Probabilistic remote preparation of a high-dimensional equatorial multiqubit with four-party and classical communication cost*

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(Received 19 October 2010; revised manuscript received 30 December 2010)

This paper presents a protocol for probabilistic remote preparation of a high-dimensional equatorial multiqubit with four-party, consisting of a sender and three receivers. The quantum channel is composed of a partial entangled high-dimensional four-particle state. We calculate the successful total probability and the total classical communication cost required for this scheme. It is shown that both the entangled resources and classical communication cost are greatly reduced.

Keywords: remote state preparation, multiqubit, classical communication cost, four-party

PACS: 03.67.Hk, 03.67.–a, 03.65.–w **DOI:** 10.1088/1674-1056/20/5/050310

1. Introduction

Recently, Lo[1] has proposed an interesting new method to transmit a pure known quantum state by using a prior shared entanglement and some classical communication when the sender fully knows the transmitted state. This communication protocol is called remote state preparation (RSP). Compared with the usual teleportation,[2] the sender Alice in RSP is assumed to know completely the transmitted state and Bob does not know about it at all, while in the teleportation neither Alice nor Bob has a knowledge of the transmitted state. On the other hand, RSP protocol is more economical and requires less classical communication than teleportation.[3] Bennett *et al.*[4] have investigated the tradeoff between the required entanglement and classical communication cost in RSP. After this, the researchers have presented various kinds of theoretical RSP schemes,[4−20] such as low-entanglement RSP,[5] optimal RSP,[6] RSP without oblivious conditions,[7] generalized RSP,[8] oblivious RSP,[9] faithful RSP,[10] and two-dimensional k-particle entangled state ($k \geq 2$).[14−21] In the meantime, some RSP schemes have been implemented experimentally by using nuclear magnetic resonance (NMR)[22] and spontaneous parametric down-conversion.[23] In addition, some authors have generalized these methods to the case of the higher-dimensional equatorial quantum state.[15,24−27] However, no scheme has yet been reported for probabilistically and remotely prepared high-dimensional equatorial multipartite entangled state with four-parties by using a partial entangled high-dimensional four particle state as quantum channel.

Here, we propose a scheme to probabilistically and remotely prepare the high-dimensional equatorial multiqubit with four-party, consisting of a sender and three receivers. The quantum channel is composed of a non-maximally entangled high-dimensional four-particle state. We calculate the successful total probability and the total classical communication cost required for this scheme. Our results show that both the entangled resources and classical communication cost are greatly reduced. For the quantum channel composed of a maximally entangled d-dimensional four-particle state, our RSP scheme can possess 100% successful probability and only consumes $7\,\mathrm{lb}d$ bits classical communication cost per multiqubit. The classical communication cost for this scheme is most economic in the RSP with four-party.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11074307 and 60974037).
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2. Probabilistic remote preparation of a high-dimensional equatorial multiqubit with four-party

Suppose that the sender Alice wants to help either of the three receivers Bob, Charlie and David, to probabilistically and remotely prepare the following high-dimensional equatorial multiqubit, i.e. d-dimensional equatorial l-particle entangled state:

$$|\phi\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \mathrm{e}^{\mathrm{i}\,\delta_j} |jj\cdots j\rangle, \tag{1}$$

where parameters δ_j ($j = 0, 1, \ldots, d-1$) are real numbers and let $\delta_0 = 0$. We suppose that the state in Eq. (1) is known completely to Alice, but unknown to Bob, Charlie and David. We also suppose that the quantum channel is composed of a partially entangled d-dimensional four-particle state

$$|\Psi\rangle_{\mathrm{ABCD}} = \sum_{j=0}^{d-1} z_j |jjjj\rangle_{\mathrm{ABCD}}, \tag{2}$$

where the real nonzero coefficients z_j ($j = 0, 1, \ldots, d-1$) satisfy $\sum_{j=0}^{d-1} |z_j|^2 = 1$ and $|z_{d-1}|$ is less than or equals to all the other coefficients' absolute value. Particle A is in the possession of the sender Alice, while particles (B, C, D) belong to the receivers Bob, Charlie and David, respectively.

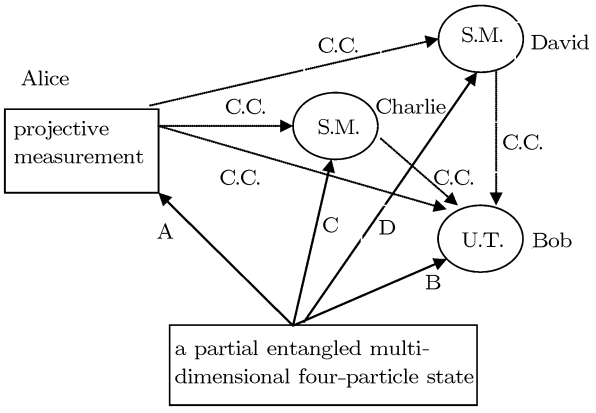


Fig. 1. A sketch of RSP of a high-dimensional equatorial multiqubit with four-party. A partially entangled multi-dimensional four-particle state (particles A, B, C, D) are used as quantum channel. Particle A is in the possession of Alice, while particles (B, C, D) belong to the receivers Bob, Charlie and David, respectively. The 'S.M.' in the round box denotes that the separate measurement on particles (C, D) Charlie and David must carry out, whereas 'U.T.' denotes that the unitary transformation Bob must perform in order to retrieve the original state, C.C. represents classical communication.

In order to help either of the three receivers Bob, Charlie and David remotely prepare the equatorial multiqubit of Eq. (1), Alice must perform a projective measurement on her particle A. The measurement basis chosen by Alice is a set of mutually orthogonal basis vectors $\{|x_k\rangle, k = 0, 1, \ldots, d-1\}$, which are related to the computation basis vectors $\{|0\rangle, |1\rangle, \ldots, |d-1\rangle\}$ by the following relation:

$$|x_k\rangle = \sum_{j=0}^{d-1} \mathrm{e}^{2\pi\,\mathrm{i}\,j\,k/d} \mathrm{e}^{-\,\mathrm{i}\,\delta_j} |j\rangle/\sqrt{d}. \tag{3}$$

Therefore, without loss of generality, the partially entangled high-dimensional state described in Eq. (2) can be written in the following form:

$$|\Psi\rangle_{\mathrm{ABCD}} = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |x_k\rangle_{\mathrm{A}} \times \sum_{j=0}^{d-1} z_j \mathrm{e}^{-2\pi\,\mathrm{i}\,j\,k/d} \mathrm{e}^{\mathrm{i}\,\delta_j} |jjj\rangle_{\mathrm{BCD}}. \tag{4}$$

Then Alice measures her particle A. For example, if Alice's projective measurement result is $|x_2\rangle_{\mathrm{A}}$, the state of particles (B, C, D), as shown by Eq. (4), will collapse into

$$|\Psi\rangle_{\mathrm{BCD}} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} z_j \mathrm{e}^{-4\pi\,\mathrm{i}\,j\,/d} \mathrm{e}^{\mathrm{i}\,\delta_j} |jjj\rangle_{\mathrm{BCD}}. \tag{5}$$

Now, we see how the original state described in Eq. (1) can be prepared remotely from the state in Eq. (5). first, Alice informs the receivers Bob, Charlie and David of her measurement outcomes via a classical communication. If Bob wants to receive the original state of Eq. (1), Charlie and David can be asked to make the following separate measurements on their particles (C, D) in the new bases $\{|\pi_k\rangle, k = 0, 1, \ldots, d-1\}$, related to the old bases $\{|0\rangle, |1\rangle, \ldots, |d-1\rangle\}$, respectively, defined by

$$|\pi_k\rangle = \sum_{j=0}^{d-1} \mathrm{e}^{2\pi\,\mathrm{i}\,j\,k/d} |j\rangle/\sqrt{d}. \tag{6}$$

Thus, from Eq. (6), the state in Eq. (2) or Eq. (4) can be rewritten as

$$|\Psi\rangle_{\mathrm{ABCD}} = \frac{1}{d\sqrt{d}} \sum_{k=0}^{d-1} |x_k\rangle_{\mathrm{A}} \sum_{k'=0}^{d-1} |\pi_{k'}\rangle_{\mathrm{C}} \sum_{k''=0}^{d-1} |\pi_{k''}\rangle_{\mathrm{D}} \times \sum_{j=0}^{d-1} z_j \mathrm{e}^{-2\pi\,\mathrm{i}\,j\,(k+k'+k'')/d} \mathrm{e}^{\mathrm{i}\,\delta_j} |j\rangle_{\mathrm{B}}. \tag{7}$$

For the state in Eq. (5), if Charlie's and David's separate measurement results on their particles (C, D) are

$|\pi_2\rangle_C$ and $|\pi_1\rangle_{\rm D}$, the state of particle B, as shown by Eq. (5), will collapse into

$$|\Psi\rangle_{\rm C} = \frac{1}{d\sqrt{d}} \sum_{j=0}^{d-1} z_j {\rm e}^{-10\pi\,{\rm i}\,j/d} {\rm e}^{{\rm i}\,\delta_j} |j\rangle_{\rm B}. \quad (8)$$

After their measurements, Charlie and David inform Bob of their measurement outcomes via a classical communication, respectively. According to Alice's, Charlie's and David's measurement results, Bob carries out the following unitary transformation U_1 on his particle B:

$$U_1 = u_{kk'k''} = \sum_{j=0}^{d-1} {\rm e}^{2\pi\,{\rm i}\,j(k+k'+k'')/d} |j\rangle\langle j|. \quad (9)$$

That is, the unitary transformation $U_1 = u_{221} = \sum_{j=0}^{d-1} {\rm e}^{10\pi\,{\rm i}\,j/d} |j\rangle\langle j|$ described in Eq. (9) will transform the state shown in Eq. (8) into

$$|\Psi\rangle_{\rm B} = \frac{1}{d\sqrt{d}} \sum_{j=0}^{d-1} z_j {\rm e}^{{\rm i}\,\delta_j} |j\rangle_{\rm B}. \quad (10)$$

Second, he introduces an auxiliary d-dimensional particle $\rm B_0$ with its initial state $|0\rangle_{\rm B_0}$ as follows:

$$|\Psi\rangle_{\rm B} \otimes |0\rangle_{\rm B_0} = \frac{1}{d\sqrt{d}} \sum_{j=0}^{d-1} z_j {\rm e}^{{\rm i}\,\delta_j} |j\rangle_{\rm B} \otimes |0\rangle_{\rm B_0}, \quad (11)$$

and performs another unitary transformation U_2 on particles B and $\rm B_0$. In order for Bob to reincarnate the original state under the bases $\{|00\rangle_{\rm BB_0}, |10\rangle_{\rm BB_0}, \ldots, |(d-1)0\rangle_{\rm BB_0}, |01\rangle_{\rm BB_0}, |11\rangle_{\rm BB_0}, \ldots, |(d-1)1\rangle_{\rm BB_0}, \ldots, |0(d-1)\rangle_{\rm BB_0}, |1(d-1)\rangle_{\rm BB_0}, \ldots, |(d-1)(d-1)\rangle_{\rm BB_0}\}$, the unitary transformation U_2 may take the form of the following $d^2 \times d^2$ matrix:

$$\boldsymbol{U}_2 = \begin{pmatrix} A_1 & A_2 & 0 & \ldots & 0 \\ A_2 & -A_1 & 0 & \ldots & 0 \\ 0 & 0 & I & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & I \end{pmatrix}, \quad (12)$$

where I is the $d \times d$ identity matrix and 0 is the $d \times d$ zero matrix, A_i ($i = 1, 2$), a kind of $d \times d$ matrices, may be written as

$$A_1 = {\rm diag}(a_0,\ a_1,\ \ldots,\ a_{d-1}), \quad (13)$$

and

$$A_2 = {\rm diag}(\sqrt{1-a_0^2}, \sqrt{1-a_1^2}, \ldots, \sqrt{1-a_{d-1}^2}), \quad (14)$$

respectively, where a_i ($i = 0,\ 1,\ \ldots,\ d-1$) ($|a_i| \le 1$) depends on the state of particle B. If we choose

$$(a_0,\ a_1,\ \ldots,\ a_{d-2},\ a_{d-1}) = \left(\frac{z_{d-1}}{z_0},\ \frac{z_{d-1}}{z_1}, \ldots,\ \frac{z_{d-1}}{z_{d-2}},\ 1\right), \quad (15)$$

the unitary transformation U_2 in Eq. (12) will transform the state shown in Eq. (11) into

$$\frac{z_{d-1}}{d\sqrt{d}} \sum_{j=0}^{d-1} {\rm e}^{{\rm i}\,\delta_j} |j\rangle_{\rm B} \otimes |0\rangle_{\rm B_0} + \frac{1}{d\sqrt{d}} \sum_{j=0}^{d-2} \sqrt{z_j^2 - z_{d-1}^2}\, {\rm e}^{{\rm i}\,\delta_j} |j\rangle_{\rm B} \otimes |1\rangle_{\rm B_0}. \quad (16)$$

Thirdly, Bob measures the state of auxiliary particle $\rm B_0$. If the receiver Bob cannot obtain the state in Eq. (16) or if the result $|1\rangle_{\rm B_0}$ is measured during the protocol, the remote preparation of the original state fails and this protocol is not successful under this condition. We would like to emphasize that our protocol is probabilistic, but not determined. Otherwise, if the measured outcome is $|0\rangle_{\rm B_0}$ in Eq. (16), quantum state $(1/\sqrt{d}) \sum_{j=0}^{d-1} {\rm e}^{{\rm i}\,\delta_j} |j\rangle_{\rm B}$ is successfully obtained and its successful probability is $|z_{d-1}|^2/d^2$.

In order to reincarnate the original state shown in Eq. (1), finally Bob again introduces $(l-1)$ auxiliary particles $\rm B_1$, $\rm B_2$, ..., $\rm B_{l-1}$, with their initial state $|00\ldots0\rangle_{\rm B_1B_2\ldots B_{l-1}}$. The quantum state of particle B and auxiliary particles $\rm B_1$, $\rm B_2$, ..., $\rm B_{l-1}$ can be written as

$$|\Psi\rangle_{\rm BB_1B_2\ldots B_{l-1}} = \frac{z_{d-1}}{d\sqrt{d}} \sum_{j=0}^{d-1} {\rm e}^{{\rm i}\,\delta_j} |j\rangle_{\rm B} \otimes |0\rangle_{\rm B_1B_2\ldots B_{l-1}}. \quad (17)$$

Bob performs $(l-1)$ d-dimensional C-NOT operations on his particles (B, $\rm B_1$, $\rm B_2$, ..., $\rm B_{l-1}$), in which particle B works as controlling qubit and all auxiliary particles ($\rm B_1$, $\rm B_2$, ..., $\rm B_{l-1}$) as target qubits. The state shown in Eq. (17) will become

$$|\Psi\rangle_{\rm BB_1B_2\ldots B_{l-1}} = \frac{z_{d-1}}{d\sqrt{d}} \sum_{j=0}^{d-1} {\rm e}^{{\rm i}\,\delta_j} |jj\ldots j\rangle_{\rm BB_1B_2\ldots B_{l-1}}. \quad (18)$$

This is the quantum state described in Eq. (1) with the probability $|z_{d-1}|^2/d^2$, that is to say that the remote preparation of the d-dimensional equatorial l-particle entangled state is successfully realized with the probability $|z_{d-1}|^2/d^2$.

After Alice performs a projective measurement on her particle A, then measures them, for states shown in Eq. (4) or Eq. (7), Bob can obtain d^3 kinds of states in all. It can easily be proven that the d^3 kinds of states have the same probability for Bob, thus the total probability of successful RSP is $d\,|z_{d-1}|^2$. If $|z_1| = |z_2| = \cdots = |z_{d-1}| = 1/\sqrt{d}$, i.e. the quantum channel is composed of a pair of d-dimensional particles in a maximally entangled state $|\Psi\rangle_{\rm ABCD} = \sum_{j=0}^{d-1} |jjjj\rangle_{\rm ABCD}/\sqrt{d}$, the total successful probability for RSP equals 100%. Clearly, in this case, the unitary transformation made by Bob is only the unitary transformation U_1 of Eq. (9) only, in which the auxiliary particle $\rm B_0$ and unitary transformation U_2 described in Eq. (12) do not need to be introduced.

3. Classical communication cost

The classical message plays an important role in the RSP process of the equatorial multiqubit. How many bits of classical information are required in the RSP process with four-party by a partially entangled high-dimensional four-particle state?

For convenience, this classical information can be divided into two transmitted processes.[21,28,29] One is the classical information sent from the sender Alice to the receiver Bob, for which Alice performs a projective measurement on particle A and informs him of her measurement result. The other is the classical information from Alice to Charlie and David and from Charlie and David to the receiver Bob. That is, it includes the classical information for which Alice performs a projective measurement on particle A and informs him of her measurement results, while Charlie and David make the separate operations on particles (C, D), then measures particles (C, D) and inform Bob of their measurement outcomes, meanwhile Bob introduces an auxiliary high-dimensional particle A with the initial state $|0\rangle_{\rm A}$ and measures the auxiliary particle A.

We first considered the classical information for the high-dimensional equatorial multiqubit state from the sender Alice to the receiver Bob. After projective measurement on particle A, Alice can obtain outcomes $|x_k\rangle$ ($k = 0,\ 1,\ \ldots,\ d-1$) with the following probabilities:

$$p_i = |z_i|^2 . \tag{19}$$

Without loss of generality, amount of the classical information required in this process from the sender Alice to the receiver Bob will become

$$S_{\rm AB} = -\sum_{i=0}^{d-1} p_i \,{\rm lb} p_i = \sum_{i=0}^{d-1} |z_i|^2 \,{\rm lb}(|z_i|^2) \text{ bits}. \tag{20}$$

The amount of the classical information needed in this process from the sender Alice to Charlie and David and from Charlie and David to the receiver Bob may be expressed as

$$S_{\rm ACB} = S_{\rm ADB} = d\,|z_{d-1}|^2 \,{\rm lb}(d^2/|z_{d-1}|^2) \text{ bits}. \tag{21}$$

Therefore, the total classical communication cost required for this reliable probabilistic remote preparation of the d-dimensional equatorial multiqubit with four-party can be written as

$$\begin{aligned} S &= S_{\rm AB} + S_{\rm ACB} + S_{\rm ADB} \\ &= \sum_{i=0}^{d-1} |z_i|^2 \,{\rm lb}(|z_i|^2) + 2d\,|z_{d-1}|^2 \\ &\quad \times {\rm lb}(d^2/|z_{d-1}|^2) \text{ bits}. \end{aligned} \tag{22}$$

If the quantum channel consists of a maximally entangled d-dimensional four-particle state, the required total amount of the classical information will be $7\,{\rm lb}d$ bits in total.

4. Conclusion

In summary, we put forward a scheme for probabilistic remote preparation of the d-dimensional equatorial multiqubit with four-party, consisting of a sender and three receivers. The quantum channel is composed of a non-maximally entangled d-dimensional four-particle state. We calculate the successful total probability and the total classical communication cost required for this scheme. It is shown that both the entangled resources and classical communication cost are greatly reduced. For the quantum channel of a pair of maximally entangled d-dimensional four-particle state, our RSP scheme can possess 100% successful probability and only consume $7\,{\rm lb}d$ bits classical communication per multiqubit. To our knowledge, few schemes have as yet been proposed for calculation of the classical communication cost required in the d-dimensional probabilistic RSP process with four-party via this partially entangled channel. This scheme may be important in the future quantum network.

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