A new metric for rotating black holes in Gauss–Bonnet gravity*

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This paper presents a new metric and studies slowly rotating Gauss–Bonnet black holes with a nonvanishing angular momentum in five dimensional anti-de Sitter spaces. Taking the angular momentum parameter a up to second order, the slowly rotating black hole solutions are obtained by working directly in the action. In addition, it also finds that this method is applicable in higher order Lovelock gravity.

Keywords: Gauss–Bonnet gravity, slow rotation, anti-de Sitter spaces

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1. Introduction

It is a general belief that Einstein's gravity is the low-energy limit of a quantum theory of gravity. Lovelock[1] extended the Einstein tensor, which is only a symmetric and conserved tensor depending on the metric, to the most general tensor. In higher dimensional spacetimes, the Lovelock theory is the most natural extension of general relativity and its field equations of motion contain the most symmetric conserved tensor with no more than a second derivative of the metric. It has been argued that the Gauss–Bonnet term appears as the leading correction to the effective low energy action of the string theory. Until now, the analytic expressions of static and spherically symmetric Gauss–Bonnet black hole solutions have been investigated in Refs. [2]–[4] and of Born–Infeld–Gauss–Bonnet models in Refs. [5]–[7]. The thermodynamics of the uncharged static spherically Gauss–Bonnet black hole solutions have been considered in Refs. [8]–[10] and of charged solutions in Ref. [11]. Konoplya *et al.*[12,13] presented an analysis of the scalar perturbations in the background of Gauss–Bonnet black hole spacetimes and the (in)stability in high dimensions. Very recently the quasinormal mode of a scalar field in five-dimensional Lovelock black hole spacetime for different angular quantum numbers l has been obtained in Ref. [14]. Liu[15] studied the electromagnetic perturbations of black holes in Gauss–Bonnet gravity.

For Gauss–Bonnet gravity, it is interesting to explore some rotating black holes. However, since the equations of motion are highly nonlinear, it is rather hard to obtain the exact analytic rotating black hole solutions. By introducing a small angular momentum as a perturbation into a non-rotating system, Kim and Cai[16] studied slowly Gauss–Bonnet rotating black hole solutions with a nonvanishing angular momentum. It is worth mentioning that the Lagrangian of Gauss–Bonnet action only involves three terms constituted by the contracted product of Ricci curvature and the Riemann curvature tensors. Meanwhile, the resulting field equations, obtained after variation with respect to the metric tensor, have seven terms. If considering a higher order Lovelock gravity, the resulting field equation for third order Lovelock gravity contains thirty-four terms.[17,18] Therefore, taking into account all the relevant terms of the Lovelock action, we find that obtaining slowly rotating black hole solutions by solving the field equations in high dimensions is a very complicated task. Note that the exact static Gauss–Bonnet black hole solutions were obtained by working directly in the action in Refs. [2], [8] and [9]. In this paper, we dedicate to investigate the slowly rotating black hole solutions in Gauss–Bonnet gravity following the same method. Apparently the lowest level contribution of rotation should be proportional to a^2, where

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a is the parameter in Gauss–Bonnet gravity. The metric demonstrated in Ref. [16] is linearly dependent on a, hence it is not applicable in this case. So, we need to find a proper metric ansatz up to a^2.

This paper is organized as follows. In Section 2, we present a new form metric and obtain slowly rotating black hole solutions by working directly in the action. Section 3 is devoted to a summary of the results.

2. Slowly rotating Gauss–Bonnet black holes in five dimensions

In this section, we analyse the slowly rotating Gauss–Bonnet black holes in five dimensional spacetimes. In order to explore slowly rotating black holes by working directly in the action, the new metric describing rotating black holes is expressed as

$$\begin{aligned}\mathrm{d}s^2 =& -\left[f(r)+\frac{a^2}{l^2}+\frac{a^2(1-f(r))\cos^2\theta}{r^2}\right]\mathrm{d}t^2\\ &+\left[\frac{1}{f(r)}+\frac{a^2(\cos^2\theta f(r)-1)}{r^2f(r)^2}-\frac{a^2}{l^2f(r)^2}\right]\mathrm{d}r^2\\ &+\left(r^2+a^2\cos^2\theta+\frac{a^2r^2}{l^2}\cos^2\theta\right)\mathrm{d}\theta^2\\ &+\left[r^2+a^2+a^2\big(1-f(r)\big)\sin^2\theta\right.\\ &\left.+\frac{2a^2r^2\sin^2\theta}{l^2}\right]\sin^2\theta\,\mathrm{d}\phi^2\\ &+2ar^2p(r)\sin^2\theta\,\mathrm{d}t\,\mathrm{d}\phi+r^2\cos^2\theta\,\mathrm{d}\varphi^2, \qquad (1)\end{aligned}$$

where the parameter a is a small quantity and the functions $f(r)$ and $p(r)$ are independent.

The action for Gauss–Bonnet theory with negative cosmological constant $\Lambda=-6/l^2$ in five dimensions is given by[9]

$$\mathscr{I}=\frac{1}{16\pi G}\int \mathrm{d}^5x\sqrt{-g}(-2\Lambda+R+\alpha\mathscr{L}_2), \qquad (2)$$

where α is the Gauss–Bonnet coefficient with dimension (length)^2 and is positive in the heterotic string theory. The second term R is the Einstein–Hilbert term and the third order term is the Gauss–Bonnet term

$$\mathscr{L}_2=R_{\gamma\delta\lambda\sigma}R^{\gamma\delta\lambda\sigma}-4R_{\gamma\delta}R^{\gamma\delta}+R^2. \qquad (3)$$

We also notice that the Lagrangian of Lovelock gravity is the sum of dimensionally extended Euler densities[19]

$$\mathscr{L}=\sum_{n=0}^{m}\alpha_n\mathscr{L}_n, \qquad (4)$$

where α_n is an arbitrary constant and $\mathscr{L}_n$ is the Euler density of a $2n$-dimensional manifold:

$$\mathscr{L}_n=\frac{1}{2^n}\delta^{a_1b_1\ldots a_nb_n}_{c_1b_1\ldots c_nd_n}R^{c_1d_1}{}_{a_1b_1}\ldots R^{c_nd_n}{}_{a_nb_n}. \qquad (5)$$

Here the generalized delta function is totally antisymmetric in both sets of indices. Usually, we set $\mathscr{L}_0=1$ and hence α_0 is just the cosmological constant. The $\mathscr{L}_1$ gives us the usual Einstein–Hilbert term and $\mathscr{L}_2$ is the Gauss–Bonnet term and then it reads

$$\mathscr{L}_2=\frac{1}{4}\delta^{a_1b_1a_2b_2}_{c_1b_1c_2d_2}R^{c_1d_1}{}_{a_1b_1}R^{c_2d_2}{}_{a_2b_2}. \qquad (6)$$

In this paper, we focus on the Lagrangian of Gauss–Bonnet term with Eq. (6), instead of the corresponding one in Eq. (3).

In principle, one can directly put the metric Eq. (1) into the action Eq. (2) and derives out the equation of motion for functions $f(r)$ and $p(r)$. However, this will become complicated, especially in higher order Lovelock gravity. Fortunately, what we need is the term proportional to a^2, including the lower terms in action. According to the ansatz of metric, all nonvanishing components of Riemann tensors (up to a^2) can be classified into three groups: (I) diagonal form $R^{\hat{i}\hat{j}}{}_{\hat{i}\hat{j}}$; (II) off-diagonal form proportional to a; (III) off-diagonal form proportional to a^2. Some key steps are given in Appendix. For group (I), the elements read

$$\begin{aligned}R^{12}{}_{12}&=\bar{R}^{12}{}_{12}+\tilde{R}^{12}{}_{12},\quad R^{1i}{}_{1i}=\bar{R}^{1i}{}_{1i}+\tilde{R}^{1i}{}_{1i},\\ R^{2i}{}_{2i}&=\bar{R}^{2i}{}_{2i}+\tilde{R}^{2i}{}_{2i},\quad R^{ij}{}_{ij}=\bar{R}^{ij}{}_{ij}+\tilde{R}^{ij}{}_{ij}. \qquad (7)\end{aligned}$$

where $3\le i<j\le 5$ and the Riemann tensors $\bar{R}^{12}{}_{12}$, $\bar{R}^{2i}{}_{2i}$ and $\bar{R}^{ij}{}_{ij}$ represent the components which are independent of parameter a, while the tensors $\tilde{R}^{12}{}_{12}$, $\tilde{R}^{2i}{}_{2i}$ and $\tilde{R}^{ij}{}_{ij}$ are proportional to a^2. Denote the Lagrangian of Gauss–Bonnet term from the contribution of the case (I) by $\mathscr{L}_d$, we can write

$$\mathscr{L}_d=\bar{\mathscr{L}}_d+\tilde{\mathscr{L}}_d, \qquad (8)$$

where

$$\begin{aligned}\bar{\mathscr{L}}_d&=\frac{1}{4}\delta^{a_1b_1a_2b_2}_{c_1b_1c_2d_2}\bar{R}^{c_1d_1}{}_{a_1b_1}\bar{R}^{c_2d_2}{}_{a_2b_2}\\ &=24\bar{R}^{12}{}_{12}\bar{R}^{34}{}_{34}+48\bar{R}^{13}{}_{13}\bar{R}^{24}{}_{24}\\ &\quad+24\bar{R}^{13}{}_{13}\bar{R}^{45}{}_{45}+24\bar{R}^{23}{}_{23}\bar{R}^{45}{}_{45},\\ \tilde{\mathscr{L}}_d&=\frac{1}{2}\delta^{a_1b_1a_2b_2}_{c_1b_1c_2d_2}\bar{R}^{c_1d_1}{}_{a_1b_1}\tilde{R}^{c_2d_2}{}_{a_2b_2}\\ &=24\ \tilde{R}^{12}{}_{12}\times\bar{R}^{34}{}_{34}+8\sum_{j=3}^{5}\tilde{R}^{1j}{}_{1j}\end{aligned}$$

$$\times (2\bar{R}^{24}{}_{24} + \bar{R}^{45}{}_{45}) + 8\sum_{j=3}^{5} \tilde{R}^{2j}{}_{2j} \times (2\bar{R}^{14}{}_{14} + \bar{R}^{45}{}_{45}) + 8(\tilde{R}^{34}{}_{34} + \tilde{R}^{35}{}_{35} + \tilde{R}^{45}{}_{45}) \times (\bar{R}^{12}{}_{12} + \bar{R}^{13}{}_{13} + \bar{R}^{23}{}_{23}). \quad (9)$$

Unlike the static and spherically symmetric metric, there are some off-diagonal Riemann tensors. With the help of the properties of Kronecker delta function, one can find that the contribution of off-diagonal Riemann tensors in case (III) vanishes. The parts of Lagrangian from the off-diagonal Riemann tensors in case (II) are obtained as

$$\mathscr{L}_{od} = \mathscr{L}_{od1} + \mathscr{L}_{od2}, \quad (10)$$

where

$$\begin{aligned} \mathscr{L}_{od1} &= 4(R^{12}{}_{34}R^{34}{}_{12} + R^{13}{}_{24}R^{24}{}_{13} + R^{14}{}_{23}R^{23}{}_{14}), \\ \mathscr{L}_{od2} &= 4(R^{12}{}_{24}R^{34}{}_{13} + R^{24}{}_{12}R^{13}{}_{34} + R^{12}{}_{24}R^{54}{}_{15} \\ &\quad + R^{24}{}_{12}R^{15}{}_{54} + R^{13}{}_{34}R^{54}{}_{15} + R^{34}{}_{13}R^{15}{}_{54}). \end{aligned}$$

Furthermore, the Ricci scalar R is given by

$$R = \bar{R} + \tilde{R}, \quad (11)$$

where $\bar{R}$ is equal to $6(1 - f(r) - rf(r)')/r^2 - f(r)''$ and $\tilde{R}$ is proportional to a^2 and given in Appendix.

Varying the action in Eq. (2) with regard to the function $p(r)$, we have

$$\begin{aligned} 0 = {} & [2\tilde{\alpha}f(r)r - 2\tilde{\alpha}r - r^3]f(r)p(r)'' \\ & + [6\tilde{\alpha}f(r) - 5r^2 + 2\tilde{\alpha}rf(r)' - 6\tilde{\alpha}]f(r)p(r)' \\ & + 3\Big[- f(r)'r^2 - 2\tilde{\alpha}f(r)' + 2r \\ & + 2\tilde{\alpha}f(r)'f(r) - 2f(r)r + \frac{4r^3}{l^2}\Big]p(r). \end{aligned} \quad (12)$$

Here we suppose that the coefficient of function $p(r)$ vanishes, and then easily obtain

$$f(r) = 1 + \frac{r^2}{2\tilde{\alpha}}\left(1 - \sqrt{1 - \frac{4\tilde{\alpha}}{l^2} + \frac{4\tilde{\alpha}m}{r^4}}\right), \quad (13)$$

where the Gauss–Bonnet coefficient α is rescaled to $\tilde{\alpha}/2$ and m is an integration constant. Hence, Equation (12) reduces to

$$\begin{aligned} & [2\tilde{\alpha}f(r)r - 2\tilde{\alpha}r - r^3]p(r)'' \\ & + [6\tilde{\alpha}f(r) - 5r^2 + 2\tilde{\alpha}rf(r)' - 6\tilde{\alpha}]p(r)' = 0. \end{aligned} \quad (14)$$

Then, the expression for function $p(r)$ can be written as

$$p(r) = -\frac{C_2}{8\tilde{\alpha}m}\sqrt{1 - \frac{4\tilde{\alpha}}{l^2} + \frac{4\tilde{\alpha}m}{r^4}} + C_1, \quad (15)$$

where C_1 and C_2 are two integration constants. Let $C_2 = 4m$ and $C_1 = 1/2\tilde{\alpha}$, Eq. (15) becomes

$$p(r) = \frac{1}{2\tilde{\alpha}}\left(1 - \sqrt{1 - \frac{4\tilde{\alpha}}{l^2} + \frac{4\tilde{\alpha}m}{r^4}}\right). \quad (16)$$

In addition, we obtain another equation for functions $f(r)$ and $p(r)$ by varying the action in Eq. (2) with respect to $f(r)$. It is worth to point out that this equation identically equals zero when $f(r)$ and $p(r)$ take the forms of Eqs. (13) and (16), respectively. Notice that the expressions for functions $f(r)$ and $p(r)$ are identical to the counterparts shown in Ref. [16].

For the slowly rotating solution, the stationary and rotationally symmetric metric in Eq. (1) admits two commuting Killing vector fields $\xi_{(t)} = \partial/\partial t$ and $\xi_{\phi} = \partial/\partial\phi$. The Killing vectors can be used to give a physical interpretation of the parameters m and a. Following the analysis given in Refs. [20]–[23], one can obtain coordinate-independent definitions for these parameters. We have the integral

$$\begin{aligned} M &= -\frac{3}{32\pi G}\oint \xi^{\mu;\nu}_{(t)}\,\mathrm{d}^3\Sigma_{\mu\nu}, \\ J &= \frac{1}{16\pi G}\oint \xi^{\mu;\nu}_{(\phi)}\,\mathrm{d}^3\Sigma_{\mu\nu}, \end{aligned} \quad (17)$$

where the integral are taken over the three-sphere at spatial infinity,

$$\mathrm{d}^3\Sigma_{\mu\nu} = \frac{1}{3!}\sqrt{-g}\epsilon_{\mu\nu\alpha\beta\gamma}\mathrm{d}x^{\alpha} \wedge \mathrm{d}x^{\beta} \wedge \mathrm{d}x^{\gamma}. \quad (18)$$

We can obtain the mass M and angular momentum J as follows:

$$M = \frac{3m\Sigma_k}{16\pi G}, \quad J = \frac{2Ma}{3}. \quad (19)$$

3. Concluding remarks

In this paper, we proposed a new metric and obtained the slowly rotating Gauss–Bonnet black hole solutions in five dimensions by working directly in the action. It is worth noting that the diagonal components of the metric also involve a^2 besides the function $f(r)$. Moreover, $g_{t\phi}$ was proportional to $r^2p(r)$ in order to make the equation for $p(r)$ much simple. By discarding any terms involving a^3 and higher powers in a in the action, we obtained the exact form for function $p(r)$, while the function $f(r)$ still kept the form of the static solutions. In addition, we described the Killing isometries of the metric and presented its mass parameter and angular momentum of the black holes.

The advantage of this method is avoiding the equations of motion to obtain the slowly rotating black holes solutions. Although we only considered the slowly rotating black holes in five dimensional spacetimes, this method is still valid for general Gauss–Bonnet gravity in higher dimensions including charge. Furthermore, in general Lovelock gravity, the Einstein equation must involve the terms derived from the action of Lovelock gravity, which becomes very hard if $n > 3$. However, in the present case, the key is to find the action, which is possible to carry out for slowly rotating and charge cases. Besides, this approach can also be used to find the slowly rotating solutions involving multiple angular momenta in different orthogonal planes of rotation. We will discuss them elsewhere.

Appendix A

From the metric in Eq. (1), we give some of the intermediate steps of the calculation.

Riemann tensors The non-vanishing Riemann tensors can be classified into three groups: (I) diagonal form $R^{\hat{i}\hat{j}}{}_{\hat{i}\hat{j}}$; (II) off-diagonal form proportional to a; (III) off-diagonal form proportional to a^2.

group (I):

$$
\begin{aligned}
R^{12}{}_{12} &= \bar{R}^{12}{}_{12} + \tilde{R}^{12}{}_{12}, \quad \bar{R}^{12}{}_{12} = -\frac{f(r)''}{2},\\
\tilde{R}^{12}{}_{12} &= a^2\Bigg\{\Bigg[\bigg(-1+\frac{rf(r)'+r^2f(r)''}{2f(r)}-\frac{r^2f(r)'^2}{4f(r)^2}\bigg)p(r)^2+\bigg(\frac{r^2f(r)'p(r)'}{f(r)}-5rp(r)'-r^2p(r)''\bigg)\frac{p(r)}{2}-\frac{r^2p(r)'^2}{4}\\
&\quad+\frac{f(r)'}{2r^3f(r)}+\frac{f(r)'^2}{4r^2f(r)^2}-\frac{f(r)''}{2r^2f(r)}\Bigg]\sin^2\theta+\bigg(\frac{-2f(r)'}{r^3}+\frac{f(r)''}{2r^2}+\frac{3f(r)-3}{r^4}\bigg)\cos^2\theta\Bigg\},\\
R^{1i}{}_{1i} &= \bar{R}^{1i}{}_{1i} + \tilde{R}^{1i}{}_{1i}, \quad \bar{R}^{1i}{}_{1i} = -\frac{f(r)'}{2r}, \quad (3\le i\le 5),\\
\tilde{R}^{13}{}_{13} &= \Bigg\{-\frac{p(r)^2}{f(r)}+\frac{1}{f(r)r^4}+\frac{f(r)'}{r^3}-\frac{f(r)}{r^4}+\Bigg[-\frac{rp(r)^2f(r)'}{2f(r)}+\frac{rp(r)p(r)'}{2}-\frac{f(r)+1}{r^4}+p(r)^2-\frac{f(r)'}{2f(r)r^3}\\
&\quad+\frac{2}{r^4f(r)}-\frac{2p(r)^2}{f(r)}+\frac{f(r)'}{r^3}\Bigg]\sin^2\theta\Bigg\}a^2,\\
\tilde{R}^{14}{}_{14} &= \Bigg\{\Bigg[-p(r)^2+\frac{f(r)+f(r)'^2}{r^4}-rp(r)'p(r)-\frac{r^2p(r)'^2}{4}-\frac{f(r)'}{2r^3f(r)}-\frac{1}{f(r)r^4}+\frac{p(r)^2}{f(r)}\\
&\quad+\frac{f(r)'}{4r^2}-\frac{f(r)'f(r)}{2r^3}+\frac{rf(r)'p(r)^2}{2f(r)}\Bigg]\sin^2\theta-\frac{p(r)^2}{f(r)}+\frac{1}{f(r)r^4}+\frac{f(r)'}{r^3}-\frac{f(r)}{r^4}\Bigg\}a^2,\\
\tilde{R}^{15}{}_{15} &= \Bigg\{\Bigg[\frac{2p(r)^2+rf(r)'p(r)^2}{f(r)}-p(r)^2-\frac{f(r)'}{2r^3f(r)'}-\frac{1}{r^4f(r)}-\frac{p(r)'p(r)}{2}\Bigg]\sin^2\theta\\
&\quad+\bigg[\frac{f(r)'}{2r^3}-\frac{f(r)}{r^4}\bigg]\cos^2\theta+\frac{1}{r^4}\Bigg\}a^2,\\
R^{2i}{}_{2i} &= \bar{R}^{2i}{}_{2i} + \tilde{R}^{2i}{}_{2i}, \quad \bar{R}^{2i}{}_{2i} = -\frac{f(r)}{2r}, \quad (3\le i\le 5),\\
\tilde{R}^{23}{}_{23} &= \tilde{R}^{25}{}_{25} = a^2\bigg[\frac{f(r)'}{r^3}+\frac{2(1-f(r))}{r^4}\bigg]\cos^2\theta,\\
\tilde{R}^{24}{}_{24} &= \Bigg\{\Bigg[\frac{f(r)'^2+2f(r)''f(r)}{4r^2}-1-\frac{r^2p(r)p(r)''}{2}-\frac{3f(r)f(r)'}{2r^3}+\frac{f(r)^2}{r^4}-\frac{r^2p(r)'^2}{4}\\
&\quad-\frac{3rp(r)p(r)'}{2}\Bigg]\sin^2\theta+4(2-2f(r)+f(r)'r)\Bigg\}a^2,\\
R^{ij}{}_{ij} &= \bar{R}^{ij}{}_{ij} + \tilde{R}^{ij}{}_{ij}, \quad \bar{R}^{ij}{}_{ij} = \frac{1-f(r)}{r^2}, \quad (3\le i<j\le 5),\\
\tilde{R}^{34}{}_{34} &= \Bigg\{\Bigg[\frac{6-5f(r)-f(r)^2}{r^4}+\frac{f(r)f(r)'}{2r^3}+\frac{10}{r^2}-\frac{rp(r)p(r)'}{2}\Bigg]\sin^2\theta-\frac{9}{r^2l^2}-\frac{6(1-f(r))}{r^4}\Bigg\}a^2,
\end{aligned}
$$

$$\tilde{R}^{35}{}_{35} = 2a^2\cos^2\theta\Big(-\frac{1}{r^2l^2}+\frac{f(r)-1}{r^4}\Big),$$

$$\tilde{R}^{45}{}_{45} = \Big\{\Big[\frac{f(r)f(r)'}{2r^3}-\frac{rp(r)p(r)'}{2}+\frac{3}{r^2l^2}+\frac{2-f(r)^2-f(r)}{r^4}\Big]\sin^2\theta+\frac{2(f(r)-1)}{r^4}-\frac{2}{r^2l^2}\Big\}a^2;$$

group (II):

$$R^{12}{}_{34} = -ar^2\sin\theta\cos\theta p(r)', \quad R^{13}{}_{24} = -\frac{a\cos\theta\sin\theta p(r)'}{2f(r)},$$

$$R^{14}{}_{32} = -\frac{a\cos\theta p(r)'}{2f(r)\sin\theta}, \quad R^{12}{}_{24} = -\frac{ar\sin^2\theta}{2}(3p(r)'+rp(r)''),$$

$$R^{13}{}_{34} = R^{15}{}_{54} = -\frac{ar\sin^2\theta}{2}p(r)';$$

group (III):

$$R^{12}{}_{13} = \Big[-\frac{f(r)'}{2f(r)r^2}-p(r)^2r+\frac{p(r)^2f(r)'r^2}{2f(r)}-p(r)p(r)'r^2+\frac{3f(r)-3}{r^3}-\frac{f(r)'}{r^2}\Big]a^2\sin\theta\cos\theta,$$

$$R^{24}{}_{34} = \Big[-p(r)p(r)'r^2+\frac{3f(r)(1-f(r))}{r^3}+\frac{3f(r)'f(r)}{2r^2}\Big]a^2\sin\theta\cos\theta.$$

With regard to Ricci scalar, the term $\tilde{R}$, which is proportional to a^2, is given by

$$\begin{aligned}\tilde{R} = \Big\{\Big[&\Big(\frac{r^2f(r)''}{f(r)}-8-\frac{r^2f(r)'^2}{2r^2f(r)^2}+\frac{4rf(r)'}{f(r)}+\frac{8}{f(r)}\Big)p(r)^2+\Big(14r-\frac{r^2f(r)'}{f(r)}\Big)p(r)'\\&-\frac{3r^2p(r)'^2}{2}-\frac{2f(r)'}{f(r)r^3}-\frac{2f(r)f(r)'}{r^3}+\frac{f(r)'^2}{r^2}-\frac{8}{r^4f(r)}+\frac{f(r)f(r)''}{r^2}-\frac{2f(r)'}{r^3}\\&+\frac{f(r)'^2}{2f(r)^2r^2}-\frac{f(r)''}{r^2}-\frac{2f(r)^2}{r^4}+\frac{20}{r^4}-\frac{f(r)''}{f(r)r^2}-\frac{10f(r)}{r^4}\Big]\sin^2\theta\frac{4p(r)^2}{f(r)}\\&-\frac{14}{r^4}+\frac{4}{f(r)r^4}+\frac{6f(r)'}{r^3}+\frac{f(r)''}{r^2}+\frac{10f(r)}{r^4}+\frac{30\sin^2\theta-26}{r^2l^2}\Big\}a^2.\end{aligned}$$

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