Entropic noise induced stability and double entropic stochastic resonance induced by correlated noises*

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For the activated dynamics of a Brownian particle moving in a confined system with the presence of entropic barriers, this paper investigates a periodic driving and correlations between two noises. Within the two-state approximation, the explicit expressions of the mean first passage time (MFPT) and the spectral power amplification (SPA) are obtained, respectively. Based on the numerical computations, it is found that: (i) The MFPT as a function of the noise intensity exhibits a maximum with the positive correlations between two noises ($\lambda > 0$), this maximum for MFPT shows the characteristic of the entropic noise induced stability (ENIS) effect. The intensity λ of correlations between two noises can enhance the ENIS effect. (ii) The SPA as a function of the noise intensity exhibits a double-peak by tuning the noise correlation intensity λ, i.e., the existence of a double-peak behaviour is the identifying characteristic of the double entropic stochastic resonance phenomenon.

Keywords: correlated noises, confined system, entropic noise induced stability, double entropic stochastic resonance

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1. Introduction

It is well known that noise can play a constructive role in nonlinear dynamical systems. After the works of Kramers[1] and Benzi *et al.*,[2] the escape problem and stochastic resonance under the influence of noise have become a subject with relevance to a wide range of fields such as physics, chemistry and biology.[3−8] All the above works focus mainly on systems with purely energetic potentials. However, in some situations, frequently found in soft condensed matter and biological systems, particles move in constrained regions such as small cavities, pores or channels whose presence and shape play an important role for the stochastic resonance dynamics.[9,10] These irregularities in the form of confining, curved boundaries, being modeled via an entropic potential, can cause noise-assisted and resonant-like behaviours in the system under consideration.[11−15] In the presence of entropic barriers, the appearance of the entropic stochastic resonance arising from entropy variations is presented when a Brownian particle moves in a confined system under the presence of a Gaussian white noise or a coloured noise and a periodic driving.[11−13] The interplay among a periodic input signal, a constant bias and intrinsic thermal noise causes a double entropic stochastic resonance (DESR) phenomenon in a geometrically confined system.[14] The entropic resonant activation, which is characteristic of the shape and size of the confinement, has been observed.[15] The behaviour of the current in a two-dimensional Büttiker–Landauer motor is investigated in the presence of entropic and energy barriers.[16] The previous investigations in the presence of entropic barriers were limited to single noise case. Yet the effect of correlations between two noises has not been reported.

In the past decade, the effect of correlations between two noises on nonlinear systems and the related phenomenon of stochastic resonance have received great attention.[17−21] Correlated noise processes have made their presence felt in a wide range of applications, such as the statistical properties for a single mode laser,[22−24] bistable kinetics,[25−27] tumor cell growth,[28−30] barrier crossing dynamics,[31,32] stochastic resonance,[33−35] steady state entropy production and flux.[36,37] In this paper, we study the effect of correlations between two noises on the transient properties and stochastic resonance of a Brow-

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nian particle moving in a confined system under the presence of entropic barriers and a periodic driving. How will the correlations between two noises impact on the confined system in the presence of a periodic driving? We find that the correlations between two noises can enhance the entropic noise induced stability (ENIS) effect and induce the DESR phenomenon.

This paper is organized as follows. In Section 2, the explicit expressions of the MFPT and the SPA are presented, respectively. The effect of correlations between two noises on the MFPT and SPA is discussed. Finally, a conclusion is given in Section 3.

2. The MFPT and stochastic resonance

2.1. The effect of correlations between two noises on MFPT

We consider the overdamped motion of a Brownian particle in a constrained geometry subjected to a sinusoidal oscillating force $\boldsymbol{F}(t)$ along the axis of the structure and to a time dependent force $\boldsymbol{G}(t)$ in the declining direction, shown in Fig. 1, described by the following Langevin equation:

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = -\boldsymbol{G}(t) - F(t)\boldsymbol{e}_x + \boldsymbol{\eta}(t), \tag{1}$$

where $\boldsymbol{r} = (x, y)$ denotes the position of the particle, $\boldsymbol{e}_x$ is the unit vector along x-direction. The longitudinal force along the axial direction is given by $F(t) = F_0 \sin(\Omega t)$, where F_0 is the amplitude and Ω is the angular frequency of the sinusoidal driving.

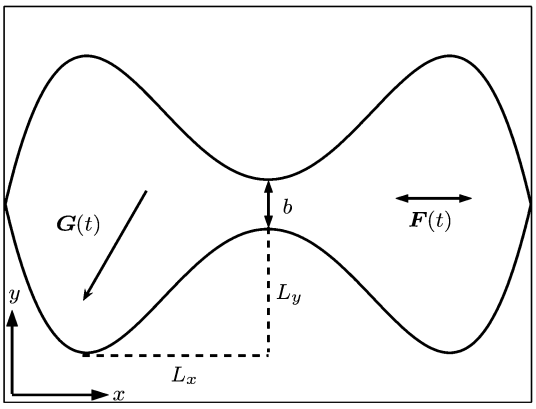


Fig. 1. Schematic illustration of the two-dimensional structure confining the overdamped motion of the Brownian particles. A time dependent force $\boldsymbol{G}(t)$ in the declining direction and a sinusoidal periodic force $\boldsymbol{F}(t)$ along the longitudinal direction drive the Brownian particles.

The $\boldsymbol{\eta}(t) = (\eta_x(t), \eta_y(t))$ is a Gaussian white noise with zero mean which obeys the fluctuation–dissipation relation $\langle \eta_i(t)\eta_j(t')\rangle = 2D_{\mathrm{A}}\delta_{ij}\delta(t - t')$, for $i, j = x, y$, where D_{A} describes the intensity of the noises $\eta_x(t)$ and $\eta_y(t)$, respectively. The time dependent force $\boldsymbol{G}(t)$ consists of a constant force G and time modulated by a stochastic process in the following form:

$$\boldsymbol{G}(t) = G(1 + \boldsymbol{\xi}(t)), \tag{2}$$

where $\boldsymbol{\xi}(t) = (\xi_x(t), \xi_y(t))$ is another Gaussian white noise with zero mean and the autocorrelations of $\xi_x(t)$ and $\eta_y(t)$ are

$$\langle \xi_i(t)\xi_j(t')\rangle = 2D_M\delta_{ij}\delta(t - t'), \quad \text{for} \quad i, j = x, y\ , \tag{3}$$

where D_{M} is the intensity of the noises $\xi_x(t)$ and $\xi_y(t)$, respectively. Substituting Eq. (2) into Eq. (1) we can rewrite Eq. (1) along x and y-directions as

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= -F(t) - G\xi_x(t) + \eta_x(t), \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= -G - G\xi_y(t) + \eta_y(t). \end{aligned} \tag{4}$$

Here, we assume that the two noises are correlated with each other and the correlation between two noises has the following form:[17,18]

$$\begin{aligned} \langle \xi_i(t)\eta_j(t')\rangle &= \langle \eta_i(t)\xi_j(t')\rangle \\ &= 2\lambda\sqrt{D_{\mathrm{M}}D_{\mathrm{A}}}\delta_{ij}\delta(t - t'), \\ &\quad \text{for} \quad i, j = x, y, \end{aligned} \tag{5}$$

where λ denotes the intensity of correlation between two noises.

Then we can equivalently transform Eq. (4) into the stochastic differential equations as

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= -F(t) + H(\lambda)\Gamma_x(t), \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= -G + H(\lambda)\Gamma_y(t), \end{aligned} \tag{6}$$

in which $\Gamma_x(t)$ and $\Gamma_y(t)$ are Gaussian white noises with zero mean and

$$\langle \Gamma_i(t)\Gamma_j(t')\rangle = 2\delta_{ij}\delta(t - t'), \quad \text{for } i, j = x, y\ , \tag{7}$$

$$H(\lambda) = \sqrt{D_{\mathrm{M}}G^2 - 2\lambda\sqrt{D_{\mathrm{M}}D_{\mathrm{A}}}G + D_{\mathrm{A}}}. \tag{8}$$

In the presence of confining boundaries, the confinement can be accounted for by imposing the reflecting boundaries on the system. The walls, as shown in Fig. 1, can be described by the following equation:

$$w_l(x) = \epsilon x^4 - 2\epsilon x^2 - b/2 = -w_u(x), \tag{9}$$

in which $w_l(x)$ and $w_u(x)$ correspond to the lower and the upper boundary functions, respectively. The $\epsilon = L_y/L_x$ is the aspect ratio, where L_x corresponds to the distance between bottleneck position and position of maximal width, L_y refers to the narrowing of the boundary functions, b is the remaining width at the bottleneck, $2w(x) = w_u(x) - w_l(x)$ is the local width of the structure.

In the absence of the periodic driving (i.e., $F(t) = 0$), the Fokker–Planck equation corresponding to the Langevin equations in two dimensions [Eq. (6)] is described by the following equation:[38]

$$\begin{aligned}\frac{\partial P(x,y,t)}{\partial t} &= \frac{\partial}{\partial x}\exp\left[\frac{-u(x,y)}{H(\lambda)}\right] \\ &\times \frac{\partial}{\partial x}\exp\left[\frac{u(x,y)}{H(\lambda)}\right]P(x,y,t) \\ &+ H(\lambda)\frac{\partial}{\partial y}\exp\left[\frac{-u(x,y)}{H(\lambda)}\right] \\ &\times \frac{\partial}{\partial y}\exp\left[\frac{u(x,y)}{H(\lambda)}\right]P(x,y,t), \quad (10)\end{aligned}$$

with reflecting boundary conditions at the confining walls, where the potential function is given by $u(x,y) = Gy$.

The overdamped motion of a Brownian particle moving along the axis of two dimensions in an enclosure of spatially varying cross section can be described by the Fick–Jacobs equation. Since we are mainly interested in the dynamics in x-direction, the two-dimensional Fokker–Planck equation can be reduced to an effective one-dimensional Fokker–Planck equation, this results in the Ficks–Jacob equation[39−41]

$$\begin{aligned}\frac{\partial P(x,t)}{\partial t} &= \frac{\partial}{\partial x}\Big[H(\lambda)\frac{\partial}{\partial x}P(x,t) \\ &+ A'(x,\lambda)P(x,t)\Big], \quad (11)\end{aligned}$$

where the prime refers to the derivative with respect to x. The steady-state distribution function is

$$P_{\rm st}(x) = N\exp\left[-\frac{A(x,\lambda)}{H(\lambda)}\right], \quad (12)$$

where N is normalization constant and the effective entropic potential experienced by the Brownian particle is given by

$$A(x,\lambda) = -H(\lambda)\ln\left[\frac{2H(\lambda)}{G}\sinh\left(\frac{Gw(x)}{H(\lambda)}\right)\right]. \quad (13)$$

The effective entropic potential $A(x,\lambda)$ for the confinement described by Eq. (13) is shown in Fig. 2, the height of the peak and the depth of the two walls decrease as λ increases.

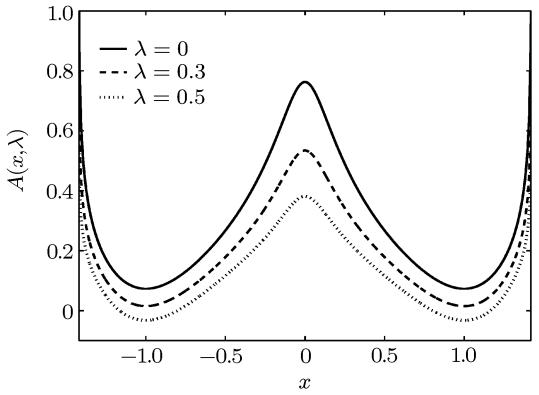


Fig. 2. The effective entropic potential $A(x,\lambda)$ versus x for different values of λ with $G = 1.0$, $\epsilon = 0.25$, $b = 0.02$, $D_{\rm M} = 0.1$ and $D_{\rm A} = 0.095$.

If the potential barrier is sufficiently high and the two wells are well separated by a barrier, the intra-well motion may be neglected and a simplified two-state description can be used to get useful insights into the full dynamics.[42] Within the two-state approximation, the MFPT of a Brownian particle from one well to the other in the absence of the periodic driving is given by

$$T(\lambda) = \frac{2\pi}{\sqrt{A''(x_-,\lambda)|A''(x_+,\lambda)|}}\exp\left[\frac{\Delta A}{H(\lambda)}\right], \quad (14)$$

where A'' is the second derivative with respect to x and x_- and x_+ indicate the position of the maximum and minimum of the symmetric potential, respectively.

For the potential given by Eq. (13) and the shape defined by Eq. (9), the corresponding MFPT for transitions from one well to the other reads[11,12,14,15]

$$\begin{aligned}&T(\lambda) \\ &= \frac{\pi}{G\epsilon}\frac{\sinh^2[G(b+2\epsilon)/2H(\lambda)]}{\sqrt{2\sinh(Gb/H(\lambda))\sinh[G(b+2\epsilon)/H(\lambda)]}}, \quad (15)\end{aligned}$$

where $H(\lambda)$ is given by Eq. (8). Above results are in agreement with those obtained in Refs. [11], [12] and [15] by setting $\lambda = 0$ and $D_{\rm M} = 0$. We carry out a numerical analysis for Eq. (15) and the results are presented in Figs. 3 and 4, respectively.

Figures 3 and 4 display the effect of the noise correlation intensity λ on the MFPT. As we can see, when there are no correlations between two noises (i.e., $\lambda = 0$ in Figs. 3 and 4), it is shown that the MFPT is monotonic and decreases with the increase of the noise intensities $D_{\rm M}$ and $D_{\rm A}$, respectively. However, when the positive correlation between two noises $\lambda > 0$ (i.e., $\lambda = 0.3$, 0.5 and 0.7 in Figs. 3 and 4), the MFPT first increases, reaches a maximum and

then decreases as the noise intensity $D_{\rm M}$ or $D_{\rm A}$ increases, a critical noise intensity $D_{\rm M}$ or $D_{\rm A}$ exists at the place where the MFPT induced by noise is the largest. This maximum for MFPT shows the characteristic of the ENIS effect. The noise induced stability effect with purely energetic potentials has been shown in previous investigations,[5–8] which implies that the stability of metastable or unstable states can be enhanced by the noise and the mean lifetime of the metastable state is longer than the deterministic decay time. This resonance-like behaviour contradicts the monotonic behaviour predicted by the Kramers theory.[1] It is clear from Figs. 3 and 4 that the maximum of MFPT increases with the increase of the noise correlation intensity λ, i.e., the increasing λ intensifies the ENIS effect.

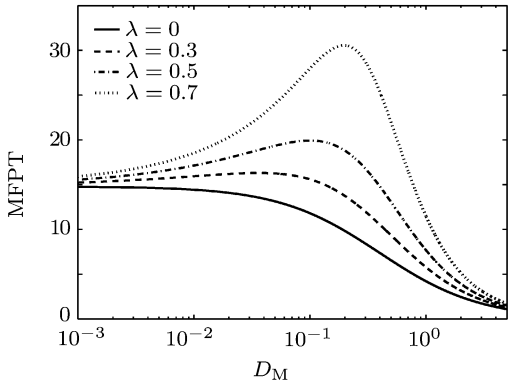


Fig. 3. The MFPT versus $D_{\rm M}$ for different values of λ with $G = 1.0$, $\epsilon = 0.25$, $b = 0.02$ and $D_{\rm A} = 0.4$.

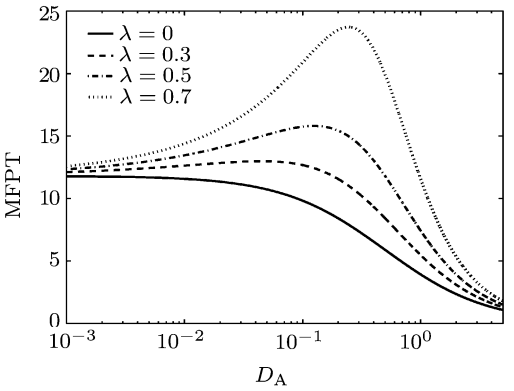


Fig. 4. The MFPT versus $D_{\rm A}$ for different values of λ with $G = 1.0$, $\epsilon = 0.25$, $b = 0.02$ and $D_{\rm M} = 0.5$.

2.2. The effect of correlations between two noises on stochastic resonance

A small sinusoidal driving leads to a modulation of the barrier height of the double well potential and consequently to a modulation of the transition rates. Applying the Kramers–Smoluchowski rate formula[1,3] for the transition rates among potential wells, we obtain the transition probability densities $W_{\pm}(t)$ out of the states $x_{\pm}$,

$$W_{\pm}(t) = r_K(\lambda) \exp\left[\pm \frac{F_0}{H(\lambda)} \sin(\Omega t)\right],$$

where $r_K(\lambda)$ is the transition rate of the unperturbed two-state system and is given by

$$\begin{aligned} r_K(\lambda) =& T^{-1}(\lambda) \\ =& \frac{G\epsilon}{\pi} \frac{\sqrt{2\sinh(Gb/H(\lambda))\sinh[G(b+2\epsilon)/H(\lambda)]}}{\sinh^2[G(b+2\epsilon)/2H(\lambda)]}, \end{aligned} \tag{16}$$

the probability densities $n_{\pm}(t)$ of the two states $x_{\pm}$ obey the following master equation:

$$\frac{\rm d}{{\rm d}t} n_{\pm}(t) = -W_{\pm}(t) n_{\mp}(t) + W_{\mp}(t) n_{\pm}(t).$$

Making use of the normalization condition $n_+(t) + n_-(t) = 1$, we obtain the differential equation for the mean value $\langle x(t)\rangle = n_+(t) - n_-(t)$ as[43]

$$\begin{aligned} \frac{\rm d}{{\rm d}t}\langle x(t)\rangle =& -[W_+(t) + W_-(t)]\langle x(t)\rangle \\ &+ W_+(t) - W_-(t). \end{aligned}$$

Next, we can expand $\langle x(t)\rangle$ into a Fourier series representation,

$$\langle x(t)\rangle = \sum_{n=0}^{\infty} \gamma_n \sin(n\Omega t + \phi_n),$$

with corresponding Fourier coefficients γ_n which depend nonlinearly on the driving frequency, driving amplitude and noise parameters, ϕ_n are phase lags. The occurrence of the stochastic resonance can be detected in the SPA,[44] which is defined by the ratio of the power stored in the response of the system at frequency Ω and the power of the driving signal, this is given by

$$\eta = \left[\frac{\gamma_n}{F_0}\right]^2.$$

Within a first order perturbation theory in the ratio of driving amplitude and noise intensity, one obtains the SPA[44]

$$\eta = \frac{1}{H^2(\lambda)} \frac{4r_K^2(\lambda)}{4r_K^2(\lambda) + \Omega^2}, \tag{17}$$

where $r_K(\lambda)$ is the transition rate of the unperturbed two-state system and is given by Eq. (16). By numerical evaluation of the expression of SPA (17), the effect

of the noise correlation intensity λ on the SPA can be shown in Figs. 5–9, respectively.

In Figs. 5 and 6 we present the SPA as a function of the noise intensity $D_{\rm M}$ and as a function of the noise intensity $D_{\rm A}$ for different values of the noise correlation intensity λ, respectively. When the noise correlation intensity λ is smaller (see $\lambda = 0.25$ in Fig. 5 and $\lambda = 0.30$ in Fig. 6), there is only a single peak in the SPA at small value of the noise intensity. The existence of a single peak in the SPA as a function of the noise intensity implies the characteristic of the entropic stochastic resonance phenomenon. The entropic stochastic resonance phenomenon is found in previous investigations.[11,12] However, when the noise correlation intensity λ is increased (see $\lambda = 0.45$ and $\lambda = 0.50$ in Fig. 5, $\lambda = 0.47$ and $\lambda = 0.50$ in Fig. 6), a second peak in the SPA appears at a larger value of the noise intensity. The existence of a double-peak behaviour in the SPA is the identifying characteristic of the DESR phenomenon. The DESR phenomenon is also shown in previous investigations,[14] where the existence of a second peak in the SPA is associated with a regime of optimal sensitivity for intra-well dynamics, but for tuning the noise correlation intensity here.

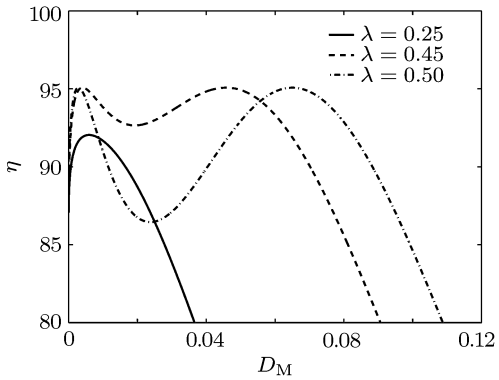


Fig. 5. The SPA versus $D_{\rm M}$ for different values of λ with $G = 1.0$, $\epsilon = 0.25$, $b = 0.02$, $\Omega = 0.01$ and $D_{\rm A} = 0.095$.

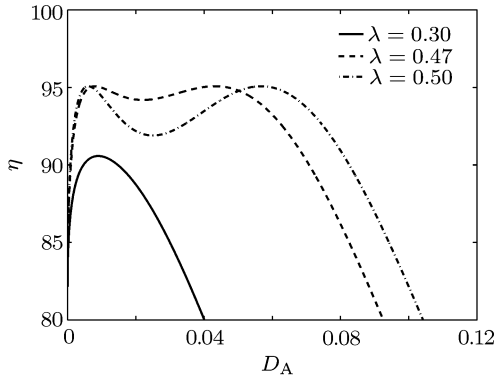


Fig. 6. The SPA versus $D_{\rm A}$ for different values of λ with $G = 1.0$, $\epsilon = 0.25$, $b = 0.02$, $\Omega = 0.01$ and $D_{\rm M} = 0.1$.

The SPA as a function of the parameter G and as a function of the parameter ϵ are shown in Figs. 7 and 8 for different values of the noise correlation intensity λ, respectively. In Fig. 7, when the noise correlation intensity $\lambda = 0$ (i.e., $\lambda = 0$ in Fig. 7), it is shown that the SPA is monotonic and decreases with the increase of G. However, when the value of the noise correlation intensity increases (i.e., $\lambda = 0.4$ in Fig. 7), the SPA as a function of the parameter G exhibits a single peak at $G < 1$. The existence of a single peak in the SPA as a function of the parameter G indicates the characteristic of the entropic parameter-induced resonance phenomenon. The parameter-induced resonance phenomenon is found in previous investigations in the presence of purely energetic barrier,[45] which characterizes that the resonance can be obtained by tuning system parameters with fixed noise. As the value of the noise correlation intensity continuously increases (i.e., $\lambda = 0.7$, $\lambda = 0.8$ and in Fig. 7), a second peak appears at $G > 1$, i.e., the existence of a double-peak in the SPA is the identifying characteristic of the double entropic parameter-induced resonance phenomenon. In Fig. 8, the SPA as a function of the parameter ϵ exhibits a resonant platform. As the value of the noise correlation intensity increases, the resonant platform gradually becomes narrower and increases, respectively.

Figure 9 displays the SPA as a function of the noise correlation intensity λ for different values of the angular frequency Ω. It is shown that the SPA as a function of λ exhibits a maximum and the maximum in the SPA decreases as Ω increases, i.e., the angular frequency Ω weakens the entropic stochastic resonance of the system.

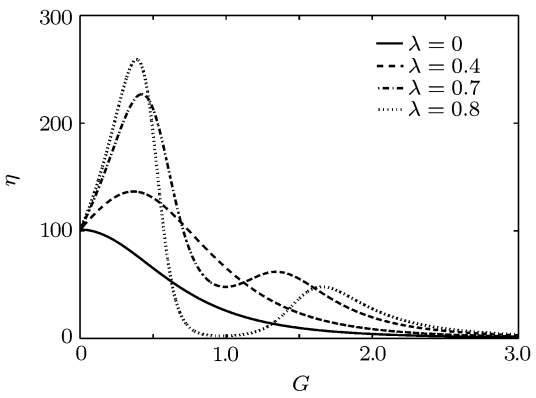


Fig. 7. The SPA versus G for different values of λ with $\epsilon = 0.25$, $b = 0.02$, $\Omega = 0.01$, $D_{\rm M} = 0.1$ and $D_{\rm A} = 0.095$.

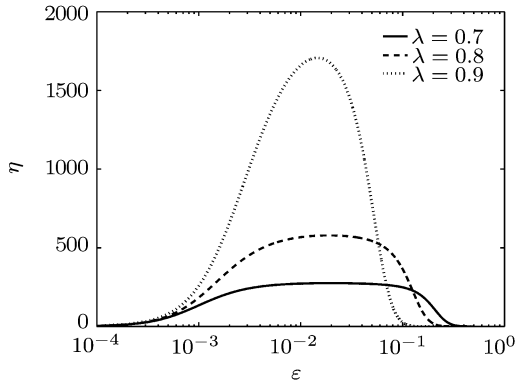


Fig. 8. The SPA versus ϵ for different values of λ with $G = 1.0$, $b = 0.02$, $\Omega = 0.01$, $D_{\rm M} = 0.1$ and $D_{\rm A} = 0.095$.

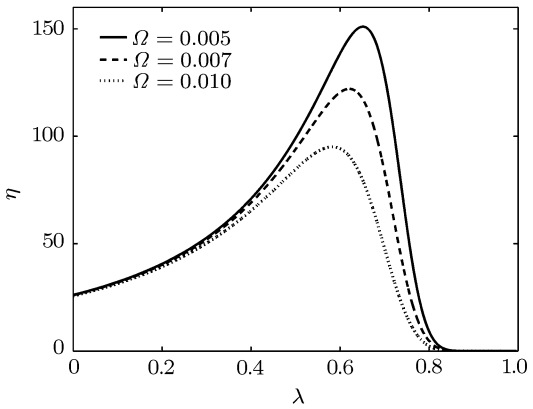


Fig. 9. The SPA versus λ for different values of Ω with $G = 1.0, \epsilon = 0.25$, $b = 0.02$, $D_{\rm M} = 0.1$ and $D_{\rm A} = 0.095$.

3. Conclusion

In this paper, we have studied the transient properties and stochastic resonance of a Brownian particle moving in a confined system under the presence of entropic barriers, a small periodic driving and correlated noises. Within the two-state approximation, the explicit expressions of the MFPT and the SPA are obtained, respectively. The constrained motion impedes the transition of a Brownian particle from one well to the other, which is described in terms of a bistable entropic potential. In the case of correlations between noises and the absence of the periodic driving, the MFPT as a function of the noise intensity exhibits a maximum, this maximum for MFPT shows the ENIS effect. The increase in the intensity of correlations between noises enhances the ENIS effect. The activated dynamics of Brownian particles in the entropic potential then results in a cooperative effect among correlations between two noises and external modulation, yielding a DESR. Both ENIS and DESR are characteristic of small-scale systems such as catalysis, biological channels and nanoporus materials. Thus the role of correlations between two noises in confined systems may constitute theoretically a useful mechanism for the manipulation and control of single molecules and nanodevices.

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