Scheme for implementing perfect quantum teleportation with four-qubit entangled states in cavity quantum electrodynamics

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Recently, Peng *et al.* [2010 *Eur. Phys. J.* D **58** 403] proposed to teleport an arbitrary two-qubit state with a family of four-qubit entangled states, which simultaneously include the tensor product of two Bell states, linear cluster state and Dicke-class state. This paper proposes to implement their scheme in cavity quantum electrodynamics and then presents a new family of four-qubit entangled state $|\Omega_4\rangle_{1234}$. It simultaneously includes all the well-known four-qubit entangled states which can be used to teleport an arbitrary two-qubit state. The distinct advantage of the scheme is that it only needs a single setup to prepare the whole family of four-qubit entangled states, which will be very convenient for experimental realization. After discussing the experimental condition in detail, we show the scheme may be feasible based on present technology in cavity quantum electrodynamics.

Keywords: entanglement, teleportation, cavity quantum electrodynamics

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1. Introduction

Quantum entanglement is a crucial resource in quantum information processing protocols such as quantum teleportation,[1] dense coding[2] and quantum information splitting.[3] In order to realize a given quantum information processing task perfectly, it may require a particular class of entangled states. For faithful teleportation of an arbitrary two-qubit state, it is known that the four-qubit Greenberger–Horne–Zeilinger (GHZ) state[4] and W state[5] are not the ideal quantum channels. Two well-known classes of genuine four-qubit entangled states, which can be used for perfect teleportation of an arbitrary two-qubit state, are the cluster-class state[6] and the $|\chi\rangle$-class state.[7] Recently, Peng *et al.*[8] have shown that the Dicke-class states are also good candidates for achieving this task. In their scheme, they present a family of four-qubit entangled states $|\overline{D}_4^{(2)}\rangle$ as follows:

$$\begin{aligned}|\overline{D}_4^{(2)}\rangle_{1234} = \frac{1}{2}(&\,\mathrm{e}^{\mathrm{i}\phi_1}|ggee\rangle + \cos\theta|gege\rangle \\ &- \sin\theta\,\mathrm{e}^{-\mathrm{i}\phi_2}|geeg\rangle \sin\theta\,\mathrm{e}^{\mathrm{i}\phi_2}|egge\rangle \\ &+ \cos\theta|egeg\rangle + \mathrm{e}^{\mathrm{i}\phi_3}|eegg\rangle)_{1234}, \quad (1)\end{aligned}$$

where θ and ϕ_i $(i = 1, 2, 3)$ are real parameters. With the bipartition 12|34, they demonstrate that $|\overline{D}_4^{(2)}\rangle$ have the maximal bipartite entanglement of two ebits. Thus, $|\overline{D}_4^{(2)}\rangle$ can be used to implement perfect teleportation of an arbitrary two-qubit state. In fact, $|\overline{D}_4^{(2)}\rangle$ will reduce to the tensor product of two Bell states $|\psi^+\rangle_{14}|\psi^+\rangle_{23}$ or $|\psi^-\rangle_{14}|\psi^-\rangle_{23}$ $[|\psi^\pm\rangle_{ij} = (1/\sqrt{2})(|ge\rangle \pm |eg\rangle)_{ij}]$ if $\theta = n\pi$ and $\phi_i = 0$ and will reduce to the four-qubit cluster-class states $|C_4^\pm\rangle = (1/2)(|ggee\rangle \mp |geeg\rangle \pm |egge\rangle + |eegg\rangle)_{1234}$ with the choice of the parameters $\theta = (n+1/2)\pi, \phi_i = 0$. Generally, they are four-qubit Dicke-class states.[9]

It is known that the key steps to implement quantum teleportation are the preparation of entangled quantum channel and the discrimination of orthogonal measurement bases. Generally speaking, a single experimental setup can just be used to prepare a particular class of entangled states only. In Ref. [10], Ye *et al.* proposed to prepare the tensor product of two Bell states in cavity quantum electrodynamics (QED). In ion trap, Wang and Yang[11] presented the preparation and discrimination scheme of the $|\chi\rangle$-class state.[7] Also, various schemes have been devoted to preparing the four-qubit linear cluster-class states in quantum optics,[12] cavity QED[13,14] and ion trap.[15] However, as one of the promising candidates for teleporting an arbitrary two-qubit state, the four-qubit Dicke-class states[8] have not been prepared in any

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physical system yet. On the other hand, Wieczorek *et al.*[16] have shown that an entire family of four-qubit entangled states can be prepared with just a single optical setup, which breaks up the inflexibility of usually preparing multipartite entangled states and brings great convenience to the practical quantum information processing. Up to now, there is no scheme to prepare all these four-qubit entangled states mentioned above with the same experimental setup.

Inspired by Refs. [8] and [16], we propose a feasible scheme to generate a family of four-qubit entangled states $|\tau_4\rangle$ with just a single setup in cavity QED. By adjusting the interaction time appropriately, $|\tau_4\rangle$ will also reduce to the tensor product of two Bell states, linear cluster state and Dicke-class state respectively just as the family of four-qubit entangled states $|\overline{D}_4^{(2)}\rangle$ in Ref. [8]. We also propose a scheme for discrimination of the orthogonal four-qubit cluster states from Dicke-class states, thus we can implement perfect quantum teleportation of an arbitrary two-qubit state in cavity QED. After discussing the experimental condition in detail, we show that based on present technology in cavity QED our proposed schemes may be realizable. Lastly, we also present a new family of four-qubit entangled states and it simultaneously includes all the well-known four-qubit entangled states which can be used to teleport an arbitrary two-qubit state.

2. Teleportation with the family of four-qubit entangled states $|\tau_4\rangle$

2.1. Preparation of the family of four-qubit entangled states $|\tau_4\rangle$

We first consider four identical two-level atoms simultaneously interacting with two single-mode cavities respectively, i.e., atoms 1, 2 interact with cavity C_1 and atoms 3, 4 interact with cavity C_2. In the interaction picture, the interaction Hamiltonian is

$$H_{\rm I1} = g\Big[\sum_{j=1,2}(\mathrm{e}^{-\mathrm{i}\delta t}a_1^\dagger S_j^- + \mathrm{e}^{\mathrm{i}\delta t}a_1 S_j^+) + \sum_{j=3,4}(\mathrm{e}^{-\mathrm{i}\delta t}a_2^\dagger S_j^- + \mathrm{e}^{\mathrm{i}\delta t}a_2 S_j^+)\Big], \quad (2)$$

where $S_j^+ = |e\rangle_{jj}\langle g|$ and $S_j^- = |g\rangle_{jj}\langle e|$, with $|e\rangle_j$ and $|g\rangle_j$ $(j = 1, 2, 3, 4)$ being the excited and ground states of the jth atom respectively, $a_k^\dagger$ and a_k $(k = 1, 2)$ are the creation and annihilation operators for the cavity mode respectively, g is the atom–cavity coupling strength, and δ is the detuning between the atomic transition frequency ω_0 and cavity field frequency ω. In the case of $\delta \gg g$, there is no energy exchange between the atomic system and the cavity field. According to Ref. [17], we can obtain the effective Hamiltonian as follows:

$$H_{\rm e1} = \lambda\Big[\sum_{j=1,2}(|e\rangle_{jj}\langle e|a_1a_1^\dagger - |g\rangle_{jj}\langle g|a_1^\dagger a_1) + (S_1^+S_2^- + S_1^-S_2^+) + \sum_{j=3,4}(|e\rangle_{jj}\langle e|a_2a_2^\dagger - |g\rangle_{jj}\langle g|a_2^\dagger a_2) + (S_3^+S_4^- + S_3^-S_4^+)\Big], \quad (3)$$

where $\lambda = g^2/\delta$. If the cavity field is initially in the vacuum state, the effective Hamiltonian will reduce to

$$H_{\rm e2} = \lambda\Big[\sum_{j=1}^{4}|e\rangle_{jj}\langle e| + (S_1^+S_2^- + S_1^-S_2^+) + (S_3^+S_4^- + S_3^-S_4^+)\Big]. \quad (4)$$

Thus, the evolution operator of the whole system is $U_1(t) = \exp(-\mathrm{i}H_{\rm e2}t)$. We assume that four atoms 1, 2, 3 and 4 are initially in the state $|gege\rangle_{1234}$, and the evolution of the system is given by

$$|gege\rangle_{1234} \to \mathrm{e}^{-2\mathrm{i}\lambda t}[\cos(\lambda t)|ge\rangle_{12} - \mathrm{i}\sin(\lambda t)|eg\rangle_{12}] \times [\cos(\lambda t)|ge\rangle_{34} - \mathrm{i}\sin(\lambda t)|eg\rangle_{34}]. \quad (5)$$

If we choose the interaction time $t_1 = \pi/(4\lambda)$, we can obtain the following evolution:

$$|gege\rangle_{1234} \to -\frac{\mathrm{i}}{2}(|ge\rangle_{12} - \mathrm{i}|eg\rangle_{12})(|ge\rangle_{34} - \mathrm{i}|eg\rangle_{34}). \quad (6)$$

Secondly, we let atoms 2 and 3 interact with the same single-mode cavity C_3. Similarly, we can obtain the effective Hamiltonian as follows:

$$H_{\rm e3} = \lambda\Big[\sum_{j=2,3}|e\rangle_{jj}\langle e| + (S_2^+S_3^- + S_2^-S_3^+)\Big]. \quad (7)$$

The evolution operator is $U_2(t) = \exp(-\mathrm{i}H_{\rm e3}t)$. For the different initial states of atoms 2 and 3, we can obtain the following evolutions:

$$|gg\rangle_{23} \to |gg\rangle_{23}, \quad (8)$$

$$|ge\rangle_{23} \to \mathrm{e}^{-\mathrm{i}\lambda t}[\cos(\lambda t)|ge\rangle_{23} - \mathrm{i}\sin(\lambda t)|eg\rangle_{23}], \quad (9)$$

$$|eg\rangle_{23} \to \mathrm{e}^{-\mathrm{i}\lambda t}[\cos(\lambda t)|eg\rangle_{23} - \mathrm{i}\sin(\lambda t)|ge\rangle_{23}], \quad (10)$$

$$|ee\rangle_{23} \rightarrow \mathrm{e}^{-2\mathrm{i}\lambda t}|ee\rangle_{23}. \quad (11)$$

Then the state of atoms 1, 2, 3 and 4 will be

$$\begin{aligned}&|\tau_4\rangle_{1234}\\ =& -\frac{\mathrm{i}}{2}\{\mathrm{e}^{-\mathrm{i}\lambda t}|ge\rangle_{14}[\cos(\lambda t)|eg\rangle - \mathrm{i}\sin(\lambda t)|ge\rangle]_{23}\\ &- \mathrm{e}^{-\mathrm{i}\lambda t}|eg\rangle_{14}[\cos(\lambda t)|ge\rangle - \mathrm{i}\sin(\lambda t)|eg\rangle]_{23}\\ &- \mathrm{i}|ee\rangle_{14}|gg\rangle_{23} - \mathrm{i}\,\mathrm{e}^{-\mathrm{i}2\lambda t}|gg\rangle_{14}|ee\rangle_{23}\}. \quad (12)\end{aligned}$$

Now we can calculate the bipartite entanglement of $|\tau_4\rangle_{1234}$ with the bipartition 14|23. It is easy to obtain $\rho_{14} = \mathrm{Tr}_{23}(|\tau_4\rangle_{1234}\langle\tau_4|) = I_{14}/4$ and $\rho_{23} = \mathrm{Tr}_{14}(|\tau_4\rangle_{1234}\langle\tau_4|) = I_{23}/4$. Using the von Neumann entropy of the reduced density operator of the subsystem, we know that there are two ebits of entanglement shared between the subsystems 1, 4 and 2, 3. In the following, we will show that $|\tau_4\rangle_{1234}$ will also reduce to the tensor product of two Bell states, cluster state and Dicke-class state respectively with the different choices of the interaction time, which is similar to the family of four-qubit entangled states $|\overline{D}_4^{(2)}\rangle$.[8]

Choosing the interaction time $t_2 = \pi/\lambda$, we can obtain the product state of two Bell states as follow:

$$|\psi_4\rangle = |\phi^-\rangle_{12} \otimes |\phi^-\rangle_{34}, \quad (13)$$

where $|\phi^-\rangle = (1/\sqrt{2})(|ge\rangle - \mathrm{i}|eg\rangle)$. Note that $|\psi_4\rangle$ is the same as that in Eq. (5).

If we choose the interaction time $t_3 = \pi/(2\lambda)$, $|\tau_4\rangle_{1234}$ will reduce to the following four-qubit entangled state:

$$\begin{aligned}|C_4\rangle_{1234} =& \frac{1}{2}(|gg\rangle_{14}|ee\rangle_{23} + \mathrm{i}|ge\rangle_{14}|ge\rangle_{23}\\ &- \mathrm{i}|eg\rangle_{14}|eg\rangle_{23} - |ee\rangle_{14}|gg\rangle_{23}). \quad (14)\end{aligned}$$

By performing the local operations $|e\rangle_2 \rightarrow \mathrm{i}|e\rangle_2$ and $|e\rangle_3 \rightarrow \mathrm{i}|e\rangle_3$ (which can be realized by letting atoms 2, 3 interact with an appropriate classical field) on atom 2 and 3 respectively, it will become the linear four-qubit cluster state.[6]

Generally, $|\tau_4\rangle_{1234}$ is a four-qubit Dicke-class state. With the choice of the interaction time $t_4 = t_1 = \pi/(4\lambda)$, we will obtain the following Dicke-class state:

$$\begin{aligned}|\overline{\tau}_4\rangle_{1234} =& \frac{1}{2\sqrt{2}}[\mathrm{i}\sqrt{2}|gg\rangle_{14}|ee\rangle_{23} + \mathrm{e}^{-\mathrm{i}\frac{3\pi}{4}}|ge\rangle_{14}(|eg\rangle\\ &- \mathrm{i}|ge\rangle)_{23} + \mathrm{e}^{\mathrm{i}\frac{\pi}{4}}|eg\rangle_{14}(|ge\rangle - \mathrm{i}|eg\rangle)_{23}\\ &- \sqrt{2}|ee\rangle_{14}|gg\rangle_{23}]. \quad (15)\end{aligned}$$

Now we discuss the experimental feasibility of our preparation schemes. In order to prepare the four-qubit entangled states $|C_4\rangle_{1234}$ and $|\overline{\tau}_4\rangle_{1234}$ in cavity QED, the atomic radiative time should be longer than the time which is needed to prepare the four-qubit entangled states $|C_4\rangle_{1234}$ and $|\overline{\tau}_4\rangle_{1234}$. For Rydberg atoms with principal quantum numbers 49, 50, 51, the radiative time is about 30 ms and the coupling constant is $g = 2\pi\times 24$ kHz.[18] With the setting $\delta = 10g$, the required atom–cavity field interaction time is $t_1 = t_4 = 0.2$ ms and $t_3 = 0.4$ ms. Then the time needed to prepare the cluster state and the Dicke-class state is about 0.6 ms and 0.4 ms, respectively, which are much shorter than the atomic radiative time. Thus, our preparation schemes based on present cavity QED techniques might be feasible.

2.2. Quantum teleportation with four-qubit cluster-class state and Dicke-class state

We suppose that Alice has two qubits a and b in the following unknown state $|\psi\rangle_{ab} = \alpha|gg\rangle_{ab} + \beta|ge\rangle_{ab} + \gamma|eg\rangle_{ab} + \delta|ee\rangle_{ab}$, where α, β, γ and δ are unknown complex numbers such that $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Alice and Bob share the general four-qubit entangled state $|\tau_4\rangle_{1234}$ where Alice has qubits 2, 3 and Bob has qubits 1, 4. Now Alice wants to teleport the unknown two-qubit state $|\psi\rangle_{ab}$ to Bob and then the whole system can be rewritten as

$$|\psi\rangle_{ab}|\tau_4\rangle_{1234} = \frac{1}{4}\sum_{i,j=0}^{3}(\sigma_a^i \otimes \sigma_b^j)|\tau_4'\rangle_{ab23}|\psi^{ij}\rangle_{14}, \quad (16)$$

where

$$\begin{aligned}&|\tau_4'\rangle_{ab23}\\ =& -\frac{\mathrm{i}}{2}\{\mathrm{e}^{-\mathrm{i}\lambda t}|ge\rangle_{ab}[\cos(\lambda t)|eg\rangle - \mathrm{i}\sin(\lambda t)|ge\rangle]_{23}\\ &- \mathrm{e}^{-\mathrm{i}\lambda t}|eg\rangle_{ab}[\cos(\lambda t)|ge\rangle - \mathrm{i}\sin(\lambda t)|eg\rangle]_{23}\\ &- \mathrm{i}|ee\rangle_{ab}|gg\rangle_{23} - \mathrm{i}\,\mathrm{e}^{-\mathrm{i}2\lambda t}|gg\rangle_{ab}|ee\rangle_{23}\}, \quad (17)\end{aligned}$$

$$|\psi^{ij}\rangle_{14} = (\sigma_1^i \otimes \sigma_4^j)|\psi\rangle_{14}, \quad (i,j = 0,1,2,3). \quad (18)$$

Here the local operation σ_a^i represents that we perform σ^i on qubit a (where $\sigma^0 = I$ is the identity operator, $\sigma^1 = \sigma_x$, $\sigma^2 = \mathrm{i}\sigma_y$ and $\sigma^3 = \sigma_z$ are the Pauli operators). In order to implement the quantum teleportation protocol, Alice needs to perform a complete projective measurement jointly on qubits a, b, 2 and 3 in the orthogonal bases $\{(\sigma_a^i \otimes \sigma_b^j)|\tau_4'\rangle_{ab23}\}$. In this section, we will mainly discuss the case of $\lambda t = \pi/2$ and $\lambda t = \pi/4$, i.e., the shared quantum channel are the four-qubit cluster state $|C_4\rangle_{1234}$ and Dicke-class state $|\overline{\tau}_4\rangle_{1234}$, respectively. It is noted that we can

transform the Dicke-class state $|\overline{\tau}_4\rangle_{1234}$ into the cluster state $|C_4\rangle_{1234}$ by letting atoms 2 and 3 undergo the evolution $U_2(t_1)$, i.e., $|C_4\rangle_{1234} = U_2(t_1)|\overline{\tau}_4\rangle_{1234}$. Thus, we just need to discuss either the case of $\lambda t = \pi/4$ or $\lambda t = \pi/2$. In the following, we will show how to discriminate the orthogonal bases $\{(\sigma_a^i \otimes \sigma_b^j)|\tau_4'\rangle_{ab23}\}$ in the case of $\lambda t = \pi/2$, i.e.,

$$|C_4^{ij}\rangle_{ab23} = (\sigma_a^i \otimes \sigma_b^j)|C_4^{00}\rangle_{ab23}, \tag{19}$$

where

$$|C_4^{00}\rangle_{ab23} = \frac{1}{2}[|ggee\rangle + \mathrm{i}|gege\rangle - \mathrm{i}|egeg\rangle - |eegg\rangle]_{ab23}. \tag{20}$$

For the case of $\lambda t = \pi/4$, we can deal with it in a similar way.

Now let us consider another model where N atoms simultaneously interact with another same single-mode cavity C_4 and at the same time N atoms are driven by a classical field. In the rotating-wave approximation, the Hamiltonian is (assuming $\hbar = 1$)

$$H = \omega_0 \sum_{j=1}^{N} S_z + \omega a^\dagger a + \sum_{j=1}^{N}[g(a^\dagger S_j + a S_j^+) + \Omega(S_j^+ \mathrm{e}^{-\mathrm{i}\omega_c t} + S_j^- \mathrm{e}^{\mathrm{i}\omega_c t})], \tag{21}$$

where $S_z = (1/2)(|e\rangle_{jj}\langle e| - |g\rangle_{jj}\langle g|)$, Ω is the Rabi frequency of the classical field and ω_c is the classical field frequency. Assuming that $\omega_0 = \omega_c$, the interaction Hamiltonian is

$$H_{I2} = \sum_{j=1}^{N}[\Omega(S_j^+ + S_j^-) + g(\mathrm{e}^{-\mathrm{i}\delta t} a^\dagger S_j^- + \mathrm{e}^{\mathrm{i}\delta t} a S_j^+)]. \tag{22}$$

In the strong driving regime $\Omega \gg \delta \gg g$, there is no energy exchange between the atomic system and the cavity. The effective Hamiltonian of this system can be rewritten as[19]

$$H_{\mathrm{e}4} = 2\Omega S_x + \lambda S_x^2, \tag{23}$$

where $S_x = (1/2)\sum_{j=1}^{N}(S_j^+ + S_j^-)$. Thus, the evolution operation of the system is given by $U_3(t) = \exp(-\mathrm{i}H_{\mathrm{e}4}t)$, which is independent of the cavity field.

In order to discriminate the orthogonal four-qubit cluster states $|C_4^{ij}\rangle_{ab23}$, first we let atoms b and 2 interact with both the single-mode cavity C_4 and the strong classical field. Choosing the interaction time $t_5 = t_3 = \pi/(2\lambda)$, we can obtain the following evolutions:

$$|C_4^{00}\rangle_{ab23} \to \frac{\sqrt{2}}{2}(|ge\rangle_{a3}|ge\rangle_{b2} - |eg\rangle_{a3}|eg\rangle_{b2}), \tag{24}$$

$$|C_4^{01}\rangle_{ab23} \to \frac{\sqrt{2}}{2}(|ge\rangle_{a3}|ee\rangle_{b2} - |eg\rangle_{a3}|gg\rangle_{b2}), \tag{25}$$

$$\vdots$$

$$|C_4^{33}\rangle_{ab23} \to \frac{\sqrt{2}}{2}(|ge\rangle_{a3}|eg\rangle_{b2} - |eg\rangle_{a3}|ge\rangle_{b2}). \tag{26}$$

After making the local operation $|e\rangle_a \to \mathrm{i}|e\rangle_a$ (which can also be realized by letting atom a interact with an appropriate classical field), Alice then lets atoms a, b, 2 and 3 undergo the same evolution $U_3(t_5 = \pi/(2\lambda))$ and can obtain

$$|C_4^{00}\rangle_{ab23} \to |eegg\rangle_{ab23}, \tag{27}$$

$$|C_4^{01}\rangle_{ab23} \to |eggg\rangle_{ab23}, \tag{28}$$

$$\vdots$$

$$|C_4^{33}\rangle_{ab23} \to |egeg\rangle_{ab23}. \tag{29}$$

Now we can find that the orthogonal bases $|C_4^{ij}\rangle_{ab23}$ are transformed into the corresponding product states $\{|gggg\rangle_{ab23}, ..., |eeee\rangle_{ab23}\}$ of four atoms a, b, 2 and 3, respectively. For simplicity, we have discarded the global phase factors in this evolution process. Thus, we can distinguish the sixteen orthogonal four-qubit cluster states $\{|C_4^{ij}\rangle_{ab23} = (\sigma_a^i \otimes \sigma_b^j)|C_4^{00}\rangle_{ab23}(i, j = 0, 1, 2, 3)\}$ by detecting four atoms a, b, 2 and 3 separately. After Alice's measurement, Bob's pair of qubits 1, 4 will collapse into the state $|\psi^{ij}\rangle_{14}$. Then Bob will always succeed in recovering an exact replica of the original state of Alice's qubits a and b by performing the corresponding local operations σ_1^i and σ_4^j on his qubits 1 and 4, while receiving four bits of classical information of the measurement result from Alice. In this way, we can implement quantum teleportation of an arbitrary two-qubit state with the four-qubit cluster state $|C_4\rangle_{1234}$.

On the other hand, we can also discriminate the four-qubit Dicke-class states similar to the case of the cluster state. The unique difference is that Alice must let atoms 2 and 3 undergo the evolution $U_2(t_1)$ first. Thus, we can also complete the teleportation protocol with the Dicke-class state $|\overline{\tau}_4\rangle_{1234}$ in cavity QED.

Now we give a brief discussion on the experimental conditions. In order to complete the whole teleportation protocol, the atomic radiative time should be longer than the time which is needed to prepare the four-qubit entangled states $|C_4\rangle_{1234}$ and $|\overline{\tau}_4\rangle_{1234}$ and to distinguish the orthogonal bases $|C_4^{ij}\rangle_{ab23}$ and $|\overline{\tau}_4^{ij}\rangle_{ab23}$. For Rydberg atoms with principal quantum numbers 49, 50, 51, the radiative time is about 30 ms

and the coupling constant is $g = 2\pi\times24$ kHz.[18] With the setting $\delta = 10g$, the required atom–cavity field interaction time is $t_1 = t_4 = 0.2$ ms and $t_3 = t_5 = 0.4$ ms. Then the time needed to prepare the cluster state and the Dicke-class state is about 0.6 ms and 0.4 ms, respectively. On the other hand, the time for discrimination of four-qubit cluster state and Dicke-class is about 0.8 ms and 1 ms, respectively. Thus, the time needed to complete the whole teleportation procedure is about 1.4 ms which is much shorter than the atomic radiative time. So our scheme based on present cavity QED techniques might be realizable.

3. A new family of four-qubit entangled states

According to Ref. [8], there are three well-known classes of genuine four-qubit entangled states,the cluster-class state,[6] the $|\chi\rangle$-class state[7] and the Dicke-class state,[8] which can be used for perfect teleportation of an arbitrary two-qubit state. In section 2, we have generated a family of four-qubit entangled states $|\tau_4\rangle$ in cavity QED. By adjusting the interaction time appropriately, they will reduce to the tensor product of two Bell states, linear cluster state and Dicke-class state respectively just like the family of four-qubit entangled states in Ref. [8]. However, the $|\chi\rangle$-class state is not included in the family of $|\tau_4\rangle$. Here, we present a new family of four-qubit entangled states $|\Omega_4\rangle$ as follows:

$$\begin{aligned}
&|\Omega_4\rangle_{1234}\\
=\ &\frac{1}{2}(\cos\theta_1|gggg\rangle - \sin\theta_1\,\mathrm{e}^{\mathrm{i}\phi_1}|ggee\rangle\\
&+\ \sin\theta_2\,\mathrm{e}^{-\mathrm{i}\phi_2}|gege\rangle - \cos\theta_2|geeg\rangle + \cos\theta_2|egge\rangle\\
&+\ \sin\theta_2\,\mathrm{e}^{\mathrm{i}\phi_2}\|egeg\rangle + \sin\theta_1\,\mathrm{e}^{-\mathrm{i}\phi_1}|eegg\rangle\\
&+\ \cos\theta_1|eeee\rangle)_{1234}, \qquad (30)
\end{aligned}$$

where θ_1, θ_2, ϕ_1, ϕ_2 are real parameters. When $\phi_1 = \phi_2 = 0$, $|\Omega_4\rangle_{1234}$ is the same as $|\overline{\chi}^{00}\rangle$.[7] In Ref. [7], Yeo and Chua demand that $\theta_1, \theta_2 \neq 0$ or $\pi/2$ in order to avoid $|\overline{\chi}^{00}\rangle$ from being reducible to a product of two Bell states. Indeed, if $\theta_1 = 0, \theta_2 = \pi/2$ or $\theta_1 = \pi/2, \theta_2 = 0$, $|\overline{\chi}^{00}\rangle$ will reduce to the following product states $|\phi^+\rangle_{13}|\phi^+\rangle_{24}$ or $|\psi^-\rangle_{13}|\psi^+\rangle_{24}$ ($|\phi^+\rangle = (1/\sqrt{2})(|gg\rangle + |ee\rangle)$). However, if $\theta_1 = \theta_2 = 0$ or $\theta_1 = \theta_2 = \pi/2$, $|\overline{\chi}^{00}\rangle$ will not reduce to the product state of two Bell states but to the cluster-class states $|C_4^1\rangle = (1/2)(|gggg\rangle - |geeg\rangle + |egge\rangle + |eeee\rangle)$ or $|C_4^2\rangle = (1/2)(-|ggee\rangle + |gege\rangle + |egeg\rangle + |eegg\rangle)$. Generally, if $\theta_1 = n\pi + \pi/2, \theta_2 \neq n\pi + \pi/2$, $|\overline{\chi}^{00}\rangle$ is just the four-qubit Dicke-class state.[8] Thus, $|\Omega_4\rangle_{1234}$ is actually a new family of four-qubit entangled states. It includes all the well-known entangled states which can be used to teleport an arbitrary two-qubit state. In order to implement the teleportation protocol with $|\Omega_4\rangle_{1234}$, we must prepare it in the practical physical system. Unfortunately, it is easy to verify that we can not generate $|\Omega_4\rangle_{1234}$ just like $|\tau_4\rangle_{1234}$ with a single experimental setup and it requires further work.

4. Conclusion

In conclusion, we have proposed to implement quantum teleportation of an arbitrary two-qubit state with four-qubit entangled states. In cavity QED, we present the feasible preparation schemes for the family of four-qubit entangled states $|\tau_4\rangle_{1234}$. By appropriately adjusting the single parameter, i.e., the interaction time, it will also reduce to the tensor product of two Bell states, linear cluster state and Dicke-class state respectively just as the family of four-qubit entangled states in Ref. [8]. Compared with the previous schemes,[10,12−15] the distinct advantage of our schemes is that we can prepare a whole family of four-qubit entangled states with the same experimental setup, which will greatly improve the efficiency of generating entangled states in experiment. In particular, we have proposed a scheme to prepare the four-qubit Dicke-class state,[8] which is one of the most promising entangled states for perfect quantum information processing, for the first time. We also propose a scheme to discriminate the orthogonal four-qubit cluster states and the Dicke-class states, thus can implement perfect quantum teleportation of an arbitrary two-qubit state in cavity QED. Lastly, we present a new family of four-qubit entangled states $|\Omega_4\rangle_{1234}$. It simultaneously includes all the well-known four-qubit entangled states which can be used to teleport an arbitrary two-qubit state. In the future, there is still some work which is valuable in this field. For example, one can investigate how to prepare the family of four-qubit entangled states $|\Omega_4\rangle_{1234}$ in cavity QED or other systems. On the other hand, since there are three well-known four-qubit entangled states which can be used to teleport an arbitrary two-qubit state, one may ask: which one is the best, especially in the noisy channel?[20]

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