# Nonequilibrium dynamics in two-dimensional Ising spin glass

Zhang Kai-Cheng(张开成)[a)†] and Zhu Yan(朱 岩)[b)]

[a)] *Department of Physics, Bohai University, Jinzhou* 121013, *Liaoning Province, China*

[b)] *Department of Physics, Nanjing University of Aeronautics and Astronautics, Nanjing* 210016, *China*

(Received 29 July 2010; revised manuscript received 24 October 2010)

This paper investigates the nonequilibrium dynamics of two-dimensional Ising spin glass by dynamical Monte Carlo simulations. A new method is developed to quantitatively measure the age of domain growth. Using this method it investigates how temperature shift affects the effective age of domain growth. It finds that the $T$-shift dependence of the effective age follows the prediction of the droplet model quite well. It also investigates the overlap length between the spin glass states as well as the correlated flips of spins, which are not consistent with the theoretical predictions. The possible reasons are discussed.

**Keywords:** nonequilibrium, effective age, site-overlap, droplet model

**PACS:** 75.10.Hk, 75.90.+w **DOI:** 10.1088/1674-1056/20/4/047501

## 1. Introduction

Despite decades of extensive investigation, Ising spin glass[1−5] still attracts much attention for its rich physics. Infinite-range spin glass, called the Sherrington–Kirkpatric model[6,7] and solved by the mean-field method,[8] was proven to be unstable in zero external fields. The mean-field theory predicts that above the critical dimension, a spin glass transition occurs in an external field. The spin-glass phase was separated from the paramagnetic phase in a phase diagram by an instability line, which is called the Almeida–Thouless (AT) line.[9] By detecting the AT line, the mean-field theory was found to be valid only for short-range (SR) spin glass with dimension $d \geq 6$,[10,11] which is unlikely to be applied to real spin glass. The SR spin glass, called Edwards–Anderson (EA) model,[12] was confirmed to have phase transition in a zero field only for $d \geq 3$.[13] For dimensions less than three, a spin glass phase was found to exist in neither Ising spin glass nor Heisenberg spin glass[14,15] at any finite temperature. Many efforts were made to explain the properties of spin glass. Besides the mean-field theory, a well-known domain growth description called the droplet theory[16,17] provided some explanations. The droplet model was based on the fundamental assumption of the droplet's free energy and gave some important predictions. For the aging effect,[18] the droplet model suggests that after the spin glass waits for some time its domains grow to a certain size. Any field applied to the spin glass causes the droplets to be slowly polarized. Only when the length of polarization approximates to or exceeds the domain size, do the nonequilibrium properties arise. The droplet theory also suggests that an overlap length exists in the equilibrium states at temperatures $T_1$ and $T_2$. Beyond this length, the domain orientations at $T_1$ significantly differ from those at $T_2$. Moreover, the droplet theory considers that at low temperatures the large-droplet excitations dominate and spin flip collectively at the droplet size scales. Such predictions are important to interpret the behaviours of spin glass. Although numerical simulations[19−21] were done to test some opinions of the droplet model, it still needs to be carefully verified.

Recently, two-dimensional (2D) spin glass attracted many investigations from both experimental[22,23] and numerical simulations.[24−26] Because the mean-field solution of Ising spin glass is not valid for 2D cases, its theoretical predictions mainly come from the phenomenological droplet model. Therefore, 2D spin glass can be used as a tool to verify the predictions of the droplet theory. In this article we investigate the nonequilibrium dynamics of 2D EA spin glass using the dynamical Monte Carlo method. We developed a new method, by calculating the site-overlap correlation, which quantitatively measures the age of domain growth. The site-overlap correlations are found to peak exactly at the waiting times, which is analogous to the experimental ob-

†Corresponding author. E-mail: kczhang@yeah.net

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servations. By this method, we investigate how the temperature shift affects the effective age of domain growth. We find that the $T$-shift dependence of the effective age obeys the predictions of the droplet model quite well. The effective age can even accumulate to higher temperatures sequentially through the warming process without significant loss. We also investigate the overlap length between the spin glass states at different temperatures by calculating the link-overlap correlation. The overlap length is found to saturate instead of diverging at small temperature intervals as previously suggested. Moreover, we also investigate the collective flips of the droplets by calculating the flip-correlation functions. The correlation decays as a power-law function of spin separation. The decaying exponent increases rapidly as the temperature decreases, which is explained by the thermal excitations of the droplets.

The article is organized as follows. In Section 2 we briefly introduce our model and simulation method as well as the physical quantities to be calculated. In Section 3 the simulation results are presented and we discuss our results and compare them with the theoretical predictions as well as the simulations of other groups. We conclude the main opinions in Section 4.

## 2. Model, method and physical quantities

We use a 2D EA model to investigate the nonequilibrium dynamics of spin glass. The lattice size takes $L \times L$ and $L$ is set to be 100, which is large enough to reduce the finite-size effects. The Hamiltonian of the EA model can be written as

$$H = -\sum_{i,j} J_{ij}\sigma_i \cdot \sigma_j. \tag{1}$$

The spin variable $\sigma$ only takes 1 or $-1$. The coupling strengths $J_{ij}$ are the random variables drawn according to a Gaussian distribution with zero mean and a variance of one. Spins are flipped according to Glauber dynamics,[27] that is, spins transit from $\sigma$ to $-\sigma$ state according to the probability, $p(\sigma \to -\sigma) = 1/(1+\mathrm{e}^{\Delta E/k_{\mathrm{B}}T})$, where $\Delta E$ is the energy variance between two states. In the dynamic simulations, time is measured as Monte Carlo sweeps (MCS) through the whole lattice and the periodic boundary condition is used. Autocorrelation functions are usually calculated for most simulations to observe the aging effect.[19,28] The larger the waiting time, the more slowly the autocorrelation decays. However, the crossover between the fast decay and the slow decay near the waiting time is very obscure and the exact age of the spin glass cannot be obtained. However, by introducing the time-dependent site-overlap correlation, the age can be exactly located at the maximum of the correlation. Hence, we calculate the site-overlap correlation function by the replica method instead of the autocorrelation function to study the aging effect. For aging at temperature $T$ the site-overlap correlation is defined as

$$Q(t, t_{\mathrm{w}}) = \left[\overline{\langle \sigma_i^a(t)\sigma_i^b(t_{\mathrm{w}})\rangle_T}\right], \tag{2}$$

in which $a$ and $b$ are the replica indices. The key point is that, initially, both of the replicas have identical spin states and coupling strengths. The overline means that the calculation takes the average over all spins. The angular brackets $\langle\cdots\rangle$ and the square brackets $[\cdots]$ mean that the calculation takes the thermal average and the disorder average, respectively. Note that the above equation is equal to the widely used autocorrelation by Reiger *et al.*,[29] if we replace $t$ by $t + t_{\mathrm{w}}$. Thus equation (2) contains the full-time evolution of the spin glass relative to the reference configuration. To investigate how the temperature shifts affect the effective age, the above site-overlap correlation is modified as

$$Q(t, t_{\mathrm{w}}) = \left[\overline{\langle \sigma_i^a(t, T_i)\sigma_i^b(t_{\mathrm{w}}, T_0)\rangle}\right], \tag{3}$$

where $T_0$ is the temperature at which the replica $b$ first ages for $t_{\mathrm{w}}$ and $T_i$ is the shifted temperature. To investigate the overlap length $L_{\Delta T}$ between the states at temperatures $T_1$ and $T_2$, the link-overlap correlation function is defined as

$$\Theta(r, T_1, T_2) = \left[\overline{\langle \sigma_i^a\sigma_j^a\rangle_{T_1}\langle \sigma_i^b\sigma_j^b\rangle_{T_2}}\right]. \tag{4}$$

The overline means that the calculation takes the average over all pairs of spins with a fixed separation $r_{ij} = r$. To survey the collective flips of spins, we also define the flip-correlation function as

$$C(r) = \left[\overline{(\langle\sigma_i\sigma_j\rangle - \langle\sigma_i\rangle\langle\sigma_j\rangle)^2}\right]. \tag{5}$$

All simulations are averaged hundreds of times in order to reduce statistical errors.

## 3. Results and discussion

Experimentally, after the spin glass was quenched below its transition temperature $T_{\mathrm{g}}$ and waited for certain time $t_{\mathrm{w}}$, its magnetization relaxation was related to its waiting history.[30] In our simulations, the site-overlap correlation is calculated to observe the aging effect according to Eq. (2), which measures the spin

configuration evolving towards the reference configuration. After the replica $b$ waits for time $t_{\rm w}$, the replica $a$ begins to wait and the correlation is calculated as a function of time. The results are shown in Fig. 1.

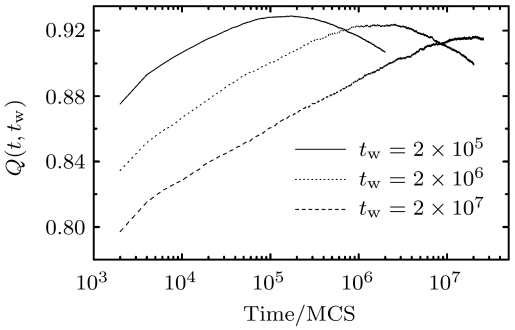



**Fig. 1.** Site-overlap correlations $Q(t,t_{\rm w})$ as a function of time $t$. The temperature takes 0.1 and the waiting time $t_{\rm w}$ takes $2\times10^5$, $2\times10^6$, $2\times10^7$ MCS respectively.

Except for the abrupt rise at beginning, the correlation functions are found to increase almost linearly as the log-scaled time increases and peaks at the corresponding $t_{\rm w}$. According to the droplet model[16,17] the physical picture can be understood as follows. After waiting for time $t$, domains of the spin glass will grow to the size

$$R_{\rm d}(t)=\left(\frac{T\ln t}{\Delta(T)}\right)^{1/\Psi}, \tag{6}$$

where $\Delta(T)$ is the barrier height of the free energy at temperature $T$. In our simulations, after waiting for $t_{\rm w}$, domains in the replica $b$ grow to the size $R_{\rm d}(t_{\rm w})$. For replica $a$, the domains continue to grow as the time increases, so the site overlap between the replicas also increases. As the domains in both replicas reach approximately the same size, the overlap reaches its maximum. After that, domains in replica $a$ continue to grow and exceed those in replica $b$. The overlap then begins to decrease. Therefore, the age of replica $a$ can be exactly determined by calculating the site-overlap correlation.

In the experiments after the spin glass had aged for $t_{\rm w}$ at $T_0$, the temperature was shifted to $T_i$ and the effective age at $T_i$ can be determined by zero-field-cool magnetization.[31] Using Eq. (3) we can determine the effective age by the following protocols. The replica $b$ first ages for $t_{\rm w}$ at $T_0$. After that the replica $a$ ages at $T_i$ and the site-overlap correlation is simultaneously calculated as a function of time. When the domains of the replica $a$ grow to the size $R_{\rm d}^a(t)\simeq R_{\rm d}^b(t_{\rm w})$, the overlap reaches maximum and the consumed time corresponds to the effective age $t=t_{\rm eff}$. For small temperature shift, it is suggested that the effective age at $T_i$ can be expressed as[16,17]

$$\frac{\ln t_{\rm w}}{\ln t_{\rm eff}}=\frac{\Delta(T_0)}{\Delta(T_i)}\frac{T_i}{T_0}. \tag{7}$$

To investigate how the temperature shifts affect the effective age we calculate the site-overlap function for both positive and negative temperature shifts. Figure 2(a) shows the effective age $t_{\rm eff}$ for different temperature shifts. Initially, replica $b$ ages for $t_{\rm w}=5\times10^5$ MCS at $T_0=0.2$, then replica $a$ is quenched to $T_i$ and ages. It is noticed that as $T_i$ shifts to higher temperatures, the site-overlap function reaches its maximum at less $t_{\rm eff}$. Scaling of the effective age versus the shifted temperature is shown in Fig. 2(b).

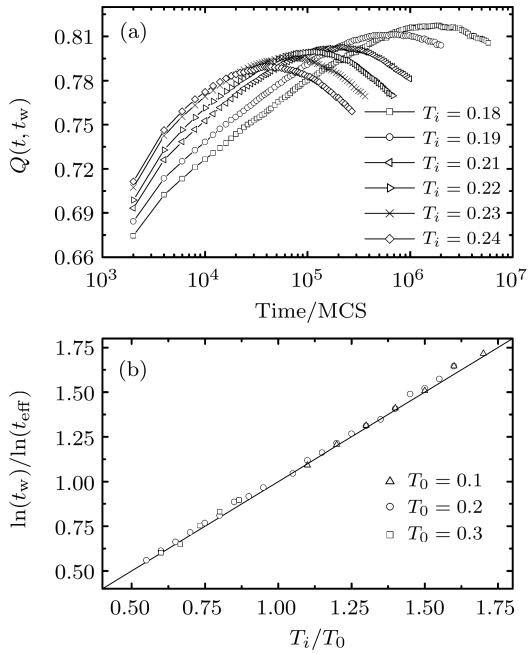



**Fig. 2.** (a) Site-overlap correlations with different temperature shifts. The replica $b$ first waits at $T_0=0.2$ for $5\times10^5$ MCS, then the replica $a$ is quenched to $T_i$ and waits. (b) Scaling plot of the effective age versus the shifted temperature. The solid line represents the case of $\Delta(T_0)=\Delta(T_i)$.

All of the data almost collapse into a line, which suggests that the barrier heights approximate to constants. In the the case of $T_0=0.2$, the data only deviates slightly from the line for larger temperature shifts, this fact is caused by the decreasing $\Delta(T_i)$ with the increasing temperature. Our results clarify two facts: (i) Domain growth obeys the logarithmic law quite well, which is manifested by the scaling plot rather than the algebraic growth law. Since Rieger *et al.*[29] suggested that the domain grows as $R_{\rm d}(t)\propto t^{\alpha(T)}$ where $\alpha(T)=a(T)\Delta(T)/T$, one immediately finds that the scaling equation should be $\ln t_{\rm w}/\ln t_{\rm eff}=(\Delta(T_i)T_0)/(\Delta(T_0)T_i)$ assuming $a(T)$ is

weakly temperature-dependent, which is just opposite to Eq. (7). (ii) The exponent $\psi$ is weakly temperature-dependent since equation (7) is only valid for constant $\psi$, which is consistent with other groups.[19,29]

Moreover, the droplet model predicts that domain growth is accumulative for small temperature shifts. To investigate the accumulation of $t_{\rm eff}$ with sequential temperature shifts, we should modify Eq. (3) as $Q(T_i, T_0) = \left[\overline{\langle \sigma_i^a(t_0, T_i)\sigma_i^b(t_{\rm w}, T_0)\rangle}\right]$, which describes that the replica $a$ evolves sequentially at the serial temperature points $\{T_i\}$ for the fixed time $t_0$. As the reference configuration, the replica $b$ first waits for $t_{\rm w}$ at $T_0$, then the replica $a$ is warmed from the base temperature at equal increments and at each increment it waits for $t_0$. The increment $\Delta T$ takes 0.01 and $t_0$ is fixed to 2000 MCS. The simulation results are shown in Fig. 3.

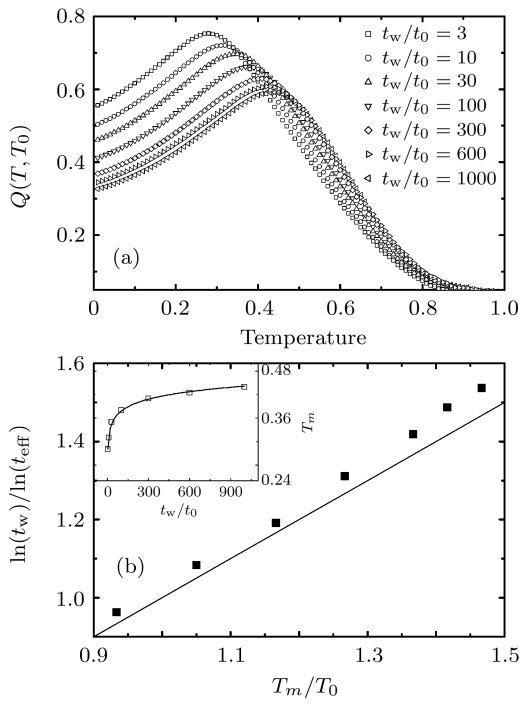



**Fig. 3.** (a) The site-overlap correlations $Q(T_i, T_0)$ as a function of $T_i$. The replica $b$ first waits at $T_0 = 0.3$ for different times $t_{\rm w}$. (b) Scaling plot of the effective age as a function of $T_m$. The line is for the case of constant $\Delta(T_i)$. The inset shows $T_m$ as a logarithmic function of $t_{\rm w}$. The curve is the logarithmic fit.

It is noticed that the correlations rise smoothly as the temperature increases and peaks at the corresponding $T_m$. The longer the waiting time, the higher $T_m$. At high temperatures all correlations decrease to zero. The inset of Fig. 3(b) clearly shows that $T_m$ behaves as a logarithmic function of $t_{\rm w}$. The data can be well fitted by the function $T_m = a_1 + b_1 \ln(t_{\rm w}/t_0)$, where $a_1$ and $b_1$ take 0.25 and 0.03, respectively. Assuming that the barrier heights remain nearly constant even for larger temperature shifts, we can express the effective age at $T_m$ for the warming process as the following equation:

$$t_{\rm eff} = \sum_{i=1}^{m} t_0^{T_i/T_m} \approx \frac{t_0}{1 - t_0^{-\Delta T/T_m}}. \tag{8}$$

Hence, the effective age at $T_m$ can be numerically solved and the scaling results are shown in Fig. 3(b). It can be seen that the effective age almost depends on $T_m$ according to Eq. (7). The data dots only significantly deviate from the theoretical prediction at higher temperatures. This means that the effective ages at each temperature increment can accumulate step by step to higher temperatures through the warming process without severe loss.

The droplet model assumed that an overlap length $L_{\Delta T}$ exists between the spin glass states at different temperatures $T_1$ and $T_2$. At equilibrium, the overlap length is supposed to behave as

$$L_{\Delta T} \sim |T_1 - T_2|^{-1/\zeta}, \tag{9}$$

where $\zeta$ is supposed to take positive value, $\zeta = d_s/2 - \theta$, $d_s$ is the fractal dimension of the droplets and $\theta$ is the stiffness exponent describing the free energy of the droplets $F_L \sim L^\theta$. By Eq. (4) we calculated the link-overlap correlation near the equilibrium states. The replicas are slowly cooled down to $T_1$ and $T_2$ respectively and wait for time $t_{\rm w}$. Temperature $T_1$ takes as high as 0.4 to facilitate the evolution towards the equilibrium states and $T_2$ is set to $T_1 + \Delta T$. The simulation results are shown in Fig. 4. The inset shows that the link-overlap correlation decays exponentially as the spin separation increases. The longer the waiting time, the larger the link-overlap correlation. Assuming that the link-overlap correlation decays as $\Theta(r, t_{\rm w}) \sim {\rm e}^{-r/L_{\Delta T}(t_{\rm w})}$, where $L_{\Delta T}$ is the time-dependent overlap length, we can express the overlap length as the integration $L_{\Delta T}(t_{\rm w}) = \int_0^\infty \Theta(r, t_{\rm w}){\rm d}r$. The overlap length as a function of the temperature interval is shown in Fig. 4. The data are fitted by the polynomials. At the same temperature interval, the overlap length increases with the increase of waiting time. For larger $t_{\rm w}$, it is noticed that the overlap length is saturated to the correlation length $\xi(T_1)$ at small $\Delta T$, but decays more rapidly as $\Delta T$ increases.

Although the overlap length decreases as the temperature interval increases, it essentially violates Eq. (9) due to its downward curvatures instead of upward curvatures. As the waiting time increases,

the system approaches the equilibrium states step by step. However, our fits contradict the negative exponent $-1/\zeta$ suggested by the droplet model. The suggested behaviour was not obtained by other groups either.[19] Since the droplet theory proposes that only one state exists in the equilibrium spin glass at the fixed temperature,[16,17] any deviation from it must cause the spin configuration to vary greatly. As $T_2 \to T_1$, both states at $T_1$ and $T_2$ should be identical, hence the overlap length approaches infinity or, in other words, it diverges. However, numerical evidence has criticized this opinion[32] and supports the theory that spin glass states are degenerate. As $T_2$ approaches to $T_1$, the spin configuration and overlap length vary progressively. Therefore, $L_{\Delta T}$ should be saturated to $\xi(T_1)$ instead of diverging as $T_2 \to T_1$.

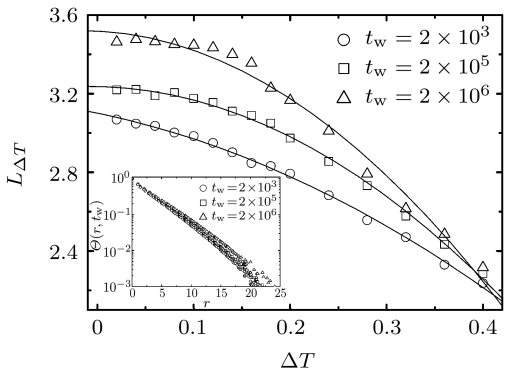



**Fig. 4.** The overlap length as a function of the temperature interval. The replicas are slowly cooled down to the goal temperatures and wait for $t_w$. The inset shows the link-overlap correlations for $T_1 = 0.4$ and $T_2 = 0.42$. The $\Theta(r, t_w)$ is logarithmically scaled.

It is thought that at low temperatures spins in the same droplet reverse collectively. Spin fluctuations are mainly dominated by those in different droplets. To investigate this behaviour we calculated the flip-correlation function of spins in nonequilibrium states by Eq. (5).

Figure 5(a) shows that the flip-correlation functions decay rapidly as the spin separation increases. When the spin separation exceeds a certain distance, the flip correlation becomes constant. For those integer separations, the flip correlations are much larger than those of noninteger separations. It manifests that the colinear spins are more inclined to reverse collectively in comparison with those noncolinear spins. The data are fitted by the power-law function $C(r) = \alpha/r^{\beta}$ where $\alpha$ and $\beta$ are the fitting parameters. Parameter $\beta$ strongly depends on temperature $T$. As shown in Fig. 5(b) $\beta$ can be well fitted by the power-law function of the temperature $T$, $\beta(T) = a_2/T^{b_2}$. For waiting times of $10^3$, $10^4$ and $10^5$ MCS, the fitting parameters $a_2$ and $b_2$ take 1.55 and 0.29, 1.63 and 0.44, 1.58 and 0.55, respectively. It is suggested by the droplet model that at low temperatures the flip correlation of equilibrium spin glass behaves as $C(r) \sim T/r^{\theta}$[16,17] in which $\theta$ should satisfy the inequality $\theta \leq (d-1)/2$. For 2D spin glass, $\theta$ should be equal to or less than 0.5. Indeed, the stiffness exponent $\theta = -0.28$ is obtained for 2D EA spin glass by several groups.[24,33] However, our results show that the exponent $\beta$ is always larger than one and increases rapidly as the temperature decreases. The discrepancy arises from the droplet-reversal types in the spin glass. The correlation $C(r) \sim T/r^{\theta}$ represents the collective droplet reversals in large scales, which are very rare at low temperatures but yield small $\theta$. In our simulations, the droplets are more inclined to reverse by sequential spin flips, which are more common than collective reversal but yield large $\theta$ and are strongly affected by temperature. Hence the predicted $\theta$ should only be considered as the lower limit of the fitting parameter $\beta$.

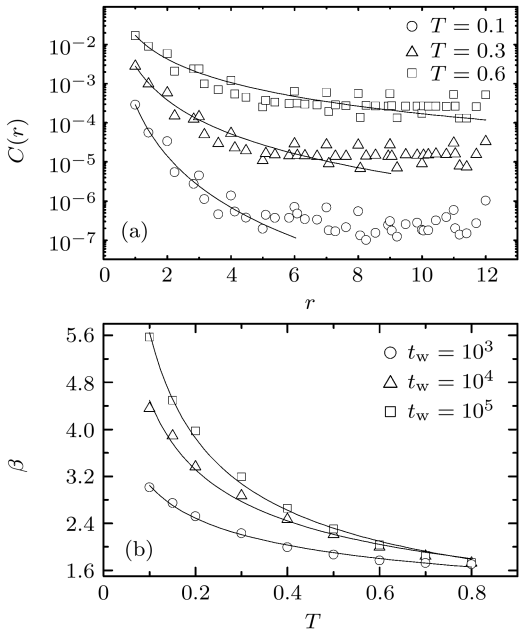



**Fig. 5.** (a) The flip-correlation functions $C(r)$ as a function of spin separation. The system is quenched directly to the goal temperatures and waits for $10^4$ MCS. The solid curves are the fits according to the power law. (b) The fitting parameter $\beta$ as a function of $T$ for different waiting times.

# 4. Conclusion

Using dynamical Monte Carlo simulations we have investigated the nonequilibrium dynamics of 2D

EA spin glass. We developed a new method for measuring the age of domain growth. We find that the effective age of domain growth obeys the prediction of the droplet model quite well, even for larger temperature shifts. By warming spin glass homogeneously, domain growth at each temperature increment can accumulate step by step to higher temperatures without severe loss. We find that the overlap length is saturated to the correlation length instead of diverging at a small temperature interval. Moreover, the spin-flip correlation decays as a power-law function of spin separation. Colinear spins are more inclined to flip collectively than noncolinear spins. Since numerical simulations always play important role in spin glass explorations, we hope that our work will be helpful for future investigations.

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