max}} [Y(i)]^2}. \tag{3}$$

The above function is calculated by changing n value and then the relationship between $F(n)$ and n can be obtained. A power-law relation between $F(n)$ and the box size n indicates the following scaling:

$$F(n) \sim n^{\alpha}. \tag{4}$$

The α is called correlation exponent or scaling exponent, representing the correlation characteristics of the signal: $\alpha = 0.5$ means that it is uncorrelated signal (i.e. white noise signal); $\alpha < 0.5$ means that it is anti-correlated signal; $\alpha > 0.5$ means that it is a long-range power-law correlated signal.

2.2. Quantization analysis method

Quantization is the process of mapping a continuous range of values by a relatively finite set of integers. The most common quantization algorithm is the uniform quantization[11] and it can be simulated as follows: for the original time series $x(i)$ $(i = 1, 2, \ldots, N_{\max})$, the partition width is set first. In our study, we define it as

$$\Delta = K \cdot \sigma. \tag{5}$$

Here, the σ is the standard deviation of the series $x(i)$ and K is a real number greater than zero with the values of $0.1 - 9$.

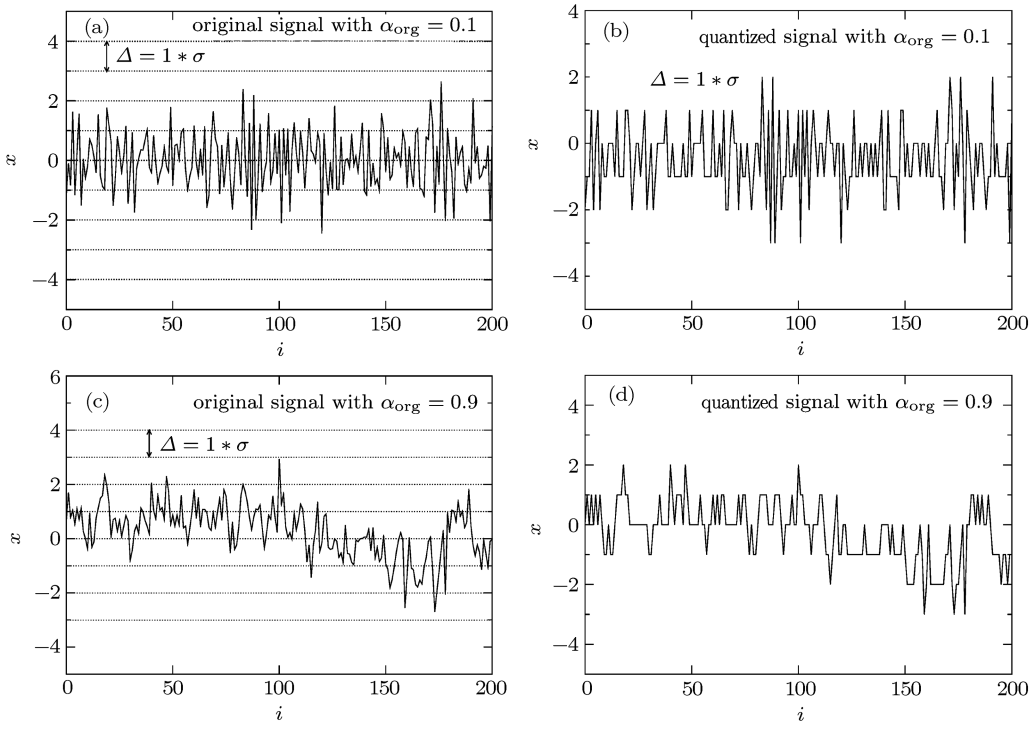


Fig. 1. Illustrations of quantization on time series. The length of series are 2^{17} and only the first two hundred data points of series are displayed here. Panel (a) is for anti-correlated original signal with scaling exponent $\alpha_{\text{org}} = 0.1$, panel (b) for quantized realization of (a) with partition width $\Delta = 1 * \sigma$, panel (c) for long-range power-law correlated original signal with scaling exponent $\alpha_{\text{org}} = 0.9$, and panel (d) for quantized realization of (c) with partition width is $\Delta = 1 * \sigma$.

We divide the series $x(i)$ according to the partition width Δ and give different integers to the data points in different ranges. Then the integer series $x_m(i)$ $i = 1, 2, \ldots, N_{\max}$, the quantized signal, can be expressed as

$$x_m = \text{floor}\left(\frac{x(i)}{\Delta}\right). \tag{6}$$

The floor is the function of getting the nearest integers towards minus infinity. That is,

$$\text{floor}(\theta) = m, \quad \text{if } m \le \theta < m+1, \quad m \in Z. \tag{7}$$

Using a modified Fourier filtering method,[13] we generate original signals with a given correlation property characterized by a given scaling exponent α_{org} and with length $N_{\max} = 2^{17}$ and standard deviation $\sigma = 1$. Figure 1 illustrates the two original time series (one is anti-correlated signal and the other is a long-range power-law correlated signal) and their quantization realizations.

3. Effects of quantization on DFA result

3.1. Effects of quantization on scaling exponent of anti-correlated signals

We generate an anti-correlated signal[13] as an original signal with the scaling exponent $\alpha_{\text{org}} = 0.1$, then quantize it using Eq. (6), where the quantization partition width is $\Delta = K * \sigma$ ($K = 0.1, 0.3, 0.5, 0.7, 0.9, 1.5, 2, 3, 4, 5, 6, 7, 9$).

The $\log - \log$ plot of $F(n) \sim n$ is shown in Fig. 2. It reveals that (1) the scaling property of anti-correlated signals is very sensitive to the quantization process, even if a small quantization partition width Δ can lead to a great change of α (slope of curve); (2) the α increases with the increase of quantization partition width Δ, so that the signals gradually lose the property of anti-correlation i.e., they gradually turn into the white noise ($\alpha = 0.5$); (3) not all the curves of $F(n) \sim n$ in Fig. 2 are exactly linear, which means different scaling ranges with different effects of quantization, or to be exact, the effects of quantization in a large scale range are greater than those of a small scale range; (4) the greater the quantization partition width Δ is, the more obvious the effect is and the greater the effect on scaling range will be.

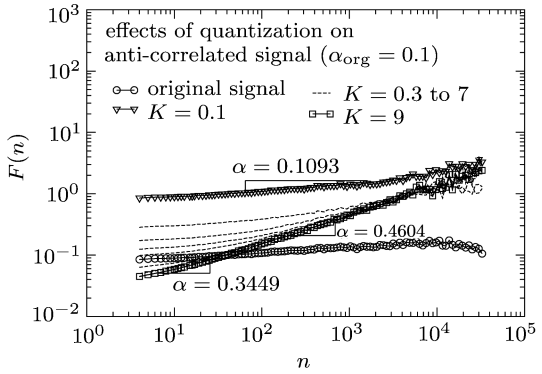


Fig. 2. Effects of quantization on scaling exponent of anti-correlated signal. We calculate the scaling exponents αs being of the original signal ($\circ$), of the quantized signal with quantization partition width $\Delta = 0.1 * \sigma$ ($\triangledown$), of the quantized signal with quantization partition width $\Delta = 9 * \sigma$ ($\square$), and of other quantized signals with different partition widths (dash line) separately. In the figure we find that the slopes of curves increase gradually with quantization partition width increasing and all curves are superposed greatly. The behaviour of a crossover point changes from strong anti-correlated to weak anti-correlated, which means that the effect of quantization in a large scale range is stronger than in a small scale range.

Quantization theory suggests that the quantization error derived from the quantization process has white noise property while the partition width $\Delta < 2$ and the white noise intensity increases with the increase of partition size Δ. The integer series(defined as Eq. (6)) can be considered as a superposition between the original series and the white noise series.[11] In Fig. 2, white noise, which is generated by quantifying an anti-correlated signal, mainly affects the correlation behaviour of original signal on a large scale. The greater the partition width Δ is, the greater the scaling range of effect due to the white noise is and the scale of effected range changes from large to small as the increase of partition width Δ.

Figure 3 illustrates the spectrum analyses of the anti-correlated signal ($\alpha_{\text{org}} = 0.1$) and its quantized signal ($\Delta = 1 * \sigma$). It shows that the spectrum of quantized anti-correlated signal in the low-frequency range with frequency less than 10^{-2} basically tends to plane like a spectrum of white noise, i.e., for anti-correlated signal, quantization has a greater effect in a low-frequency range. Contrarily, the spectrum of quantized anti-correlated signal in the high-frequency range with frequency larger than 10^{-1} almost coincides with that of the original signal. It indicates that quantization for anti-correlated signal has a lesser effect in a high-frequency range. In fact, for an anti-correlated signal its energy concentrates in the high-

frequency range. However, the energy of white noise is uniformly distributed in the whole frequency domain and thus the effect of the white noise generated by quantization on the spectrum is more obvious in the low-frequency range than in the high-frequency range.

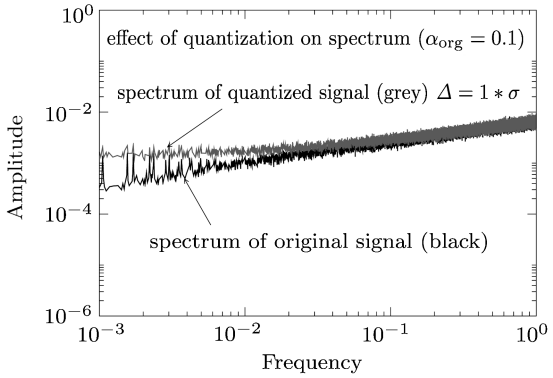


Fig. 3. Comparison between spectrum (FFT) obtained from anti-correlated original signal (black, $\alpha_{\rm org} = 0.1, N_{\max} = 2^{17}$) and its quantized signal (grey, $\Delta = 1*\sigma$). The spectrum comes from the average of 25 simulations of anti-correlated original signals with different seeds and their quantized signals. Most spectrum areas in high frequency of original signal are covered with the spectrum of quantized signal. Amplitude and frequency in the spectrum are normalized.

3.2. Effects of quantization on scaling exponent of long-range power-law correlated signals

We generate a long-range power-law correlated signal as the original series with the scaling exponent $\alpha = 0.9$, then quantize it using Eq. (6), where the quantization partition width is $\Delta = K * \sigma$ (K is the same as that in the case of Section 3.1.).

The $\log - \log$ plot of $F(n)$ versus n is shown in Fig. 4. It reveals that (i) the scaling property of correlated signals is very robust for the quantization process since only very large quantization partition width Δ ($K > 1$) can lead to a little change of α (slope of curve); (ii) the effects of quantization in a small scale range are greater than those in a large scale range.

In addition, it is noteworthy that DFA results no longer change with partition width Δ when quantization is over coarse. In fact, the quantized series have turned into a sign series (i.e. binary series with only 0, 1) in this condition. It is amazing that the sign series still basically retains the scaling property of the original signal.

Figure 5 illustrates the spectrum analyses of the long-range power-law correlatted signal ($\alpha_{\rm org} = 0.9$) and its quantized signal ($\Delta = 1 * \sigma$).

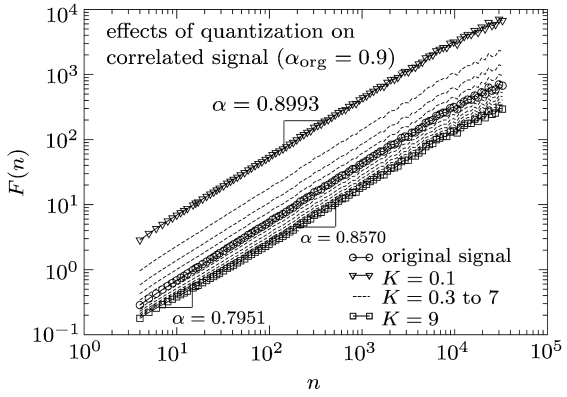


Fig. 4. Effects of quantization on the scaling exponent of long-range power-law correlated signals. We calculate the result of the scaling exponent of the original signal ($\circ$) as being of the quantized signal with quantization partition width $\Delta = 0.1*\sigma$ ($\bigtriangledown$), of the quantized signal with quantization partition width $\Delta = 9 * \sigma$ ($\square$), and other quantized signals with different partition widths (dashed line). In the figure we find that the slopes of curves decrease slightly with quantization partition width increasing. When the quantization partition width is larger ($\Delta > 1 * \sigma$), the slopes of curves in small scale($n = 4 - 30$) and large scale ($n = 30 - 10000$) have a little bit difference, i.e., there are crossover points in different ranges, but their positions do not vary significantly.

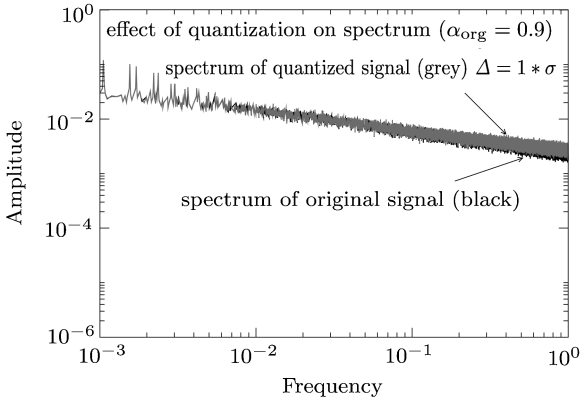


Fig. 5. Comparison between spectrum (FFT) obtained from long-range power-law correlated original signal (black, $\alpha_{\rm org} = 0.9, N_{\max} = 2^{17}$) and its quantized signal (grey, $\Delta = 1*\sigma$). The spectrum comes from the average of 25 simulations of long-range power-law correlated original signals with different seeds and their quantized signals. Almost whole spectrum areas of original signal are covered with spectrum of quantized signal. Amplitude and frequency in the spectrum are normalized.

Figure 5 shows the effect of quantization on long-range power-law correlated signals is negligible. Compared with the spectrum of the original signal, the spectrum of the quantized signal slightly increases only in a high frequency range (more than 10^{-1}), i.e.,

there is a weak effect only on a small scale. For long-range power-law correlated signal, the energy of signal concentrates in the low frequency range. However, the energy of white noise uniformly is distributed in the whole frequency domain and thus the effect of the white noise generated by quantization on the spectrum is obvious in the high-frequency range.

3.3. Effects of quantization on scaling behaviour of signals

We generate anti-correlated, uncorrelated and long-range power-law correlated signals[13] with scaling exponents $\alpha_{\rm org} = 0.1, 0.2, \ldots, 0.9$, then quantize the signals using Eq. (6) in the case of the varied quantization partition width $\Delta = K * \sigma$, $(K = 0.1 - 9)$. The DFA results of these signals are shown in Fig. 6.

Figure 6 shows that the effects of quantization on DFA results of noisy signals with different scaling exponents are totally different. For anti-correlated signals, in particular the scaling exponent of the original series is very small (such as $\alpha_{\rm org} \leq 0.2$) and the effects of quantization are very obvious. When the partition width reaches $\Delta = 1 * \sigma$, deviation of α from $\alpha_{\rm org}$ can be more than 300%. However, for long-range power-law correlated signals ($\alpha_{\rm org} > 0.5$), the effects of quantization are very slender. Even if the partition width reaches $\Delta = 1 * \sigma$, deviation of α from $\alpha_{\rm org}$ is still lower than 1%.

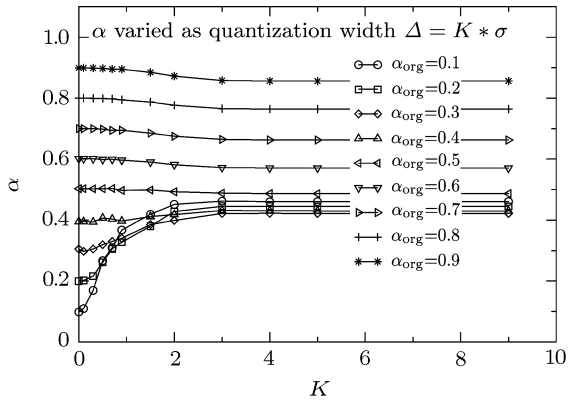


Fig. 6. Effects of quantization on DFA results of anti-correlated, uncorrelated and long-range power-law correlated signals. $\alpha_{\rm org}$s are the scaling exponents of original signals. Horizontal coordinate K is the parameters of quantization partition width ($\Delta = K * \sigma$). Each curve represents the variation of DFA result of quantized signal derived from original signal with $\alpha_{\rm org}$. The αs are obtained by linear regression in the scale range $n = 10 - 10^4$. The data points with $K = 0$ in all curves refer to the scaling exponents of original signals (not quantized signals).

The above results have an important guiding significance in selecting the precision of instruments used in data collection. Another significance is that the time series after over coarse ($K > 4$) will only be of two symbols (0, 1), i.e. sign series. For an anti-correlated signal, the sign series has lost the correlated information, almost turns into a random series. However, for a long-range power-law correlated noise, the sign series contains the main correlation information about the original series.

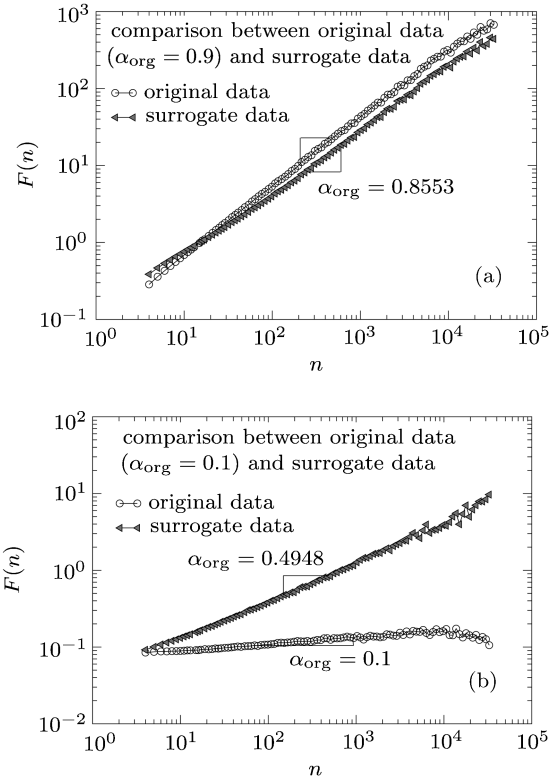


Fig. 7. Comparison between DFA results with different correlation property series and their surrogate data series. (a) Comparison between a long-range power-law correlated series $\alpha_{\rm org} = 0.9$ and its surrogate data series. The surrogate data series still has the strong correlated property. The difference of DFA results of the surrogate data series and the original series is only 5%. (b) Comparison between an anti-correlated series $\alpha_{\rm org} = 0.1$ and its surrogate data series. The surrogate data series almost tends to random and the difference between the DFA results of it and the original series is 395%. Surrogate data series are generated by: keeping the order of sign of the original data series, but random sorting the magnitude of data series. That is, the sign of ith surrogate datum point is the same as the ith original datum point, but the magnitude is reallocates with another datum point at random.

To further study whether the signs (not its magnitude) of data points in long-range power-law correlated series contain the main correlation information, we use a long-range power-law correlated series and an

anti-correlated series to generate their surrogate data series and compare DFA results of the originals and the their surrogate data series. The DFA results are shown in Figs. 7(a) and 7(b).

For long-range power-law correlated signals, the difference in the DFA result between the surrogate data series and original series is within 5%, i.e. their DFA results are essentially the same. This reveals that the signs of long-range power-law correlated signals play an important role in the correlation behaviour of the signals. Different results are obtained while a similar analysis is performed for an anti-correlated signal, shown in Fig. 7(b). The DFA result of the surrogate data series of the anti-correlated signal behaves as a random noise, which indicates that not only the sign but also the magnitude of the anti-correlation series contains important correlation information.

4. Conclusion

The effects of quantization on DFA results of long-range power-law correlated, uncorrelated and anti-correlated signals are totally different. For long-range power-law correlated signals, the effects of quantization on scaling behaviour are inappreciable. Only if the quantization is very rough, can the quantization make the scaling exponents slightly decline and the signals tend only a little to white noise. Moreover, the effects of quantization on a small scale range are greater than on a large scale range. When the quantization partition width is increased to make the long-range power-law correlated data series into a sign series, the change of the scaling exponent is still less than 5%. This illustrates that the signs of long-range power-law correlated signals contain main correlation information.

The effects of quantization on uncorrelated signals are next to nothing. Only in the case of large quantization partition width $\Delta > 1 * \sigma$, will the scaling exponents of these signals fall slightly and have the trend of deviating from white noise to anti-correlated noise.

The effects of quantization on anti-correlated signals are the greatest. The scaling exponents of signals increase rapidly with the increase of quantization partition width and attain to the scaling exponent of random white noise fleetly. The effects of quantization on anti-correlated signals are different in different scale ranges. The effects of quantization in a large scale range are greater than in a small scale range, so there are crossover points in $\log - \log$ plot of $F(n) - n$. When the quantization partition width is lager, the crossover points will shift to the small scale range, i.e., the affected scaling range increases with the increase of the quantization partition width, spreads gradually from large scale range to the whole scaling range.

The results of analysis in this paper can be used for guiding data acquisition. For the anti-correlated signals, because their scaling exponents are very sensitive to quantization partition width, resolution of instrument for data acquisition must be as high as possible. On the contrary, for close-to-random or long-range power-law correlated signals, the scaling exponents are robust to the quantization partition width so that the resolution of the instrument can be low.

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