Some applications of a new coherent-entangled state in two-mode Fock space*

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(Received 19 June 2010; revised manuscript received 4 January 2011)

Using the technique of integration within an ordered product of operators, we find a new kind of coherent-entangled state (CES), which exhibits both coherent and entangled state properties. The set of CESs makes up a complete and partly nonorthogonal representation. Using a beam splitter, we propose a simple experimental scheme to produce the CES. Finally, we present some applications of CESs in quantum optics.

Keywords: new coherent-entangled state, technique of integration within an ordered product of operators, beam splitter, squeezing operator

PACS: 03.65.Ud, 42.79.Fm **DOI:** 10.1088/1674-1056/20/5/050303

1. Introduction

Coherent state (CS) and entangled state (ES) are two extraordinary important concepts in many fields of physics. It is well known that the CS, defined by using a unitary displacement operator $D(\alpha) = \exp(\alpha a^{\dagger} - \alpha^{*} a)$ acting on the vacuum state $|0\rangle$, i.e. $|\alpha\rangle = D(\alpha)\,|0\rangle$, was introduced by Glauber[1] when he found them to be eigenstates of the annihilation operator of the harmonic oscillator. Since CSs do not exhibit quantum effects but a superposition of CSs shows second-order squeezing, higher-order squeezing and sub-Poissonian statistics. Various CSs,[2−5] such as even and odd CSs, photon-added CSs, nonlinear CSs and pair CS, have been successively introduced and widely discussed in quantum optics theory. However, quantum ES was first proposed by Einstein, Podolsky and Rosen (EPR)[6] through using the commutative properties of relative coordinate $X_1 - X_2$ and total momentum $P_1 + P_2$ between two particles, where $X_i = (a_i^{\dagger} + a_i)/\sqrt{2}$, $P_i = (a_i - a_i^{\dagger})/(\mathrm{i}\sqrt{2})$, $(i = 1, 2)$. Since then, EPR ES has been a research subject of the non-locality of quantum mechanics. Following EPR's idea of quantum entanglement,[6] Fan and Klauder[7] constructed the common eigenvector of the operators $X_1 - X_2$ and $P_1 + P_2$ in two-mode Fock space, i.e. $|\xi\rangle = \exp[-\frac{1}{2}|\xi|^2 + \xi a_1^{\dagger} - \xi^{*} a_2^{\dagger} + a_1^{\dagger} a_2^{\dagger}]\,|00\rangle$, $\xi = \xi_1 + \mathrm{i}\,\xi_2$, where $|00\rangle$ is the two-mode vacuum state, a_i and $a_i^{\dagger}$ are two-mode Bose annihilation and creation operators in Fock space. The state $|\xi\rangle$ obeys the eigenvector equations $(X_1 - X_2)\,|\xi\rangle = \sqrt{2}\xi_1\,|\xi\rangle$, $(P_1 + P_2)\,|\xi\rangle = \sqrt{2}\xi_2\,|\xi\rangle$ and possesses the overcomplete relation $\int \frac{\mathrm{d}^2\xi}{\pi}\,|\xi\rangle\langle\xi| = 1$, as well as the orthogonal property $\langle\xi|\,\xi'\rangle = \pi\delta(\xi - \xi')\delta(\xi^{*} - \xi'^{*})$.

In this work we aim to propose a new coherent-entangled state (CES) $|\alpha, x\rangle$ distinguished from $|\xi\rangle$ in two-mode Fock space by noticing the commutator $[X_1 + P_2, a_1 + \mathrm{i}a_2] = 0$. We pay attention to $|\alpha, x\rangle$ because it exhibits both an entangled state property and coherent state behaviours, which differs from the usual CS and ES. According to the new CES $|\alpha, x\rangle$ we find that its superposition in a certain way may form an EPR bipartite entangled state. We also show that the newly explored CES is able to make up a new quantum mechanical representation so that it may be useful to propose the teleportation network, construct some new unitary operators corresponding to complex optical transforms in classical optics and establish the relations between classical optical transforms and unitary operators in quantum optics.

The rest of this paper is arranged as follows. In Section 2 we derive the explicit state vector for $|\alpha, x\rangle$ in two-mode Fock space. In Section 3 we reveal its partly orthogonal property, which exhibits both a co-

*Project supported by the National Natural Science Foundation of China (Grant No. 10574060), the Natural Science Foundation of Shandong Province, China (Grant Nos. Y2008A23 and ZR2010AQ027), and the Shandong Provincial Higher Educational Science and Technology Program, China (Grant Nos. J09LA07 and J10LA15).

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herent property and entangled behaviour. In Section 4 we discuss how to use a beam splitter (BS)[8] to produce such a state. Section 5 is devoted to discussing some applications of $|\alpha, x\rangle$ in quantum optics, including generating some new ESs and obtaining some new squeezing operators and squeezed states. Finally, we draw some conclusions in Section 6.

2. Construction of new CES

In order to obtain a new CES in two-mode Fock space, we first construct the normally ordered Gaussian operator as follows:

$$\begin{aligned} &: \exp\left\{ -\left(x - \frac{X_1 + P_2}{\sqrt{2}}\right)^2 \right. \\ &\left. -\frac{1}{2}\left[\alpha^* - (a_1^\dagger - \mathrm{i}a_2^\dagger)\right]\left[\alpha - (a_1 + \mathrm{i}a_2)\right]\right\} : \\ &\equiv O(\alpha, x), \end{aligned} \tag{1}$$

where : : means the normal ordering. Using the method of integration within an ordered product (IWOP) of operators,[7] we have

$$\begin{aligned} &\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt{\pi}} \int \frac{\mathrm{d}^2\alpha}{2\pi} O(\alpha, x) \\ &= \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt{\pi}} : \exp\left[-\left(x - \frac{X_1 + P_2}{\sqrt{2}}\right)^2 \right] : \\ &= \int \frac{\mathrm{d}^2\alpha}{2\pi} : \exp\left\{ -\frac{1}{2}\left[\alpha^* - (a_1^\dagger - \mathrm{i}a_2^\dagger)\right] \right. \\ &\left. \times \left[\alpha - (a_1 + \mathrm{i}a_2)\right]\right\} : \\ &= 1. \end{aligned} \tag{2}$$

Thus we can use the normal ordered product of the two-mode vacuum projector[9,10]

$$|00\rangle\langle 00| =: \exp\left[-a_1^\dagger a_1 - a_2^\dagger a_2\right] : \tag{3}$$

to decompose $O(\alpha, x)$ as the following projector operator:

$$O(\alpha, x) = |\alpha, x\rangle\langle\alpha, x|. \tag{4}$$

According to Eqs. (1) and (4) the state $|\alpha, x\rangle$ is expressed as

$$\begin{aligned} |\alpha, x\rangle = \exp\Bigg[&-\frac{|\alpha|^2}{4} - \frac{x^2}{2} + \frac{1}{2}(2x + \alpha)a_1^\dagger \\ &+ \frac{\mathrm{i}}{2}(2x - \alpha)a_2^\dagger - \frac{1}{4}(a_1^\dagger + \mathrm{i}a_2^\dagger)^2 \Bigg] |00\rangle. \end{aligned} \tag{5}$$

Thus

$$\int \frac{\mathrm{d}^2\alpha}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt{\pi}} |\alpha, x\rangle\langle\alpha, x| = 1, \tag{6}$$

so the set of $|\alpha, x\rangle$ can be composed as a new CES representation.

Using the bosonic commutative relation $[a_i, a_j] = \delta_{i,j}$ and $[a_i, f(a_i, a_i^\dagger)] = \frac{\partial}{\partial a_i^\dagger} f(a_i, a_i^\dagger)$, we have

$$a_1 |\alpha, x\rangle = \left[\frac{1}{2}(2x + \alpha) - \frac{1}{2}(a_1^\dagger + \mathrm{i}a_2^\dagger)\right] |\alpha, x\rangle, \tag{7}$$

$$a_2 |\alpha, x\rangle = \left[\frac{\mathrm{i}}{2}(2x - \alpha) - \frac{\mathrm{i}}{2}(a_1^\dagger + \mathrm{i}a_2^\dagger)\right] |\alpha, x\rangle. \tag{8}$$

Combining Eqs. (7) and (8), we obtain the eigenvalue equations as

$$\begin{aligned} (X_1 + P_2) |\alpha, x\rangle &= \sqrt{2}x |\alpha, x\rangle, \\ (a_1 + \mathrm{i}a_2) |\alpha, x\rangle &= \alpha |\alpha, x\rangle, \end{aligned} \tag{9}$$

which shows that the state $|\alpha, x\rangle$ possesses partly the property of CS, and partly the property of ES.

3. Properties of $|\alpha, x\rangle$

Now we investigate whether the state $|\alpha, x\rangle$ is mutual orthogonal. From Eq. (9) we obtain

$$\begin{aligned} &\langle\alpha', x'| (X_1 + P_2) |\alpha, x\rangle \\ &= \sqrt{2}x \langle\alpha', x' |\alpha, x\rangle = \sqrt{2}x' \langle\alpha', x' |\alpha, x\rangle, \end{aligned} \tag{10}$$

which leads to

$$\langle\alpha', x' |\alpha, x\rangle (x - x') = 0. \tag{11}$$

In Eq. (5) performing differentiation $\partial/\partial\alpha$ on $|\alpha, x\rangle$ yields

$$2\left(\frac{\partial}{\partial\alpha} + \frac{\alpha^*}{4}\right) |\alpha, x\rangle = (a_1^\dagger - \mathrm{i}a_2^\dagger) |\alpha, x\rangle. \tag{12}$$

It then follows from Eq. (12) that

$$\begin{aligned} &\langle\alpha', x'| (a_1 + \mathrm{i}a_2) |\alpha, x\rangle = \alpha \langle\alpha', x' |\alpha, x\rangle \\ &= 2\left(\frac{\partial}{\partial\alpha'^*} + \frac{\alpha'}{4}\right) \langle\alpha', x' |\alpha, x\rangle. \end{aligned} \tag{13}$$

Taking the complex conjugate of formula (13) and the interchanging α', x' with α, x, we have

$$\frac{\partial}{\partial\alpha} \langle\alpha', x' |\alpha, x\rangle = \left(\frac{\alpha'^*}{2} - \frac{\alpha^*}{4}\right) \langle\alpha', x' |\alpha, x\rangle. \tag{14}$$

On the other hand, acting differentiation $\partial/\partial\alpha^*$ on $|\alpha, x\rangle$ yields

$$\frac{\partial}{\partial\alpha^*} |\alpha, x\rangle = -\frac{\alpha}{4} |\alpha, x\rangle, \tag{15}$$

so

$$\frac{\partial}{\partial\alpha'}\langle\alpha',x'\,|\alpha,x\rangle=-\frac{\alpha'^*}{4}\langle\alpha',x'\,|\alpha,x\rangle\,. \tag{16}$$

Through solving the partial differential Eqs. (14) and (16), and considering Eqs. (6) and (11), we finally derive

$$\begin{aligned}&\langle\alpha',x'\,|\alpha,x\rangle\\&=\sqrt{\pi}\exp\left(-\frac{|\alpha|^2}{4}-\frac{|\alpha'|^2}{4}+\frac{\alpha\alpha'^*}{2}\right)\delta(x-x').\end{aligned} \tag{17}$$

In Eq. (17) the term $\alpha\alpha'^*$ reflects the non-orthogonal property as that of the usual CS, while $\delta(x-x')$ reflects the orthogonal property of the ES. So we conclude that the state $|\alpha,x\rangle$ exhibits the properties of both CS and ES. In particular, when $\alpha=\alpha'$

$$\langle\alpha',x'\,|\alpha,x\rangle=\sqrt{\pi}\delta(x-x'). \tag{18}$$

On the other hand, by making a Fourier transformation for $|\alpha=\alpha_1+\mathrm{i}\alpha_2,x\rangle$ with respect to α_2 we have

$$\begin{aligned}&\frac{1}{2\pi}\int\mathrm{d}\alpha_2\,|\alpha=\alpha_1+\mathrm{i}\alpha_2,x\rangle\,\mathrm{e}^{\mathrm{i}u\alpha_2}\\&=\exp\left[-\frac{1}{8}(2u-\alpha_1)^2\right]\left|y=\frac{1}{\sqrt{2}}\left(x-u+\frac{\alpha_1}{2}\right)\right\rangle_1\\&\quad\otimes\left|p=\frac{1}{\sqrt{2}}\left(x+u-\frac{\alpha_1}{2}\right)\right\rangle_2,\end{aligned} \tag{19}$$

where the two single-mode states $|y\rangle_1$ and $|p\rangle_2$ are respectively the set of coordinate eigenvectors and momentum eigenvectors, i.e.,

$$|y\rangle_1=\pi^{-1/4}\exp\left(-\frac{1}{2}y^2+\sqrt{2}ya_1^\dagger-\frac{1}{2}a_1^{\dagger2}\right)|0\rangle\,, \tag{20}$$

and

$$|p\rangle_2=\pi^{-1/4}\exp\left(-\frac{1}{2}p^2+\mathrm{i}\sqrt{2}pa_2^\dagger+\frac{1}{2}a_2^{\dagger2}\right)|0\rangle\,. \tag{21}$$

Further, performing the inverse Fourier transformation we have

$$\begin{aligned}&|\alpha=\alpha_1+\mathrm{i}\alpha_2,x\rangle\\&=\int\mathrm{d}u\exp\left[-\frac{1}{8}(2u-\alpha_1)^2-\mathrm{i}u\alpha_2\right]\\&\quad\times\left|y=\frac{1}{\sqrt{2}}\left(x-u+\frac{\alpha_1}{2}\right)\right\rangle_1\\&\quad\otimes\left|p=\frac{1}{\sqrt{2}}\left(x+u-\frac{\alpha_1}{2}\right)\right\rangle_2.\end{aligned} \tag{22}$$

Equation (22) is actually the Schmidt decomposition of $|\alpha=\alpha_1+\mathrm{i}\alpha_2,x\rangle$, so we conclude that the state $|\alpha=\alpha_1+\mathrm{i}\alpha_2,x\rangle$ is an ES according to the standard Schmidt decomposition theory.[11]

4. Generation scheme of $|\alpha,x\rangle$

Now, we discuss how to produce $|\alpha,x\rangle$ by an experiment setup. As we know, BSs can play a role in producing quantum entanglement. The role of a BS operator with free-phase can be expressed by[8]

$$B=\exp\left[-\mathrm{i}\frac{\theta}{2}(a_1^\dagger a_2+a_1a_2^\dagger)\right]. \tag{23}$$

Letting the ideal single-mode coordinate eigenstate expressed by $|x=0\rangle_1\equiv\exp[-\frac{1}{2}a_1^{\dagger2}]\,|0\rangle_1$ in mode 1 and the vacuum state $|0\rangle_2$ in mode 2 respectively enter the two input ports of a BS and get overlapped, when $\theta=\pi/2$, then the output state is

$$B\,|x=0\rangle_1\otimes|0\rangle_2=\exp\left[-\frac{1}{4}(a_1^\dagger+\mathrm{i}a_2^\dagger)^2\right]|00\rangle\,. \tag{24}$$

Then, we operate two local oscillator laser beams, denoted by two displacement operators,

$$D_1\left(\varepsilon_1=\frac{x+\alpha}{2}\right)=\exp(\varepsilon_1a_1^\dagger-\varepsilon_1^*a_1) \tag{25}$$

and

$$D_2\left(\varepsilon_2=\mathrm{i}\frac{x-\alpha}{2}\right)=\exp(\varepsilon_2a_2^\dagger-\varepsilon_2^*a_2) \tag{26}$$

on the output state $\exp[-\frac{1}{4}(a_1^\dagger+\mathrm{i}a_2^\dagger)^2]\,|00\rangle$, and the result is

$$\begin{aligned}&D_1(\varepsilon_1)D_2(\varepsilon_2)\exp\left[-\frac{1}{4}(a_1^\dagger+\mathrm{i}a_2^\dagger)^2\right]|00\rangle\\&=\exp\left\{-\frac{|\varepsilon_1|^2+|\varepsilon_2|^2}{2}-\frac{(\varepsilon_1^*+\mathrm{i}\varepsilon_2^*)^2}{4}-\frac{1}{4}(a_1^\dagger+\mathrm{i}a_2^\dagger)^2\right.\\&\quad\left.+\left[\frac{\varepsilon_1^*+\mathrm{i}\varepsilon_2^*}{2}+\varepsilon_1\right]a_1^\dagger+\left[\frac{\mathrm{i}\varepsilon_1^*-\varepsilon_2^*}{2}+\varepsilon_2\right]a_2^\dagger\right\}|00\rangle\\&=|\alpha,x\rangle\,.\end{aligned} \tag{27}$$

Thus the ideal CES $|\alpha,x\rangle$ can be produced because we can also implement the displacement $D_1(\varepsilon_1)$ (or $D_2(\varepsilon_2)$) experimentally by reflecting the light field from a partially reflecting mirror (say 99% reflection and 1% transmission) and adding through the mirror a field that has been phase and amplitude modulated based on the values α and x in $D_1(\varepsilon_1)$ (or $D_2(\varepsilon_2)$).

5. Some applications

5.1. Conjugate state of $|\alpha,x\rangle$

Through making a Fourier transform of $|\alpha,x\rangle$ as follows:

$$|\beta,p\rangle=\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}px}\int_{-\infty}^{\infty}\frac{\mathrm{d}^2\alpha}{4\pi}\,|\alpha,x\rangle$$

$$\times \exp[(\alpha^*\beta - \alpha\beta^*)/4]$$
$$= \exp\left\{ -\frac{p^2}{2} - \frac{|\beta|^2}{4} + \frac{1}{2}(\beta + \mathrm{i}2p)a_1^\dagger - \frac{1}{2}(\mathrm{i}\beta + 2p)a_2^\dagger + \frac{1}{4}(a_1^\dagger + \mathrm{i}a_2^\dagger)^2 \right\} |00\rangle\,, \quad (28)$$

we find that the state $|\beta, p\rangle$ is the common eigenstate of $P_1 - X_2$ and $a_1 + \mathrm{i}\,a_2$, i.e.

$$(P_1 - X_2)\,|\beta, p\rangle = \sqrt{2}p\,|\beta, p\rangle\,,$$
$$(a_1 + \mathrm{i}a_2)\,|\beta, p\rangle = \beta\,|\beta, p\rangle\,. \quad (29)$$

Employing the IWOP technique we also prove the following completeness relation:

$$\int \frac{\mathrm{d}^2\beta}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}p}{\sqrt{\pi}}\,|\beta, p\rangle\,\langle\beta, p| = 1, \quad (30)$$

which shows that the set of $|\beta, p\rangle$ is also composed as a new CES representation. Thus $|\beta, p\rangle$ is the conjugate state of $|\alpha, x\rangle$.

5.2. Generating some new ESs

In this section we examine the possible superpositions of $|\alpha, x\rangle$. Using Eq. (5) and, performing the integral over $\mathrm{d}\alpha_1$ for $|\alpha, x\rangle$, we obtain

$$\frac{1}{2\sqrt{\pi}} \int \mathrm{d}\alpha_1\,|\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle$$
$$= \exp\left\{ -\frac{\alpha_2^2}{4} - \frac{x^2}{2} + \frac{1}{2}\left[(2x + \mathrm{i}\alpha_2)a_1^\dagger + (\alpha_2 + \mathrm{i}2x)a_2^\dagger\right] - \mathrm{i}a_1^\dagger a_2^\dagger \right\} |00\rangle$$
$$= |\alpha_2, x\rangle\,, \quad (31)$$

which is the common eigenstate of the two commuting operators $X_2 + P_1$ and $X_1 + P_2$, i.e.

$$(X_2 + P_1)\,|\alpha_2, x\rangle = \frac{\alpha_2}{\sqrt{2}}\,|\alpha_2, x\rangle\,,$$
$$(X_1 + P_2)\,|\alpha_2, x\rangle = \sqrt{2}x\,|\alpha_2, x\rangle\,. \quad (32)$$

So the superposition of the CES $|\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle$ along the α_1 axis yields a new EPR ES, which is different from previous ESs in construction.[10]

On the other hand, carrying out the integral over $\mathrm{d}\alpha_1$ for the projection operator $|\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle\,\langle\alpha = \alpha_1 + \mathrm{i}\alpha_2, x|$ yields

$$\frac{1}{\sqrt{2\pi}} \int \mathrm{d}\alpha_1\,|\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle\,\langle\alpha = \alpha_1 + \mathrm{i}\alpha_2, x|\,00\rangle$$
$$= \exp\left\{ -x^2 - \frac{\alpha_2^2}{2} + \frac{1}{4}\left[a_1^{\dagger 2} - a_2^{\dagger 2} - \frac{3}{4}(a_1^\dagger + \mathrm{i}a_2^\dagger)^2\right] + \frac{1}{2}(2x + \mathrm{i}\alpha_2)a_1^\dagger + \frac{\mathrm{i}}{2}(2x - \mathrm{i}\alpha_2)a_2^\dagger \right\} |00\rangle$$
$$\equiv |squ.\rangle\,. \quad (33)$$

In particular, when $\alpha_2 = x = 0$

$$|squ.\rangle = \exp\left\{ \frac{1}{4}\left[a_1^{\dagger 2} - a_2^{\dagger 2} - \frac{3}{4}(a_1^\dagger + \mathrm{i}a_2^\dagger)^2\right] \right\} |00\rangle\,, \quad (34)$$

which is a special entangled-squeezed state. So from Eq. (34) we conclude that the superposition of the projection operator

$$|\alpha = \alpha_1 + \mathrm{i}\alpha_2, x\rangle\,\langle\alpha = \alpha_1 + \mathrm{i}\alpha_2, x|_{\alpha_2 = x = 0}$$

along the α_1 axis leads to a special entangled-squeezed operator.

5.3. Two-mode generalized squeezing operator derived by $|\alpha, x\rangle$

As an important application of $|\alpha, x\rangle$, in this section we derive some new squeezing operators and squeezed states. Through constructing the asymmetric ket-bra integration

$$S = \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt{k\pi}} \int \frac{\mathrm{d}^2\alpha}{2\pi}\,|\alpha, x/k\rangle\,\langle\alpha, x| \quad (35)$$

and using the following formula[12,13]

$$:\exp\left[(\mathrm{e}^f - 1)a^\dagger a\right]: \;= \mathrm{e}^{fa^\dagger a}, \quad (36)$$

we directly obtain

$$S = \mathrm{sech}^{1/2}\gamma \exp\left[-\frac{\tanh\gamma}{2}R^{\dagger 2}\right] \times \exp\left[R^\dagger R \ln\mathrm{sech}\,\gamma\right] \exp\left[\frac{\tanh\gamma}{2}R^2\right], \quad (37)$$

where $k = \mathrm{e}^\gamma$ and $R^\dagger = (a_1^\dagger + \mathrm{i}a_2^\dagger)/\sqrt{2}$. Since $R^{\dagger 2}$, R^2 and $R^\dagger R$ compose an SU(1,1) Lie algebra

$$\left[R, R^\dagger\right] = 1, \quad \left[\frac{R^2}{2}, \frac{R^{\dagger 2}}{2}\right] = R^\dagger R + \frac{1}{2}, \quad (38)$$

S is a new squeezing operator with squeezing parameter k. From Eqs. (35) and (37) we know that S squeezes $|\alpha, x\rangle$ in a natural way

$$S\,|\alpha, x\rangle = \frac{1}{\sqrt{k}}\,|\alpha, x/k\rangle\,. \quad (39)$$

It then follows that

$$S\,|00\rangle = \mathrm{sech}^{1/2}\gamma \exp\left[-\frac{\tanh\gamma}{4}(a_1^\dagger + \mathrm{i}a_2^\dagger)^2\right] |00\rangle\,, \quad (40)$$

which is a two-mode generalized squeezed state. The type of squeezed state can be the output state of a BS when the two input states are a one-mode squeezed state and a vacuum state.[14,15]

6. Conclusion

We have introduced a new kind of CES, which can be generated by a BS. We then analyse some properties of the new state, i.e. the completeness relation and partial orthogonality. It is found that the superpositions of $|\alpha, x\rangle$ can result in some new ESs and squeezed states. So we believe that the state $|\alpha, x\rangle$ can provide us with a direct way to construct some new quantum states in quantum optics.

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