# Consensus pursuit of heterogeneous multi-agent systems under a directed acyclic graph*

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This paper is concerned with the cooperative target pursuit problem by multiple agents based on directed acyclic graph. The target appears at a random location and moves only when sensed by the agents, and agents will pursue the target once they detect its existence. Since the ability of each agent may be different, we consider the heterogeneous multi-agent systems. According to the topology of the multi-agent systems, a novel consensus-based control law is proposed, where the target and agents are modeled as a leader and followers, respectively. Based on Mason's rule and signal flow graph analysis, the convergence conditions are provided to show that the agents can catch the target in a finite time. Finally, simulation studies are provided to verify the effectiveness of the proposed approach.

**Keywords:** heterogeneous multi-agent systems, pursuit, consensus, leader–follower

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## 1. Introduction

Recently, the study of decentralized control of multi-agent systems has appeared as a new challenging area of research in different disciplines, such as communications, control, and mechanics.[1−4] These systems can potentially consist of a large number of agents, such as unmanned underwear vehicles (UUVs), unmanned aerial vehicles (UAVs), unmanned ground vehicles (UGVs), and so on. The advantages of multi-agent systems over single agent include reduced cost, increased efficiency, improved robustness, etc. Due to these advantages, there exist numerous potential engineering and military applications in intelligent transportation systems, home and building automation, health monitoring, ocean exploration, military surveillance, and rescue missions.

One of the interesting research topics for these agents is pursuit control for accomplishing predetermined goals and requirements. The history related to pursuit problem can be dated back to two hundred years ago. In 1732, a famous French scientist Pierre Bouguer first studied the mathematics of pursuit curves. Recently, a lot of research work has been carried out on the pursuit problem. Lin[5] *et al.* considered the pursuit problem as one of the three strategies for rendezvous. Furthermore, they pointed out that more general formations can be achieved if convergence to a point was feasible, and relevant work can be found in Refs. [6]–[9]. In Ref. [10], a methodology for group coordination and cooperative control of $n$ agents to achieve a target-capturing task in three-dimensional (3D) space was provided. The proposed approach is based on a cyclic pursuit strategy, where agent $i$ simply pursues agent $i+1$ modulo $n$. More recently, Ding[11] *et al.* investigated collective motions and formations of pursuit systems on directed acyclic graph for both single-integrator kinematics and double-integrator dynamics. In Ref. [12], a discrete-time pursuit problem was addressed in the plane with sensing capabilities restricted to a finite distance. Meanwhile, consensus-based approach has also been used to solve pursuit control problem of multi-agent systems.[13] In multi-agent systems, "consensus" means reaching an agreement regarding a certain quantity of interest that depends on the states of all agents. The concept of solvability of consensus problem was formally introduced by Olfati-Saber and Murray.[14] On the assumption that the dynamics of each agent is a simple scalar continuous-time integrator, and three consensus problems were discussed. Meanwhile, Chen and Duan[15] investigated the con-

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sensus problem by using graph theory, and other revelent work can be found in Refs. [16] and [17]. In Ref. [18], a control strategy based on cooperative game theory was applied to the problem of consensus for multi-agent systems. To achieve the consensus problem, a linear combination of the locally defined cost functions was considered as the term cost function.

In all the aforementioned references, the model of agents can be transformed into single-integrator or double-integrator dynamics through feedback linearization,[19] i.e., these agents are homogeneous.[20] In homogeneous multi-agent systems, each agent has a similar ability. But in the real world, effective systems usually consist of many heterogeneous agents, i.e., each agent has a different ability in effective systems. Thus the controller (or control algorithm) should be applicable to various heterogeneous agents, and the controller (or control algorithm) should assign suitable role to each agent, based on the differences of the agents. In this paper, we attempt to overcome the limitation in the aforementioned references by using a consensus-based approach. First, a pursuit problem is described, in which multiple agents and a single target are connected by a directed acyclic graph. The agents will pursue the target once they detect the existence of target, and the target would randomly move to escape the agents at the same time. Our motivation in this paper is to develop a control law for heterogeneous multi-agent systems. Based on Mason's rule and signal flow graph analysis, we provide the convergence conditions for heterogeneous multi-agent systems, which ensures that the states of agents and target converge to a common value, i.e., agents can catch up with the target in a finite time. Finally, simulation studies are provided to verify the effectiveness of the proposed approach.

The organization of the rest of this paper is as follows. In Section 2, preliminaries for multi-agent systems are described. The main results are presented in Section 3. In Section 4 simulation results are presented to validate the proposed method. Finally, a conclusion is given in Section 5.

# 2. Preliminaries

## 2.1. Directed acyclic graph

A directed graph consists of a vertex set $V$ and a directed edge set $E$, where a directed edge is an ordered pair of distinct vertices. An edge $(i, j)$ in a directed graph is said to be incoming with respect to $j$ and outgoing with respect to $i$. Such an edge has vertex $i$ as a tail and vertex $j$ as a head. The in-degree of a vertex in a directed graph is defined as the number of edges that have this vertex as a head. If $(i, j)$ is an edge, then $i$ and $j$ are adjacent. A directed cycle is a connected directed graph where every vertex is incident with one incoming and one outgoing edge. An directed acyclic graph is a directed graph with no cycle. Given a node $i$, the set of nodes from which it can receive information is called a neighbour set $N_i$, that is

$$\begin{aligned} &\forall i = 0, 1, \ldots, N, \\ &N_i = \{j \in [0, 1, \ldots, N], \quad (j, i) \in E\}. \end{aligned} \tag{1}$$

If node $j$ is a neighbour of node $i$, we define $a_{ji} = 1$, otherwise, $a_{ji} = 0$.

Similar to Ref. [21], the topology relationship between the agents and the target is represented by a directed acyclic graph $G(V, E)$ in this paper, and it is assumed that the directed acyclic graph is connected. The target, which is not head of arbitrary edge in the directed acyclic graph, can be considered as the leader, and all the agents are the followers.

**Remark 1** In this paper, we assume that the target and the agents are connected by a directed acyclic graph. The inspiration for this action comes from certain aspects of bird behaviours and engineering applications. Take birds that are foraging or traveling in a $V$-formation for example, these birds do not look backwards to form a cycle in their interaction topology when keeping their desired formation. In some engineering applications, multiple autonomous mobile robots cooperate to finish a complicated task. The robots communicate with each other using installed sensors and these sensors are only capable of looking forward by using cameras.[22] Thus a multiple autonomous mobile robot platform can form a directed acyclic graph. An example of a directed acyclic graph for the pursuit problem is given in Fig. 1.

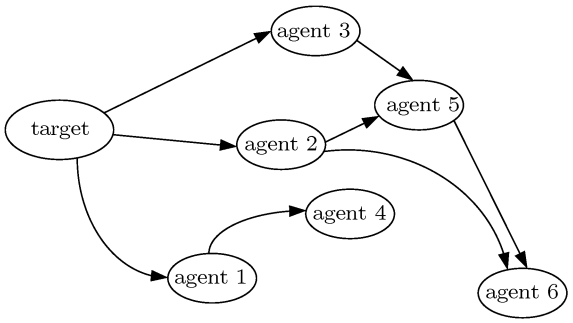



**Fig. 1.** Example of a directed acyclic graph for the pursuit problem.

### 2.2. Signal flow graph

A signal flow graph (or Mason graph)[11,23] is a network of directed branches, which connects at nodes. Branch $jk$ originates at node $j$ and terminates upon node $k$, the direction from $j$ to $k$ being indicated by an arrowhead on the branch. Each branch $jk$ is associated with a quantity called $W_{kj}$, which is a transfer function from agent $j$ to $k$ in this paper. Each node $j$ has an associated quantity called the node signal $j$, which is the state of agent $j$ in this paper. To simplify the analysis in Section 3, we define the quantity by $W_{kj}$ but not $W_{jk}$. A node performs two functions: (i) addition of the signals on all incoming branches; (ii) transmission of the total node signal to all outgoing branches. In a signal flow graph, sources are nodes having only outgoing branches, sinks are nodes having only incoming branches, and mixed nodes have both outgoing and incoming branches. A forward path is a path from source to sink along which no node is encountered more than once. A path gain is the product of the branch gains along that path. A feedback loop is a path that forms a closed cycle along which each node is encountered once per cycle. Loops are nontouching if they have no common node.

## 3. Main results

Define a set of heterogeneous agents as $\varnothing = \{1, 2, \ldots, N\}$, where $N$ is the number of agents. Initially, target and agents disperse (or hide) randomly in $n$-dimensional workplace. The target follows a reactive rabbit model:[24] moves only after being detected for the first time. Let $t_0$ denote this specified time instant when target is detected. The goal of agents is to pursue the target in a finite time, i.e., the states of agents and target should converge to a common value.

At time $t$, the positions of target and agent $i$ ($i \in \varnothing$) are denoted as $z_0(t) \in R^n$ and $z_i(t) \in R^n$, respectively. Thus the Laplace transforms of $z_0(t)$ and $z_i(t)$ can be represented by $Z_0(s)$ and $Z_i(s)$, respectively. Figure 2 shows the relation between inputs and output for agent $i$. Thus the model describing the relation between inputs and output for agent $i$ is given as

$$Z_i(s) = \sum\nolimits_{j \in N_i} a_{ji} Z_j(s) G_{ic}(s) G_i(s), \tag{2}$$

where $G_i(s)$ is a transfer function, which is used to describe the heterogeneous agent $i$. $G_{ic}(s)$ is the designed controller which satisfies different requirements.

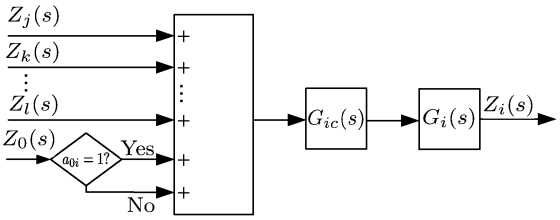



**Fig. 2.** Relation between inputs and output for agent $i$.

In this paper, the aim is to design a consensus-based controller by using a feedback strategy. Meanwhile, we use a signal flow graph to investigate the pursuit problem. In a signal flow graph, the nodes represent the states of the agents and the target, the edges denote the directions of signal flows, and the gain of each edge is a transfer function from one state to another state. Moreover, the target and agents are connected by a directed acyclic graph. Thus the target can be considered as a source node, and the agents can be considered as sink or mixed nodes. An example of a signal flow graph, which is related to Fig. 1, can be given in Fig. 3. First, we recall Mason's rule to calculate the overall transfer function from one source to one sink, i.e., the overall function from the target to an arbitrary agent.

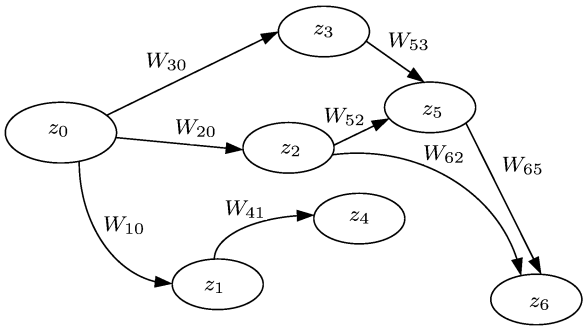



**Fig. 3.** Example of a signal flow graph related to Fig. 1.

**Lemma 1** (**Mason's rule**)[11,23] In a signal flow graph containing only one source and one sink, the overall gain $T$ is given by

$$T = \frac{\sum T_i \Delta_i}{\Delta} \tag{3}$$

with

$$\Delta = 1 - \sum\nolimits_m P_{m1} + \sum\nolimits_m P_{m2} - \sum\nolimits_m P_{m3} + \cdots, \tag{4}$$

where $\Delta$ is the graph determinant, $P_{mr}$ is the gain product of the $m$-th possible combination of $r$ nontouching loops ($m, r \in N$), $\Delta_i$ is the determinant of

the remaining subgraph when the $i$-th forward path is removed, and $T_i$ is the forward path gain.

The control objective is to design a controller for arbitrary agent $i$. Then the states (or positions) for all the agents can reach a common value with the one for the target, namely,

$$\lim_{s\to 0} sZ_0(s) = \lim_{s\to 0} sZ_1(s) = \cdots = \lim_{s\to 0} sZ_N(s), \quad (5)$$

or

$$\lim_{t\to t^*} z_0(t) = \lim_{t\to t^*} z_1(t) = \cdots = \lim_{t\to t^*} z_N(t), \quad (6)$$

where $t^* > t$ is a finite time.

To achieve a pursuit (or consensus) task, a feedback control strategy is applied to the heterogeneous multi-agent systems. This feedback control strategy can be described by a block diagram, as shown in Fig. 4.

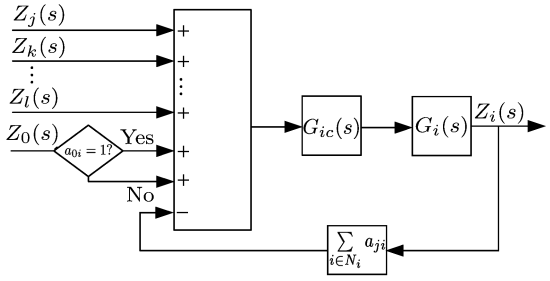



**Fig. 4.** Feedback control strategy, which is used to achieve the pursuit task.

**Lemma 2 (Routh's stability criteria)**[25] For system stability the primary requirement is that all of the roots of the characteristic equation have negative real parts. Routh's stability criterion states that all of the roots of the characteristic polynomial have negative real parts if, and only if, the following conditions are satisfied: the elements in the first column of the stability table have the same sign.

Thus the convergence condition of the pursuit (or consensus) task for heterogeneous multi-agent systems can be given as follows.

**Theorem 1** Consider the heterogeneous multi-agent systems shown in Fig. 1. The systems are stable where the states (or positions) for all agents can reach a common value as the one for the target, if all the elements in the first column of the stability table for characteristic polynomial $(c+1)G_{ic}(s)G_i(s)+1=0$ have the same sign and $G_{ic}(s)G_i(s)$ contains the factor $1/s$, where $G_i(s)$ is used to describe the heterogeneous agent $i$, $G_{ic}(s)$ is the designed controller which satisfies different requirements, $c=\sum_{j\in N_i} a_{ji}$, $i\in\varnothing$ and $j\in\{0,1,\ldots,N\}$.

**Proof** From Fig. 4, we can obtain the following equation

$$\frac{Z_i(s)}{\sum_{j\in N_i} a_{ji}Z_j(s)} = \frac{G_{ic}(s)G_i(s)}{1+\sum_{j\in N_i} a_{ji}G_{ic}(s)G_i(s)}. \quad (7)$$

Then the characteristic polynomial for systems (2) is constructed as follows:

$$(c+1)G_{ic}(s)G_i(s)+1=0, \quad (8)$$

where $c=\sum_{j\in N_i} a_{ji}$, $i\in\varnothing$ and $j\in\{0,1,\ldots,N\}$.

Based on Routh's stability criterion in Lemma 2, we can obtain the following conclusion: system (2) is stable if and only if all the elements in the first column of the stability table for characteristic polynomial $(c+1)G_{ic}(s)G_i(s)+1=0$ have the same sign.

Re-arranging Eq. (7), we have

$$\begin{aligned} Z_i(s) &= \frac{\sum_{j\in N_i} a_{ji}Z_j(s)G_{ic}(s)G_i(s)}{1+\sum_{j\in N_i} a_{ji}G_{ic}(s)G_i(s)} \\ &= \frac{\sum_{j\in N_i} a_{ji}Z_j(s)}{1/G_{ic}(s)G_i(s)+\sum_{j\in N_i} a_{ji}} \\ &= \sum\nolimits_{j\in N_i} \frac{a_{ji}}{1/G_{ic}(s)G_i(s)+c}Z_j(s). \end{aligned} \quad (9)$$

In a directed acyclic graph, there is no cycle. Thus there is no loop in the signal flow graph, namely, the determinant of the remaining subgraph and the graph determinant are always 1. Consider the signal flow graph with the target as its source and agent $k$ as its sink. According to Mason's rule in Lemma 1, the overall gain of this signal flow graph is given as

$$T = \sum T_p\Delta_p/\Delta = \sum T_p = \sum\nolimits_p \prod\nolimits_q \tilde{W}_{pq}, \quad (10)$$

where $\Delta_p = 1$ is the determinant of the remaining subgraph when the $p$-th forward path is removed, $T_p$ is the forward path gain, $\Delta$ is the graph determinant, which equals 1, $\tilde{W}_{pq}$ is the $q$-th edge on the $p$-th forward path in the signal flow graph. According to the superposition principle, the output $Z_k(s)$ is given by

$$Z_k(s) = TR(s), \quad (11)$$

where $R(s)$ is the input of the target. Without loss of generality, considering the impulse function for the target, we have

$$Z_k(s) = \sum\nolimits_p \prod\nolimits_q \tilde{W}_{pq}. \quad (12)$$

If the $q$-th edge on the $p$-th forward path in the signal flow graph is labeled as $(j,i)$, then

$$\tilde{W}_{pq} = W_{ij} = \frac{a_{ji}}{1/G_{ic}(s)G_i(s)+c}.$$

Moreover, equation (12) can be rearranged into

$$Z_k(s) = \sum_p \prod_q \frac{a_{ji}}{1/G_{ic}(s)G_i(s)+c} \tag{13}$$

and

$$T = \prod_q \frac{a_{ji}}{1/G_{ic}(s)G_i(s)+c}.$$

The impulse function for agent 1 can be denoted as

$$Z_1(s) = \frac{a_{01}}{1/G_{1c}(s)G_1(s)+a_{01}}, \tag{14}$$

where Laplace transform of the target is 1. To pursue the target, the transfer function should be 1 as $s \to 0$, namely

$$\lim_{s\to 0} \frac{a_{01}}{1/G_{1c}(s)G_1(s)+a_{01}} = 1.$$

Thus we can obtain the following conclusion: to pursue the target for agent 1, $G_{1c}(s)G_1(s)$ should contain the factor $1/s$. Applying this argument to an arbitrary agent in the group, we know the following argument: $G_{ic}(s)G_i(s)$ should contain a factor of $1/s$ for all agents. Thus we can obtain the following conclusion: the states (or positions) for all the agents can reach a common value with as one for the target, if all the elements in the first column of the stability table for characteristic polynomial $(c+1)G_{ic}(s)G_i(s)+1=0$ have the same sign and $G_{ic}(s)G_i(s)$ contains a factor $1/s$.

**Remark 2** In this paper, we only consider the objective of rendezvousing at a common point. The main reason associated with this action can be summarized as follows: if convergence to a point is feasible, then more general consensus-based problems, such as formation and flocking, are achievable too.

# 4. Simulations

In this section, we present the simulation studies of the proposed control scheme on a group of four heterogeneous agents and one target. Initially, the target and the agents are randomly distributed in the workplace. Once detected by agents at time $t_0$, the target will immediately move to escape from capture by the agents. Furthermore, we assume this special time instant be $t_0 = 0$ in this section. Under the consensus-based control strategy in Section 3, the states for all the agents can reach a common value as the one for the target in a finite time. The state (or the position state) for the target can be designed as

$$z_0(t) = \begin{cases} \dfrac{1}{20}t, & t \in [0, 60s), \\ 3, & \text{otherwise.} \end{cases}$$

The signal flow graph for the heterogeneous multi-agent systems can be given in Fig. 5. Some parameters are given as follows: $G_{1c}(s) = 1/(s^2+2s+3)$, $G_1(s) = 1/s$, $G_{2c}(s) = (s^2+2s+1)/(s^2+s)$, $G_2(s) = 1/(s^2+2s+1)$, $G_{3c}(s) = (s^2+2s+1)/(s^2+s)$, $G_3(s) = 1/(s^2+2s)$, $G_{4c}(s) = (s+1)/[s(s^2+2s+4)]$, and $G_4(s) = 1/s$. The characteristic polynomial for agent 1 can be deduced as follows: $s^3+2s^2+3s+1=0$, and all the elements in the first column of the stability table for agent 1 can be given in Table 1. Similarly, we can obtain that all the elements in the first column of the stability table for the other agents have the same sign, namely, the systems are stable. Furthermore, these parameters satisfy the requirements in Theorem 1, namely, all the agents can reach a common value as shown in Fig. 6. Let $\mathrm{err}_i(t) = z_i(t) - z_0(t)$ denote the state (or position) error for arbitrary agent $i$ as shown in Fig. 7. It is easy to find that the states (or positions) for all the agents can reach a common value as the one for the target.

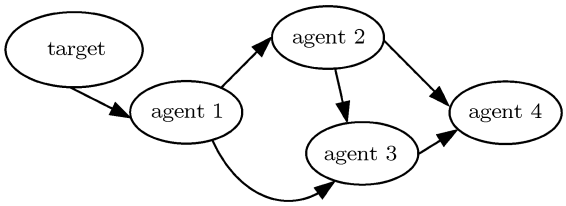



**Fig. 5.** Signal flow graph for the heterogeneous multi-agent systems in Section 4.

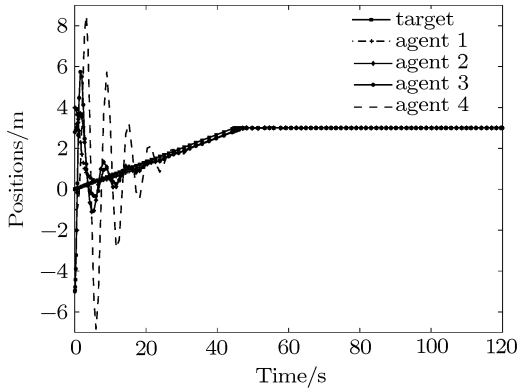



**Fig. 6.** Positions of agents and target.

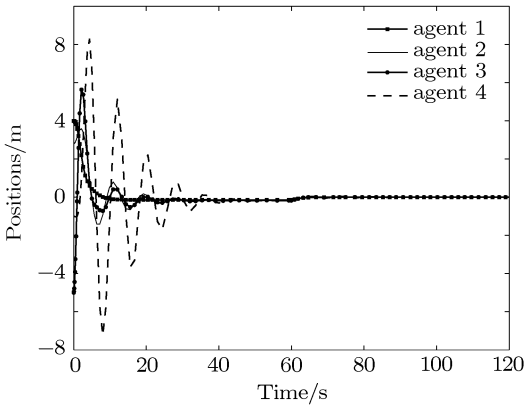



**Fig. 7.** Position errors between agents and target.

**Table 1.** Elements in the first column of the stability table for agent 1, where $c_1$ belongs to the first case, $c_2$ for the second case, $c_3$ for the third situation, and $\times$ denotes no parameter.

| | | | |
|---|---|---|---|
| $c_1$ | 1 | 2 | 2 |
| $c_2$ | 1 | 2 | $\times$ |
| $c_3$ | 1 | 2 | $-4$ |

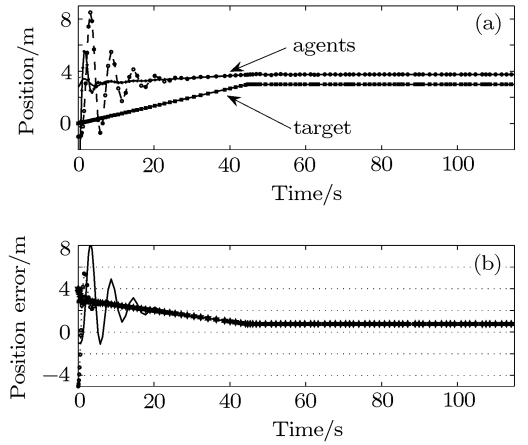



**Fig. 8.** (a) Positions of agents and the target where the pursuit task is not finished, and (b) position errors between agents and the target.

To clarify this, we can test Theorem 1 by adjusting parameters as follows: $G_{1c}(s) = s/(s^2 + 2s + 3)$ and the others are fixed. Obviously, $G_{1c}(s)$ and $G_1(s)$ do not contain the factor $1/s$, and all the elements in the first column of the stability table for agent 1 can be given in Table 1. Similarly, we can obtain that all the elements in the first column of the stability table for the other agents have the same sign, namely, the systems are stable, but all the agents cannot reach a common value as the one for the target, which can be verified by Fig. 8. In Fig. 8, the systems are stable, but the agents reach a common value 3.75, which is different with the desired value 3.

In the same way, the parameters can be adjusted as $G_{1c}(s) = 1/(s^2 + 2s - 3)$ and the others are fixed. Then the first column of the stability table for agent 1 can be given in Table 1 and we can obtain that all the elements in the first column of the stability table do not have the same sign, namely, the systems are unstable. Figure 9 can verify this argument. In Fig. 9, the states of the agents converge to an infinitesimal value in a finite time.

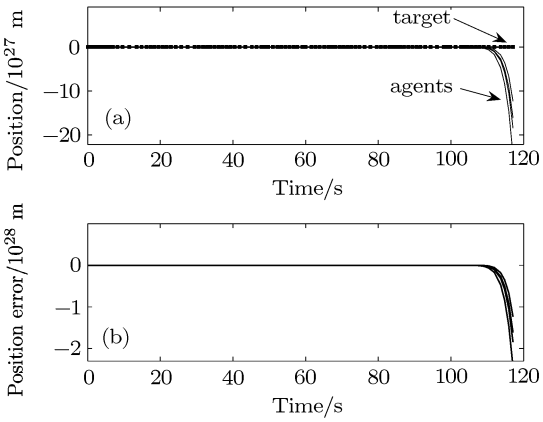



**Fig. 9.** (a) Positions of agents and target where the convergence cannot be guaranteed, and (b) position errors between agents and target.

## 5. Conclusion

We investigated the pursuit problem of heterogeneous multi-agent systems on a directed acyclic graph. According to the transform function method, a consensus based control strategy is proposed. In this control strategy, each agent pursues its neighbours with a certain pursuit controller. The convergence condition of the pursuit (or consensus) task for heterogeneous multi-agent systems can be given using Mason's rule and the Routh stability criterion. As the ability of each agent may be different, the multi-agent systems are modified from homogeneous ones to heterogeneous ones.

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