Scheme to generate and discriminate a type of multipartite maximally entangled states in ion-trap*

Xue Li(薛 丽), Huang Shou-Sheng(黄寿胜), Wu Lie(吴 烈),
Ji Yong-Yun(季永运), and Jiang Nian-Quan (姜年权)†

College of Physics and Electric Information, Wenzhou University, Wenzhou 325035, *China*

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This paper proposes a scheme to generate, in an ion-trap, a type of multipartite maximally entangled state which was first introduced by Chen *et al.* [Chen P X, Zhu S Y and Guo G C 2006 *Phys. Rev.* A **74** 032324]. The maximum entanglement property of these states is examined. It also demonstrates how to discriminate among these states in the ion-trap.

Keywords: multipartite maximally entanglement, ion-trap

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1. Introduction

In recent years, entanglement, owing to its importance in quantum information science,[1,2] has attracted much attention. Up to now, bipartite entanglement has been completely explored and well understood,[3] but multipartite entanglement has been still under extensive explorations[4−11] and much attention has been focused on it.[12] Recently, some types of multipartite entangled states, such as the GHZ state,[13] W state[14] and linear cluster state[15] have been explored and prepared in different systems.[16−20] Besides these states, in researching for faithful teleportation of an arbitrary two-qubit state, Yeo and Chua[21] introduced a genuine four-qubit entangled state:

$$\begin{aligned}&\left|\chi^{00}\right\rangle_{3214}\\ =&\frac{1}{2\sqrt{2}}(|0000\rangle-|0011\rangle-|0101\rangle+|0110\rangle\\ &+|1001\rangle+|1010\rangle+|1100\rangle+|1111\rangle)_{3214}.\end{aligned}$$

This state does not belong to the well-known three types of multipartite entangled states mentioned above and has many interesting entanglement properties.[21] Importantly, it has many applications in quantum information processing, such as teleportation and dense coding.[21] Thus, many researches have focused on looking for concrete systems for generating this state. For example, Shen *et al.*[22] put forward a scheme for generating it with linear optical elements and a photon detector, Wang and Yang[23] proposed a scheme to obtain it in an ion trap system, He and Jiang,[24] and Lin[25] gave a scheme to produce it in the cavity QED system. Later, to faithfully teleport an arbitrary N-qubit state, Chen, Zhu and Guo[26] extended such a type of state to $2N$-qubit case. In their protocol, the $2N$-qubit maximally entangled channel state, which cannot be reducible to the tensor product of N Bell states, takes the following form:

$$\begin{aligned}&|\mathrm{MES}\rangle_{A_1,\ldots,A_N;B_1,\ldots,B_N}\\ =&\frac{1}{\sqrt{2^N}}\sum_{J=1}^{2^N}|J\rangle_A|J'\rangle_B,\end{aligned}\tag{1}$$

or

$$\begin{aligned}&|\mathrm{MES}\rangle_{A_1,\ldots,A_N;B_1,\ldots,B_N}\\ =&U_1(AN)\,U_2(BN)\times|G_{1,\ldots,1}\rangle_{A_1,\ldots,A_N;B_1,\ldots,B_N},\end{aligned}\tag{2}$$

where states $|J\rangle_A$ and states $|J'\rangle_B$ are orthogonal bases of the Hilbert spaces of Alice's N qubits and Bob's N qubits, respectively. The $U_1(AN)$ and $U_2(BN)$ are unitary operators acting on Alice's N qubits and Bob's N qubits, respectively. $|G_{1,\ldots,1}\rangle_{A_1,\ldots,A_N;B_1,\ldots,B_N}$ is a tensor product of N Bell states,[27] $|G_{1,\ldots,1}\rangle_{A_1,\ldots,A;B_1,\ldots,B_N}=|\phi\rangle_{A_1B_1}\otimes|\phi\rangle_{A_2B_2}\otimes\cdots\otimes|\phi\rangle_{A_NB_N}$, where $|\phi\rangle=(1/\sqrt{2})(|00\rangle+|11\rangle)$.

In this paper, we propose a method for creating such a type of $2N$-qubit entangled states as shown in Eq. (2) in an ion-trap system. The paper is organized as follows: in Section 2, we show the scheme to create

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†Corresponding author. E-mail: jiangnq@wzu.edu.cn
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the $2N$-qubit maximally entangled channel state and study its entanglement property by using the criterion for genuine multipartite entanglement quantum channels introduced in Ref. [28]. Then in Section 3, the scheme for generating and distinguishing a set of orthonormal multipartite maximally entangled channel states is presented. Finally, the paper ends with a discussion and conclusion.

2. Multipartite maximally entangled state created in ion-trap and its entanglement property

We consider that $2N$ identical two level ions are confined in a linear Paul trap. The ground state and the excited state of each ion are denoted as $|0\rangle$ and $|1\rangle$ respectively. We drive the neighbouring two ions, i.e., the lth and $(l+1)$th ($l=1,2,\ldots,(2N-1)$) ions, by two classical homogeneous lasers with frequencies $\omega+\nu+\delta$ and $\omega-\nu-\delta$, respectively. Here ω is the frequency of the transition $|1\rangle \leftrightarrow |0\rangle$, ν is the frequency of the centre-of-mass mode of the collective motion of the ions and δ is the detuning. Assuming $\delta \ll \nu$ and considering that ν is much larger than other characteristic frequencies, in the Lamb–Dicke regime we find that the Hamiltonian in the interaction picture is[16]

$$H^{\rm i}_{l,l+1} = {\rm i}\eta\Omega\,{\rm e}^{-{\rm i}\varphi}\sum_{j=l}^{l+1}\sigma_j^+(a^+\,{\rm e}^{-{\rm i}\delta t}+a\,{\rm e}^{{\rm i}\delta t})+{\rm H.c.}, \quad (3)$$

where $a^+(a)$ denotes the creation (annihilation) operator for the vibrational mode, $\sigma_j^+ = |1_j\rangle\langle 0_j|$ and $\sigma_j^- = |0_j\rangle\langle 1_j|$ are the spin flip operators, Ω and φ are the Rabi frequency and phase of the laser fields and $\eta = \sqrt{k^2/4\nu m}$ is the Lamb–Dicke parameter with k being the effective wave vector of the laser fields along the direction of the string of the ions and m being the mass of an ion. When $\delta \gg \eta\Omega$, the resonant transitions are $|1_l 1_{l+1} n\rangle \leftrightarrow |0_l 0_{l+1} n\rangle$ and $|0_l 1_{l+1} n\rangle \leftrightarrow |1_l 0_{l+1} n\rangle$. The Rabi frequencies for $|1_l 1_{l+1} n\rangle \leftrightarrow |0_l 0_{l+1} n\rangle$ mediated by $|0_l 1_{l+1} n\pm 1\rangle$ and $|1_l 0_{l+1} n\pm 1\rangle$ and for$|0_l 1_{l+1} n\rangle \leftrightarrow |1_l 0_{l+1} n\rangle$ mediated by $|1_l 1_{l+1} n\pm 1\rangle$ and $|0_l 0_{l+1} n\pm 1\rangle$ are given by $\lambda = 2(\Omega\eta)^2/\delta$. Then the effective Hamiltonian can be described by[16]

$$H^{\rm e}_{l,l+1} = \frac{\lambda}{2}\sum_{j=l}^{l+1}(|1_j\rangle\langle 1_j| + |0_j\rangle\langle 0_j|) + \lambda(\sigma_l^+\sigma_{l+1}^+ + \sigma_l^+\sigma_{l+1}^- + {\rm H.c.}). \quad (4)$$

Noticing $|1_j\rangle\langle 1_j| + |0_j\rangle\langle 0_j| = I$ and using Eq. (4), we can express the evolution operator of the system as follows (let $\hbar = 1$):

$$U_{l,l+1}(t) = {\rm e}^{-{\rm i}\lambda t\sigma_l^x\sigma_{l+1}^x} = \cos(\lambda t) - {\rm i}\sin(\lambda t)\,\sigma_l^x\sigma_{l+1}^x, \quad (5)$$

where $\sigma_l^x = \sigma_l^+ + \sigma_l^-$ and a common phase factor ${\rm e}^{-{\rm i}\lambda t}$ has been discarded. So, the evolution of a pair of neighbouring ions l and $l+1$ obeys

$$\begin{aligned} U(t)\,|00\rangle &= \cos(\lambda t)\,|00\rangle - {\rm i}\sin(\lambda t)\,|11\rangle,\\ U(t)\,|01\rangle &= \cos(\lambda t)\,|01\rangle - {\rm i}\sin(\lambda t)\,|10\rangle,\\ U(t)\,|10\rangle &= \cos(\lambda t)\,|10\rangle - {\rm i}\sin(\lambda t)\,|01\rangle,\\ U(t)\,|11\rangle &= \cos(\lambda t)\,|11\rangle - {\rm i}\sin(\lambda t)\,|00\rangle. \end{aligned} \quad (6)$$

Assume that the initial state of the system of $2N$ ions is $|00\ldots0\rangle_{1,2,\ldots,2N}$. We drive every pair of neighboring two ions by the above mentioned lasers with duration $\tau = \pi/4\lambda$, respectively. Then the state of the ionic system evolves into

$$|\psi_{00\ldots0}\rangle_{1,2,\ldots,2N} = U_{2N-1,2N}(\tau)\ldots U_{23}(\tau)U_{12}(\tau)\,|00\ldots0\rangle_{1,2,\ldots,2N}, \quad (7)$$

where $U_{l,l+1}(\tau) = (I - {\rm i}\sigma_l^x\sigma_{l+1}^x)/\sqrt{2}$. This state can also be expressed as

$$\begin{aligned} &|\psi_{00\ldots0}\rangle_{1,2,\ldots,2N}\\ &= V_{2N-2,2N}\otimes\cdots\otimes V_{4,6}\otimes V_{2,4}\left[\frac{1}{\sqrt{2}}(|00\rangle - {\rm i}\,|11\rangle)_{1,2}\right]\otimes\left[\frac{1}{\sqrt{2}}(|00\rangle - {\rm i}\,|11\rangle)_{3,4}\right]\otimes\cdots\\ &\quad\otimes\left[\frac{1}{\sqrt{2}}(|00\rangle - {\rm i}\,|11\rangle)_{2N-1,2N}\right]\\ &= V_{2N-2,2N}\otimes\cdots\otimes V_{4,6}\otimes V_{2,4}\otimes S_{2N-1}\otimes\cdots\otimes S_3\otimes S_1\,|G_{1,\ldots,1}\rangle\underbrace{1,3,\ldots,2N-1}_{A};\underbrace{2,4,\ldots,2N}_{B}\\ &= U_1'(AN)\,U_2'(BN)\,|G_{1,\ldots,1}\rangle\underbrace{1,3,\ldots,2N-1}_{A};\underbrace{2,4,\ldots,2N}_{B}, \end{aligned} \quad (8)$$

where

$$U_1'(AN) = S_{2N-1} \otimes \cdots \otimes S_3 \otimes S_1$$

with

$$S_j = |0_j\rangle\langle 0_j| - \mathrm{i}\,|1_j\rangle\langle 1_j|,$$
$$(j = 1, 3, 5, \ldots, 2N-1);$$

$$U_2'(BN) = V_{2N-2,2N} \otimes \cdots \otimes V_{4,6} \otimes V_{2,4}$$

with

$$V_{2l,2(l+1)}(\tau) = \left(I + \mathrm{i}\sigma_{2l}^x \sigma_{2(l+1)}^y\right)/\sqrt{2},$$

$$\sigma_{2(l+1)}^y = \mathrm{i}\left(\sigma_{2(l+1)}^+ - \sigma_{2(l+1)}^-\right),$$
$$(l = 1, 2, \ldots, N-1);$$

and the state $|G_{1,\ldots,1}\rangle\underbrace{1,3,\ldots,2N-1}_{A};\underbrace{2,4,\ldots,2N}_{B}$ is a tensor product of N Bell states $|\phi\rangle_{1,2} \otimes |\phi\rangle_{3,4} \otimes \cdots \otimes |\phi\rangle_{2N-1,2N}$, with $|\phi\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$. Equation (8) shows that $|\psi_{00\ldots0}\rangle_{1,2,\ldots,2N}$ is just the type of $2N$-qubit maximally entangled channel state as that in Eq. (2). The following, we will use the criterion for genuine multipartite entanglement quantum channels introduced in Ref. [28] to examine the maximum entanglement property of this state.

According to Ref. [28], Eq. (7) can also be expressed as

$$\begin{aligned} &|\psi_{00\ldots0}\rangle_{1,2,\ldots,2N} \\ &= \sum_{i_1,i_2,\ldots,i_N=0}^{1} \boldsymbol{M}_{12\ldots N}\,|i_1 i_2 \ldots i_N\rangle\,|i_1 i_2 \ldots i_N\rangle \\ &= \sum_{K=0}^{2^{2N}-1} a_K \left|\tilde{K}\right\rangle_{12\ldots2N}, \end{aligned} \tag{9}$$

where the operator $\boldsymbol{M}_{12\ldots N}$ operates on particles 1,2 ,..., N. The $\tilde{K}$ is the binary form of integer K, namely, $K = 2^{2N-1}i_1 + 2^{2N-2}i_2 + \cdots + 2^0 i_{2N}$ and $\tilde{K} = i_1 i_2 \ldots i_{2N}$. The complex number a_K satisfies the condition $\sum_{K=0}^{2^{2N}-1} |a_K|^2 = 1$. The matrix form of operator $\boldsymbol{M}_{12\ldots N}$ (The states $|0\rangle$ and $|1\rangle$ take the matrix representations $|0\rangle = (1\ 0)^{\mathrm{T}}$ and $|1\rangle = (0\ 1)^{\mathrm{T}}$) is

$$\boldsymbol{M}_{12\cdots N} = \begin{pmatrix} a_0 & a_1 & a_2 & \cdots & a_{2^N-1} \\ a_{1\times 2^N} & a_{1\times 2^N+1} & a_{1\times 2^N+2} & \cdots & a_{1\times 2^N+2^N-1} \\ a_{2\times 2^N} & a_{2\times 2^N+1} & a_{2\times 2^N+2} & \cdots & a_{2\times 2^N+2^N-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{(2^N-1)\times 2^N} & a_{(2^N-1)\times 2^N+1} & a_{(2^N-1)\times 2^N+2} & \cdots & a_{(2^N-1)\times 2^N+2^N-1} \end{pmatrix}.$$

From the matrix $\boldsymbol{M}_{12\ldots N}$, we can derive matrix $\boldsymbol{M}_{j_1 j_2 \ldots j_N}$ which operates on arbitrary N particles $j_1, j_2, \ldots, j_N$, where $j_k \in \{1, 2, \ldots, 2N\}$, $k = 1, 2, \ldots, N$. For convenience, we let $j_1 < j_2 < \cdots < j_N$ and $j_{N+1} < j_{N+2} < \cdots < j_{2N}$. Then the matrix element b_m of the matrix $\boldsymbol{M}_{j_1 j_2 \ldots j_N}$ is related to the matrix element a_K of the matrix $\boldsymbol{M}_{12\ldots N}$ by

$$b_m = a_K, \tag{10}$$

where the subscripts m and K are related as follows: if the binary form of the integer m is $\lambda_1\lambda_2 \ldots \lambda_{2N}$ (i.e., $m = 2^{2N-1}\lambda_1 + 2^{2N-2}\lambda_2 + \ldots + 2^0\lambda_{2N}$, $\lambda_i \in \{0, 1\}$), then the integer $K = \sum_{i=1}^{2N} \lambda_i 2^{2N-j_i}$. Using the matrix $\boldsymbol{M}_{j_1 j_2 \ldots j_N}$, we can examine the maximum entanglement between the subsystems of state $|\psi_{00\ldots0}\rangle_{1,2,\ldots,2N}$. If $\boldsymbol{M}_{j_1 j_2 \ldots j_N}\boldsymbol{M}_{j_1 j_2 \ldots j_N}^{\dagger} = \boldsymbol{I}/2^N$, there exists maximum entanglement between particles $j_1, j_2, \ldots, j_N$ and $j_{N+1}, j_{N+2}, \ldots, j_{2N}$. If $\boldsymbol{M}_{j_1 j_2 \ldots j_N}\boldsymbol{M}_{j_1 j_2 \ldots j_N}^{\dagger} \neq \boldsymbol{I}/2^N$, particles $j_1, j_2, \ldots, j_N$ and $j_{N+1}, j_{N+2}, \ldots, j_{2N}$ are not maximally entangled, but the entanglement between them is quantified by von Neumann entropy[29] as $E(\boldsymbol{\rho}_{j_1 j_2 \ldots j_N}) = -\sum_i \lambda_i \log \lambda_i$, where λ_i is the eigenvalue of the matrix $\boldsymbol{\rho}_{j_1 j_2 \ldots j_N} = \boldsymbol{M}_{j_1 j_2 \ldots j_N}\boldsymbol{M}_{j_1 j_2 \ldots j_N}^{\dagger}$. For example, when $N = 3$, the state in Eqs. (7)–(9) reduces to $|\psi_{00\ldots0}\rangle_{1,2,\ldots,6}$. For this state, we can obtain $\boldsymbol{M}_{135}\boldsymbol{M}_{135}^{\dagger} = \boldsymbol{M}_{136}\boldsymbol{M}_{136}^{\dagger} = \boldsymbol{M}_{145}\boldsymbol{M}_{145}^{\dagger} = \boldsymbol{M}_{146}\boldsymbol{M}_{146}^{\dagger} = \boldsymbol{I}/2\sqrt{2}$, so there exist maximum entanglements between ions $(1, 3, 5)$ and $(2, 4, 6)$, $(1, 3, 6)$ and $(2, 4, 5)$, $(1, 4, 5)$ and $(2, 3, 6)$, $(1, 4, 6)$ and $(2, 3, 5)$. There does not exist maximum entanglements between the remaining pairs of the state, the von Neumann entropy between these pairs are $E(\rho_{123}) = 1$ and $E(\rho_{124}) = E(\rho_{125}) = E(\rho_{126}) = E(\rho_{134}) = E(\rho_{156}) = 2$.

3. Generating and distinguishing a set of orthonormal multipartite maximally entangled states

We consider that the initial state of $2N$ identical ions, which are confined in the linear Paul trap mentioned above, is an arbitrary state $|m_1 m_2 \ldots m_{2N}\rangle_{1,2,\ldots,2N}$, where $m_j \in \{0,1\}, j = 1,2,\ldots,2N$. We drive every pair of neighbouring ions, respectively, by the above mentioned lasers with duration $\tau = \pi/4\lambda$. Then the evolved state of the ionic system will be

$$\begin{aligned}
&|\psi_{m_1 m_2 \ldots m_{2N}}\rangle_{1,2,\ldots,2N} \\
&= U_{2N-1,2N} \ldots U_{23} U_{12} \, |m_1 m_2 \ldots m_{2N}\rangle_{1,2,\ldots,2N} \\
&= U_{2N-1,2N} \ldots U_{23} U_{12} (\sigma_{2N}^x)^{m_{2N}} \ldots (\sigma_2^x)^{m_2} (\sigma_1^x)^{m_1} \\
&\quad \times |00\ldots0\rangle_{1,2,\ldots,2N} \\
&= (\sigma_{2N}^x)^{m_{2N}} \ldots (\sigma_2^x)^{m_2} (\sigma_1^x)^{m_1} U_{2N-1,2N} \ldots U_{23} U_{12} \\
&\quad \times |00\ldots0\rangle_{1,2,\ldots,2N} \\
&= (\sigma_{2N}^x)^{m_{2N}} \ldots (\sigma_2^x)^{m_2} (\sigma_1^x)^{m_1} \, |\psi_{00\ldots0}\rangle_{1,2,\ldots,2N} . \quad (11)
\end{aligned}$$

It is not difficult to find that ${}_{1,2,\ldots,2N}\langle\psi_{m_1' m_2' \ldots m_{2N}'}| \psi_{m_1 m_2 \ldots m_{2N}}\rangle_{1,2,\ldots,2N} = \delta_{m_1 m_1'} \delta_{m_2 m_2'} \ldots \delta_{m_{2N} m_{2N}'}$, so $\{|\psi_{m_1 m_2 \ldots m_{2N}}\rangle_{1,2,\ldots,2N}, \; m_j \in (0,1)\}$ constitute a set of 2^{2N} orthonormal basis states. For these states, the entanglement is the same as the state in Eq. (8), they are all the type of $2N$-qubit maximally entangled channel state as those in Eq. (2). Next, we show how to deterministically distinguish among these basis states in the ion-trap mentioned above. Assume that the $2N$ trapped ions are initially in the state $|\psi_{m_1 m_2 \ldots m_{2N}}\rangle_{1,2,\ldots,2N}$, we drive the neighbouring two ions respectively by the above mentioned lasers with duration $\tau = \pi/4\lambda$, then the state of the ionic system will be evolved as follows:

$$\begin{aligned}
&|\psi_{m_1 m_2 \ldots m_{2N}}\rangle_{1,2,\ldots,2N} \\
&\to \mathrm{i}(\sigma_1^x)^{m_1} (\sigma_{2N}^x)^{m_{2N}} \, |m_1 m_2 \ldots m_{2N}\rangle_{1,2,\ldots,2N} . \quad (12)
\end{aligned}$$

Equation (12) shows that each of the 2^{2N} orthonormal basis states $\{|\psi_{m_1 m_2 \ldots m_{2N}}\rangle_{1,2,\ldots,2N}, \; m_j \in (0,1)\}$ can be transformed into a corresponding product state of $2N$ ions. So, a deterministic measurement for these basis states can be achieved by individual detection of the related ions.

4. Experimental feasibility

Now we give a brief discussion on the experimental matters. Based on the current ion-trap techniques,[30] we can use one Zeeman level of the $S_{1/2}$ ground state of $^{40}Ca^+$ ions as the state $|0\rangle$ and one Zeeman level of the metastable $D_{2/5}$ state as the state $|1\rangle$. The lifetime of the metastable $D_{2/5}$ state is $\tau_l \approx 1.16$ s. With the accessible centre-of-mass mode frequency $\upsilon = 1.2$ MHz,[31] we can choose $\delta = 0.1\upsilon$, $\eta = 0.1$, $\Omega = 0.1\upsilon$, thus the condition $\Omega\eta \ll \delta \ll \upsilon$ can be satisfied. Then the mean phonon number n of the centre-of-mass mode should be less than 3 in order to satisfy $\eta\sqrt{n+1} \ll 1$, i.e., the Lamb–Dicke criterion. According to recent experiments,[30–33] these parametric values are available, with which the duration of our schemes is only $3\tau = \pi\delta/(8\eta^2\Omega^2) \approx 1\mu$ s, much shorter than τ_l. In addition, it is experimentally achievable to precisely control the laser-illuminating time and locate the ions in a trap.[29–32] Thus our schemes are realizable with the present ion-trap techniques.

5. Conclusion

In conclusion, the multipartite maximally entangled states[26] have many interesting properties and possible applications in quantum information processing (QIP) and fundamental tests of quantum physics.[22] Therefore, it is meaningful to explore them in concrete systems. In this paper, we propose a simple scheme for generating such a type of multipartite entangled states in an ion-trap system. We examine the entanglement property of these states by using the criterion for genuine multipartite entanglement quantum channels[26] and the von Neumann entropy.[30] Moreover, we demonstrate how to discriminate among this type of 2^{2N} basis state in an ion-trap. It has been shown that these 2^{2N} basis states can be deterministically distinguished.

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