Control of slow light in a ring-out-ring structure*

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This paper proposes a ring-out-ring structure of coupled optical resonators to yield coupled-resonator-induced transparency (CRIT). Considering the insertion loss of the coupler, it theoretically deduces the transmission and the effective phase shift. The influences of the insertion loss of the coupler on the transmittance, the effective phase shift, the group index and the CRIT linewidth are fully studied. We find that the increase in multiple m can effectively enhance the normal dispersion and the group index of the proposed structure. Moreover, the specific expression of the group index at resonance is theoretically deduced and discussed for the proposed structure with two rings. The result shows that the multiple m between the lengths of ring 1 and ring 2 can enhance the group index to m times that of the structure with two equal-sized rings at resonance. The control of slow light in the proposed structure is important for applications of highly sensitivity gyroscopes, optical delay lines and optical buffers, etc.

Keywords: ring-out-ring structure, dispersion, group index, slow light

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1. Introduction

Recently, the precise control of slow light propagation has made much progress.[1−19] The significant reduction in the group velocity of light can be achieved with a large material dispersive in a dispersive medium, e.g., atomic vapour,[1−4] photorefractive crystal,[5,6] ruby[7,8] and fibres,[9] or with a large structural dispersion in a dispersive structure, e.g., coupled optical resonators[10−15] and various photonic crystals.[16−20] The first direct demonstration of slow light propagation was achieved via electromagnetically induced transparency (EIT),[21] which is based on destructive quantum interference, to render high transparency with high dispersion for the probe light when a strong coupling light is applied. Because of the inherent constraints associated with the conversion of the optical signals to coherent electronic states, the dispersive medium is less attractive for practical implementations of slow-light devices.[22] From the perspective of dispersion engineering, quantum-information storage and rotation sensing, the dispersive structure appears to be an alternative promising system.[22−29] Consequently, significant efforts were focused on controlling the speed of light by using dispersive structures (e.g. coupled resonator) for their practical advantages.

Based on mode splitting and classical destructive interference in coupled resonators, Smith *et al.*[10] firstly introduced a coupled-resonator-induced transparency (CRIT) effect by analogy to the EIT effect. Because the coupled resonators, which can carry the CRIT effect, usually present a solid state in structure and can be designed in size and shape to be operated for any wavelength of light, much attention is now directed to the development of slow light based on the CRIT effect. Recently, some experiments have been reported for observations of slow light based on the CRIT effect in a double microsphere system[14] and coupled erbium-doped fibre resonators.[15] Moreover, the slow light application based on the CRIT effect has made some progress, such as high-precision measurement of rotation. Peng *et al.* theoretically proposed a two-identical-ring planar structure[27] and a two-identical-ring folded structure,[28] which can be employed to construct highly sensitive gyroscopes. In our previous work,[29] we also theoretically proposed a highly dispersive ring-in-ring structure to construct a highly sensitive gyroscope. The sensitivities of the above proposed gyroscopes scale directly with the group indexes of the corresponding systems. The above theoretical studies were implemented in the ideal case, which ignores the insertion loss of the coupler. However, the insertion loss of the coupler is inevitable in practice, and can significantly affect the

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group index of the whole slow light system.

In this paper, we theoretically study the control of slow light in the proposed ring-out-ring structure. Wherein, the influences of the insertion loss of coupler and the parameters of the proposed structure on the slow light effect were discussed in detail. An effective way to enhance the normal dispersion and the group index of the ring-out-ring structure was proposed. The control of slow light in the proposed structure could be favourable to the practical applications of highly sensitivity gyroscopes, optical buffers and optical delay lines, etc.

2. The CRIT in the ring-out-ring structure

We consider the unidirectional propagation and partial coupling of light from a straight waveguide into the ring-out-ring structure with N rings, as shown in Fig. 1(a). Wherein, the arrowheads indicate the directions of light propagation, the numbers

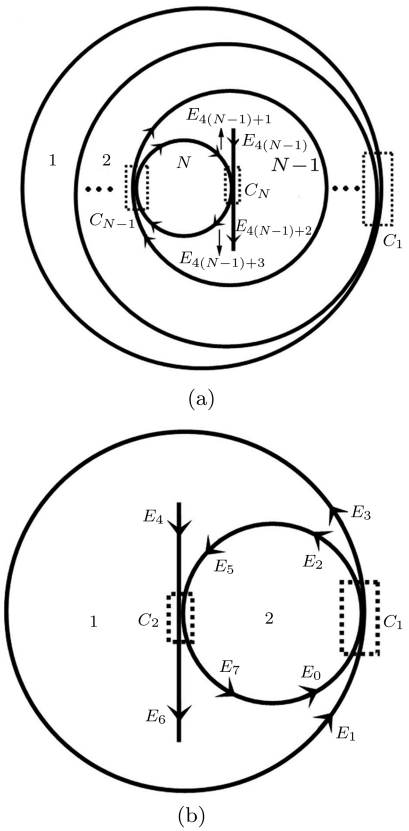


Fig. 1. (a) Illustration of ring-out-ring structure consisted of N rings. (b) Illustration of ring-out-ring structure consisted of two rings.

(1, 2, ..., N) and the signs (C_1, C_2, ..., C_N) represent the order numbers of the ring and coupler in the structure, respectively. To analyse the CRIT effect in the proposed structure considering the insertion loss of the coupler, we will employ an iterative method.[30] Primarily, a ring-out-ring structure with two rings, as shown in Fig. 1(b), is considered. For the first ring, the reflected and transmitted electric fields are respectively

$$E_2 = \sqrt{1-\gamma_1}\,(r_1 E_0 + \mathrm{i} t_1 E_1)\,, \tag{1a}$$

and

$$E_3 = \sqrt{1-\gamma_1}\,(\mathrm{i} t_1 E_0 + r_1 E_1)\,, \tag{1b}$$

where r_1, t_1 and γ_1 are the reflection coefficient, transmission coefficient, and insertion loss of the coupler C_1, respectively. Propagation of E_3 around ring 1 with length L_1 is

$$E_1 = a_1\,\mathrm{e}^{\mathrm{i}\phi_1} E_3, \tag{2}$$

where $\phi_1 = \beta_1 L_1$ and $a_1 = \exp(-\alpha_1 L_1/2)$ are the phase shift and attenuation factor in ring 1, respectively. Wherein, β_1 and α_1 are the propagation constant and the absorption coefficient of ring 1, respectively. According to Eqs. (1a), (1b) and (2), the ratio of the output field to the input field (E_2/E_0, i.e., transmission) across ring 1 can be deduced as

$$\tilde{\tau}_1(\phi_1) = \frac{r_1\sqrt{1-\gamma_1} - a_1(1-\gamma_1)\,\mathrm{e}^{\mathrm{i}\phi_1}}{1 - r_1 a_1\sqrt{1-\gamma_1}\,\mathrm{e}^{\mathrm{i}\phi_1}}. \tag{3}$$

At the output port of ring 1, due to the fact that the light must traverse coupler C_1 twice, acquiring π phase shift between the light coming out from ring 1 and the initially uncoupled light, and then a destructive interference occurs. As a result, the output intensity E_2 from ring 1 decreased obviously, leading to $|\tilde{\tau}_1(\phi_1)|^2$ (i.e. transmittance T_1) far less than 1.

For the second ring, the reflected and transmitted electric fields are, respectively,

$$E_6 = \sqrt{1-\gamma_2}\,(r_2 E_4 + \mathrm{i} t_2 E_5)\,, \tag{4a}$$

and

$$E_7 = \sqrt{1-\gamma_2}\,(\mathrm{i} t_2 E_4 + r_2 E_5)\,, \tag{4b}$$

where r_2, t_2 and γ_2 are the reflection coefficient, transmission coefficient, and insertion loss of the coupler C_2, respectively. Propagation of E_7 around ring 2 with length L_2 is

$$E_5 = a_2\,\mathrm{e}^{\mathrm{i}\phi_2}\tilde{\tau}_1 E_7, \tag{5}$$

where $\phi_2 = \beta_2 L_2$ and $a_2 = \exp(-\alpha_2 L_2/2)$ are the phase shift and the attenuation factor in ring 2, respectively. Wherein, β_2 and α_2 is the propagation

constant and the absorption coefficient of ring 2, respectively. According to Eqs. (4a), (4b) and (5), the ratio of the output field to the input field, (E_6/E_4, i.e., the transmission) across the structure, can be deduced as

$$\tilde{\tau}_2(\phi_2,\phi_1)=\frac{r_2\sqrt{1-\gamma_2}-a_2\tilde{\tau}_1(1-\gamma_2)\,\mathrm{e}^{\mathrm{i}\phi_2}}{1-r_2a_2\tilde{\tau}_1\sqrt{1-\gamma_2}\,\mathrm{e}^{\mathrm{i}\phi_2}}. \quad (6)$$

Compared with the case at the output port of ring 1, at the output port of ring 2, due to the fact that the light must traverse couplers C_1 and C_2 twice, thus acquiring a 2π phase shift between the light coming out from ring 2 and the initially uncoupled light, then we find that an instructive interference occurs. As a result, the output intensity E_6 of ring 2 increased obviously, leading to $|\tilde{\tau}_2(\phi_2,\phi_1)|^2$ (i.e. transmittance T_2) close to 1. Therefore, the transparency of light for the whole structure occurs. This coherence phenomenon is referred to as CRIT,[11] where the insertion losses of couplers were especially considered.

The transmittance T_2 of the whole structure is defined as $T_2=|\tilde{\tau}_2|^2$, and then the absorptance of the whole structure can be obtained by

$$\begin{aligned}A_2(\varphi_2,\varphi_1)&\equiv 1-T_2\\&=\frac{\tilde{A}_2^{\mathrm{env}}}{1+\tilde{F}_2\sin^2\left(\left(\varphi_1^{\mathrm{eff}}+\varphi_2\right)/2\right)}, \quad (7)\end{aligned}$$

where $\tilde{A}_2^{\mathrm{env}}(\phi_1)=1-(1-\gamma_2)\left(r_2-a_2|\tilde{\tau}_1|\sqrt{1-\gamma_2}\right)^2/\left(1-r_2a_2|\tilde{\tau}_1|\sqrt{1-\gamma_2}\right)^2$ is an envelope function, $\tilde{F}_2(\phi_1)=4r_2a_2|\tilde{\tau}_1|\sqrt{1-\gamma_2}/\left(1-r_2a_2|\tilde{\tau}_1|\sqrt{1-\gamma_2}\right)^2$ is a function related to finesse, and $\phi_1^{\mathrm{eff}}(\phi_1)\equiv\arg(\tilde{\tau}_1)$ is the effective phase shift of the first ring. Under the assumption of $a_1=1$, the ϕ_2 dependent term in A_2 can be rewritten as $\sin^2[(\tilde{\phi}_1^{\mathrm{eff}}+\phi_2)/2]=(\cos\phi_2+1)(r_1+1-2\cos\phi_2)^2/2(1-2r_1\cos 2\phi_2+r_1^2)$. For small detuning $\cos\phi_2\approx 1-\phi_2^2/2$, and sufficiently weak coupling between rings $(1-r_1)^2\ll r_1\phi_2^2$, equation (7) can be simplified as

$$\tilde{A}_2(\delta)=\frac{\tilde{A}_2^{(\mathrm{env})}}{1+\dfrac{4(\cos\varphi_2+1)}{\Gamma^2}\left[\delta-(\Delta\omega/2)^2/\delta\right]^2}, \quad (8)$$

where $\Gamma=2\sqrt{2}\left(1-r_2a_2\sqrt{1-\gamma_2}\right)/\sqrt{\eta}\tau_{\mathrm{R}}$ is related to the linewidth, $\tau_{\mathrm{R}}=n_0L_2/c$ is the second ring round-trip time, n_0 is the refractive index of ring medium, $\eta=r_2a_2\sqrt{1-\gamma_2}/r_1$, $\Delta\omega=2\sqrt{(1-r_1)}/\tau_{\mathrm{R}}$ is the frequency difference between the split modes, and $\delta=\phi_2/\tau_{\mathrm{R}}$ is the detuning.

By analogy to the form of the transmission expression of the ring-out-ring structure with two rings, we can easily obtain the transmission expression of the ring-out-ring structure with N rings as

$$\begin{aligned}&\tilde{\tau}_N(\phi_N,\phi_{N-1},\ldots,\phi_1)\\&=\frac{r_N\sqrt{1-\gamma_N}-a_N\tilde{\tau}_{N-1}(1-\gamma_N)\,\mathrm{e}^{\mathrm{i}\phi_{N-1}}}{1-r_Na_N\tilde{\tau}_{N-1}\sqrt{1-\gamma_N}\,\mathrm{e}^{\mathrm{i}\phi_N}}, \quad (9)\end{aligned}$$

where r_N and γ_N are respectively the reflection coefficient and the insertion loss of coupler C_N; $a_N=\exp(-\alpha_NL_N/2)$, α_N, L_N, $\phi_N=\beta_NL_N$ and β_N are respectively the attenuation factor, the absorption coefficient, the length, the phase shift, and the propagation constant of ring N. In the following, we assume that the insertion loss of all couplers is equivalent for simplifying, i.e., $\gamma_1=\gamma_2=\cdots=\gamma_N=\gamma$. Note that equation (9) can also be written as $\tilde{\tau}_N=|\tilde{\tau}_N|\exp(\mathrm{i}\phi_N^{\mathrm{eff}})$, where ϕ_N^{eff} is the effective phase shift of the whole structure, and is defined as

$$\phi_N^{\mathrm{eff}}(\phi_N,\phi_{N-1},\ldots,\phi_1)=\arg(\tilde{\tau}_N). \quad (10)$$

The slope of effective phase shift ϕ_N^{eff} changed by phase shift ϕ_N of ring N represents the dispersion of the whole structure. For the occurrence of the CRIT effect in the proposed structure, the large slope of the effective phase shift can be induced, and then a strong normal dispersion presents in the proposed structure, which results in considerably slow light. For the slow light induced by the CRIT effect, the group delay of the whole structure is $\tau(\omega)=\mathrm{d}\phi_N^{\mathrm{eff}}/\mathrm{d}\omega$, and the corresponding group velocity is $\upsilon_{\mathrm{g}}=L/\tau(\omega)$, where L is the length of ring N. Thus, the group index can be obtained,

$$n_{\mathrm{g}}=n_0\,\mathrm{d}\varphi_N^{\mathrm{eff}}/\mathrm{d}\varphi_N, \quad (11)$$

where $\mathrm{d}\phi_N^{\mathrm{eff}}/\mathrm{d}\phi_N$ can be defined as the relative group index of the whole structure.

For a ring-out-ring structure with two rings, assuming the radii of the two rings in the ring-out-ring structure, R_1 and R_2, satisfy the relation of $R_1=2R_2$, then we can obtain the relation of $\phi_1=2\phi_2$ from $\phi=\beta L$. According to Eq. (6), figure 2(a) shows the transmittance T_2 of the whole structure versus the insertion loss γ of the coupler and phase shift ϕ_2 of ring 2. In this figure, a narrow transparency window appears and the transmittance reaches a maximum at resonance, which originates from coherent coupling between the rings, analogous to AC-Stark shift and quantum interference occurring in three-level atomic

systems. Moreover, it is clear that the maximum of the transmittance decreases with the insertion loss, and the width of the transparency window increases with the insertion loss.

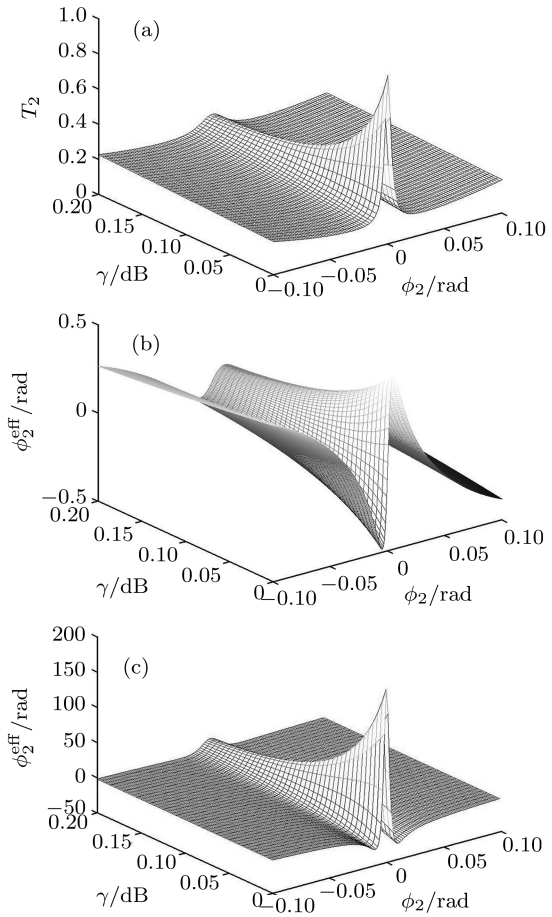


Fig. 2. (a) Transmittance of the whole structure versus insertion loss γ and phase shift ϕ_2 of ring 2. (b) Effective phase shift of the whole structure versus insertion loss γ and phase shift ϕ_2 of ring 2. (c) The relative group index of the whole structure versus insertion loss γ and phase shift ϕ_2 of ring 2. The parameters are $a_1 = 0.9999$, $a_2 = 0.8$, $r_1 = 0.999$, $r_2 = 0.9$, and the phase shift of two rings satisfies the relation of $\phi_1 = 2\phi_2$.

According to Eq. (10), figure 2(b) shows the effective phase shift $\phi_2^{\rm eff}$ of the whole structure versus the insertion loss γ of the coupler and the phase shift ϕ_2 of ring 2. In this figure, the steep normal dispersion (i.e., the slope of $\phi_2^{\rm eff}$) occurs in the resonance region that results in slow light, and the normal dispersion decreases with the insertion loss. In addition, the linewidth of the normal dispersion, which corresponds to the width of the transparency window, increases with the insertion loss.

According to Eq. (11), figure 2(c) shows the relative group index of the whole structure versus the insertion loss γ of the coupler and the phase shift ϕ_2 of ring 2. One can see that, in the narrow transparency window, the relative group index varies rapidly and reaches maximum at resonance. Moreover, it is clear that the maximum of the relative group index decreases with the insertion loss, and the linewidth of the group index increases correspondingly.

From what has been discussed above, the transmittance, the normal dispersion and the group index reach a maximum at resonance and decrease with the insertion loss, but the linewidth increases correspondingly. Briefly, the insertion loss directly weakened the CRIT effect of the ring-out-ring structure.

3. Analysis and discussion

As discussed above, the ring-out-ring structure presents highly normal dispersion and a large group index in the resonance region, but the normal dispersion and the group index are rapidly weakened by the insertion loss, thus this case is less significant for practical application. Therefore, it is necessary to suitably change the parameters of the proposed structure to overcome the unfavourable influence of insertion loss.

One way to obtain large normal dispersion is to use more rings to construct the ring-out-ring structure, as shown in Fig. 1(a). Assume that the radii of ring 1 and ring N in the ring-out-ring structure, R_1 and R_N, satisfy the relation of $R_1 = NR_N$. According to Eq. (11), figures 3(a) and 3(b), for insertion loss $\gamma = 0$ dB and $\gamma = 0.1$ dB, respectively, depict the relative group index of the whole structure versus the number N of rings and phase shift ϕ_N of ring N. For the ideal case (no insertion loss, i.e., $\gamma = 0$ dB), it can be seen from Fig. 3(a) that the relative group index reaches a maximum at resonance for a given number N, and the maximum increase with number N that is compatible with the above expectation. However, for the practical case, the insertion loss of the coupler inevitably presents in the proposed structure. As an example, we choose $\gamma = 0.1$ dB. It is clear from Fig. 3(b) that the maximum of the relative group index decreased with the number N. This result can be explained as follows. When light traverses from one ring to the adjacent ring, its intensity is decayed by the insertion loss of the coupler existing in the two adjacent rings, that is to say, insertion losses of all couplers eliminate the contribution of the number N to the normal dispersion of the structure. Therefore,

the use of more rings in the proposed structure cannot effectively enhance the normal dispersion for the practical case.

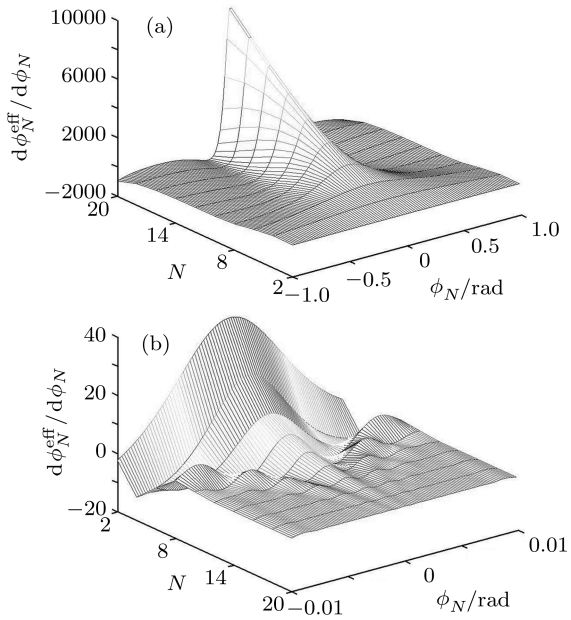


Fig. 3. (a) The relative group index of the whole structure versus the ring N phase shift and number N of rings with $\gamma = 0$ dB. (b) The relative group index of the whole structure versus the ring N phase shift and number N of rings with $\gamma = 0.1$ dB. The parameters are $a_1 = 0.9999$, $a_2 = 0.8$, $r_1 = 0.999$, $r_2 = 0.9$, and the phase shift between N rings satisfies the relation of $\phi_1 = N\phi_N$, $N = 2, 4, 6, \ldots, 20$.

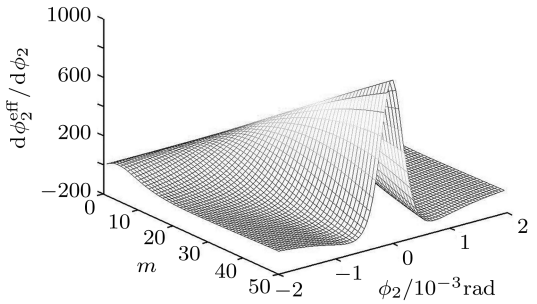


Fig. 4. The relative group index of the whole structure versus the multiple m and phase shift ϕ_2 of ring 2 with $\gamma = 0.1$ dB. The parameters are $a_1 = 0.9999$, $a_2 = 0.8$, $r_1 = 0.999$, $r_2 = 0.9$, and the phase shift between two rings satisfies the relation of $\phi_1 = m\phi_2$.

Consequently, it is evident that the number N should be determined as a small number. Thus the ring-out-ring structure with two rings is proposed, and then the large multiple between lengths of the two rings is considered to effectively enhance the normal dispersion of the proposed structure. Assume that the radii of the two rings in the proposed structure, R_1 and R_2, satisfy the relation $R_1 = mR_2$. According to Eq. (11), figure 4 shows the dependence of the relative group index on the multiple m and phase shift ϕ_2 of ring 2, for a given insertion loss $\gamma = 0.1$ dB. In the figure, the relative group index reaches the maximum at resonance, and significantly increases with the multiple m, as we expected.

As analysed above, to achieve highly normal dispersion considered insertion loss, the increase in multiple m is more feasible than that in number N in practical application. Subsequently, we theoretically deduce the group index of the structure with two rings (shown in Fig. 1(b)). Based on Eq. (6), the derivative of the effective phase shift is

$$\frac{\mathrm{d}\phi_2^{\mathrm{eff}}}{\mathrm{d}\phi_2} = \frac{1}{|\tilde{\tau}_2|^2}\left[\mathrm{Re}\ \tilde{\tau}_2 \frac{\mathrm{d}\,(\mathrm{Im}\ \tilde{\tau}_2)}{\mathrm{d}\phi_2} - \mathrm{Im}\ \tilde{\tau}_2 \frac{\mathrm{d}\,(\mathrm{Re}\ \tilde{\tau}_2)}{\mathrm{d}\phi_2}\right]. \tag{12}$$

On resonance, the result of $\mathrm{Im}\ \tilde{\tau}_2 = 0$ can be easily obtained. As a result,

$$\frac{\mathrm{d}\phi_2^{\mathrm{eff}}}{\mathrm{d}\phi_2} = \frac{\mathrm{Re}\ \tilde{\tau}_2}{|\tilde{\tau}_2|^2} \frac{\mathrm{d}\,(\mathrm{Im}\ \tilde{\tau}_2)}{\mathrm{d}\phi_2}. \tag{13}$$

The total differentiation of the transmission is

$$\mathrm{d}\tilde{\tau}_2 = \frac{\partial \tilde{\tau}_2}{\partial \phi_2}\mathrm{d}\phi_2 + \frac{\partial \tilde{\tau}_2}{\partial \tilde{\tau}_1}\mathrm{d}\tilde{\tau}_1. \tag{14}$$

Here $\partial\tilde{\tau}_2/\partial\varphi_2 = -\mathrm{i}\tilde{\tau}_1\xi_2$ and $\partial\tilde{\tau}_2/\partial\tilde{\tau}_1 = -\xi_2$ can be obtained from Eq. (6), and $\mathrm{d}\tilde{\tau}_1 = \partial\tilde{\tau}_1\,\mathrm{d}\varphi_1/\partial\varphi_1$ and $\partial\tilde{\tau}_1/\partial\varphi_1 = -\mathrm{i}\xi_1$ can be obtained from Eq. (3), where $\xi_2(\phi_2) = a_2(1 - r_2^2)(1 - \gamma_2)\,\mathrm{e}^{\mathrm{i}\phi_2}/(1 - r_2a_2\sqrt{1-\gamma_2}\tilde{\tau}_1\,\mathrm{e}^{\mathrm{i}\phi_2})^2$, $\xi_1(\phi_1) = a_1(1 - r_1^2)(1 - \gamma_1)\,\mathrm{e}^{\mathrm{i}\phi_1}/(1 - r_1a_1\sqrt{1-\gamma_1}\tilde{\tau}_1\,\mathrm{e}^{\mathrm{i}\phi_1})^2$. Hence equation (14) can be written as

$$\mathrm{d}\tilde{\tau}_2 = -\mathrm{i}\tilde{\tau}_1\xi_2\,\mathrm{d}\phi_2 + \mathrm{i}\xi_2\xi_1\,\mathrm{d}\phi_1. \tag{15}$$

It is noteworthy that $\tilde{\tau}_1$, ξ_1 and ξ_2 are real quantities at resonance, and then $\mathrm{d}\tilde{\tau}_2$ is an imaginary quantity at resonance. Based on the relation of $\phi_1 = m\phi_2$, the differentiation of the imaginary part of the transmission can be written as

$$\frac{\mathrm{d}\,(\mathrm{Im}\ \tilde{\tau}_2)}{\mathrm{d}\phi_2} = m\xi_2\,(0)\,\xi_1\,(0) - \xi_2\,(0)\,\tilde{\tau}_1\,(0)\,. \tag{16}$$

Substituting Eq. (16) into Eq. (13), we can obtain the specific expression of the group index at resonance as

$$n_{\mathrm{g}} = n_0\frac{\mathrm{d}\phi_2^{\mathrm{eff}}}{\mathrm{d}\phi_2} = \frac{n_0 m\xi_2\,(0)\,(\xi_1\,(0) - \tilde{\tau}_1\,(0))}{|\tilde{\tau}_2|}, \tag{17}$$

where m is any positive real number. Obviously, the group index n_{g} is the function of the reflection coefficients (r_1 and r_2), multiple m and insertion loss γ.

If $m = 1$, this ring-out-ring structure is reduced to the case of two equal-sized rings. From Eq. (17), it is clearly demonstrated that the group index of the ring-out-ring structure with two different-sized rings (the length of ring 1 is m times that of ring 2) is m times that of the structure with equal-sized rings.

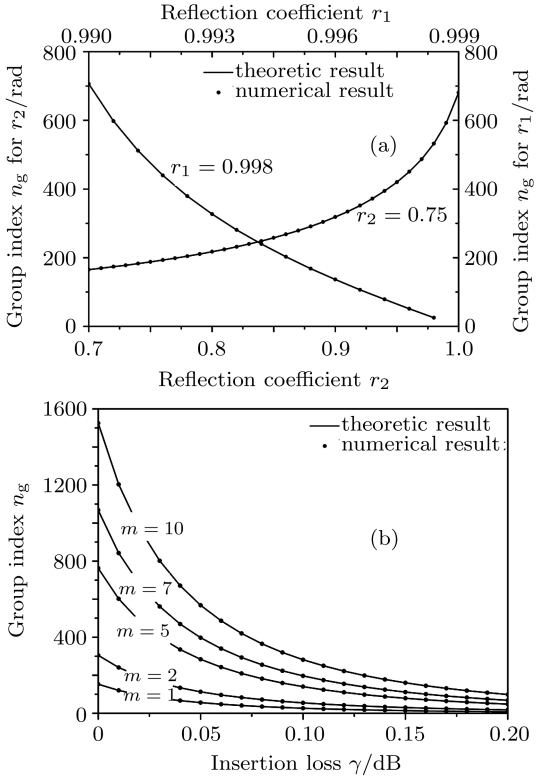


Fig. 5. (a) The group index n_g of the whole system versus reflection coefficients r_1, r_2 at resonance with $a_1 = 0.9999$, $a_2 = 0.8$, $\gamma = 0.1$ dB, $m = 5$. (b) The group index n_g of the whole system versus insertion loss γ for different m at resonance with $a_1 = 0.9999$, $a_2 = 0.8$, $r_1 = 0.999$, $r_2 = 0.9$.

Figure 5(a) shows the group index n_g at resonance versus reflection coefficients r_1 and r_2, and figure 5(b) shows the group index n_g at resonance versus insertion loss γ for different multiple m. Wherein, the dotted line in Fig. 5 indicates the numerical group index calculated from Eq. (6), and the solid line in Fig. 5 indicates the theoretical group index calculated from Eq. (17). One can see that the theoretical results agree well with the numerical result. From Fig. 5(a), it is clear that n_g increases with increasing r_1 and decreasing r_2, that is to say, the group velocity increases with decreasing r_1 and increasing r_2. From Fig. 5(b), n_g decreases with the insertion loss γ, and significantly increases with the multiple m, that is to say, the group velocity increases with the insertion loss γ, and significantly decreases with the multiple m. The analysis for the multiple m in Fig. 5(b) is compatible with that in Fig. 4.

As is well known, for atomic systems in EIT, a larger group index is associated with smaller EIT linewidths. The same case inevitably occurs for a ring-out-ring structure in CRIT, and then the full width at half maximum (FWHM) linewidth of the ring-out-ring structure should be discussed. As an example, we choose the structure with two rings where the size of ring 1 is two times that of ring 2, the FWHM linewidth δ_{FWHM} can be obtained through Eqs. (7) and (8) by $\tilde{A}_2(\delta) = \tilde{A}_2^{(\mathrm{env})}/2$ and $\delta \to 0$ as

$$\begin{aligned}\delta_{\mathrm{FWHM}} &= \frac{\sqrt{2}\Delta\omega^2}{\Gamma} \\ &= \frac{2(1-r_1)\sqrt{r_2 a_2\sqrt{1-\gamma}/r_1}}{\left(1-r_2 a_2\sqrt{1-\gamma}\right)\tau_{\mathrm{R}}}. \end{aligned} \quad (18)$$

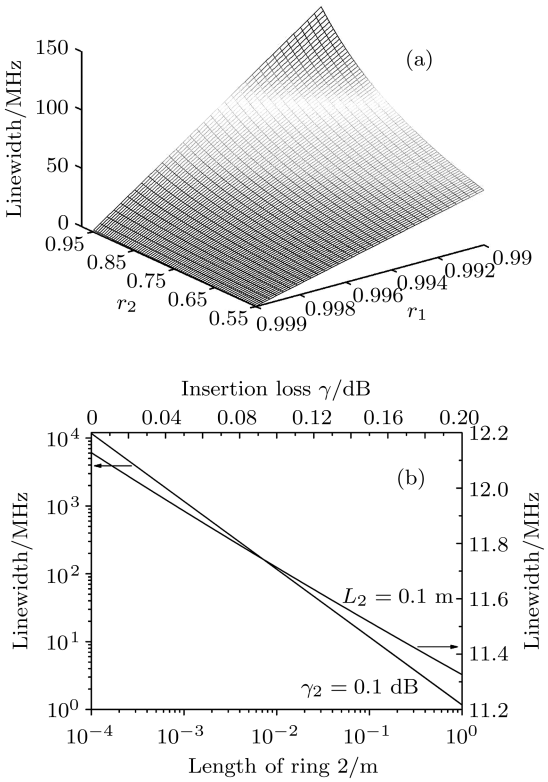


Fig. 6. (a) The FWHM linewidth versus the reflection coefficients r_1, r_2 for the ring-out-ring structure: $a_1 = 0.9999$, $a_2 = 0.8$, $\gamma = 0.1$ dB, $m = 2$. (b) The FWHM linewidth versus the length L_2 of ring 2 and insertion loss γ for the ring-out-ring structure: $a_1 = 0.9999$, $a_2 = 0.8$, $r_1 = 0.999$, $r_2 = 0.9$, $m = 2$.

Figure 6(a) shows the FWHM linewidth of the ring-out-ring structure versus reflection coefficients r_1 and r_2. It is obvious that the FWHM linewidth increases with r_2 and decreases with r_1, whose trend is contrary to that of n_g in Fig. 5(a) and compatible with the above description. Figure 6(b) shows the

FWHM linewidth versus the length L_2 of ring 2 and the insertion loss γ. It is evident that the FWHM linewidth decreases with the length L_2, but increases with the insertion loss γ, and the latter is consistent with the linewidth analysis in Section 2. Briefly, the FWHM linewidth can be tuned by reflection coefficients r_1 and r_2, length L_2 of ring 2, and insertion loss γ. For a general case of a ring-out-ring structure with two rings where the size of ring 1 is m times that of ring 2, because the multiple m can effectively enhance the group index, we can easily deduce that the FWHM linewidth decreases correspondingly with the multiple m.

4. Conclusion

In this paper, we have described a highly dispersive ring-out-ring structure with a CRIT property considering the insertion loss γ of a coupler. The theoretical analysis shows that the slow light can be controlled by insertion loss γ, number N of rings, multiple m between the lengths of ring 1 and ring 2, and reflection coefficients r_1 and r_2. Wherein, the insertion loss γ weakened directly the group index of the whole structure, the increase in N can enhance the group index only for the ideal case (no insertion loss) but not for the practical case. Also, we have demonstrated that the increase in m is an effective way to enhance the normal dispersion and the group index of the whole structure for the practical case. For the proposed structure with two rings, the specific expression of the group index at resonance is theoretically deduced and discussed, and the result shows that the multiple m can enhance the group index to m times that of the structure with two equal-sized rings.

References

[1] Hau L V, Harris S E, Dutton Z and Behroozi C H 1999 *Nature* **397** 594

[2] Budker D, Kimball D F, Rochester A M and Yashchuk V V 1999 *Phys. Rev. Lett.* **83** 1767

[3] Wang L R, Zhao Y T, Ma J, Zhao J M, Xiao L T and Jia S T 2006 *Chin. Phys.* **15** 365

[4] Chen A X, Deng L and Wu Q P 2007 *Chin. Phys.* **16** 3386

[5] Podivilov E, Sturman B, Shumelyuk A and Odoulov S 2003 *Phys. Rev. Lett.* **91** 083902

[6] Zhang G, Fang B, Rong D and Xu J 2004 *Phys. Rev. Lett.* **93** 133903

[7] Bigelow M S, Lepeshkin N N and Boyd R W 2003 *Phys. Rev. Lett.* **90** 113903

[8] Zhang Y D, Fan B H, Yuan P and Ma Z G 2004 *Chin. Phys. Lett.* **21** 87

[9] Schweinsberg A, Lepeshkin N N, Bigelow M S, Boyd R W and Jarabo S 2006 *Europhys. Lett.* **73** 218

[10] Smith D, Chang H, Fuller K A, Rosenberger A T and Boyd R W 2004 *Phys. Rev.* A **69** 063804

[11] Yanik M F and Fan S 2004 *Phys. Rev. Lett.* **92** 083901

[12] Xu Q, Sandhu S, Povinelli M L, Shakya J, Fan S and Lipson M 2006 *Phys. Rev. Lett.* **96** 123901

[13] Wang N, Zhang Y, Wang J, Tian H, Wang H, Zhang X, Zhang J and Yuan P 2009 *Acta Phys. Sin.* **58** 7672 (in Chinese)

[14] Totsuka K, Kobayashi N and Tomita M 2007 *Phys. Rev. Lett.* **98** 213904

[15] Dumeige Y, Nguyen T K N, Ghisa L, Trebaol S and Feron P 2008 *Phys. Rev.* A **78** 013818

[16] Povinelli M L, Johnson S G and Joannopoulos J D 2005 *Opt. Express* **13** 7145

[17] Finlayson C E, Cattaneo F and Perney N M B 2006 *Phys. Rev.* E **73** 016619

[18] Krauss T F 2007 *J. Phys.* D: *Appl. Phys.* **40** 2666

[19] Baba T and Mori D 2007 *J. Phys.* D: *Appl. Phys.* **40** 2659

[20] Tian K, Arora W, Takahashi S, Hong J and Barbastathis G 2009 *Phys. Rev.* B **80** 34305

[21] Boller K J, Imamoglu A and Harris S E 1991 *Phys. Rev. Lett.* **66** 2593

[22] Scheuer J and Yariv A 2006 *Phys. Rev. Lett.* **96** 3901

[23] Vahala K J 2003 *Nature* **424** 839

[24] Xu Q, Schmidt B, Pradhan S and Lipson M 2005 *Nature* **435** 235

[25] Rakovich Y P, Boland J J and Donegan J F 2005 *Opt. Lett.* **30** 2775

[26] Poon J K S, Zhu L, DeRose G A and Yariv A 2006 *Opt. Lett.* **31** 456

[27] Peng C, Li Z and Xu A 2007 *Opt. Express* **15** 3864

[28] Peng C, Li Z and Xu A 2007 *Appl. Opt.* **46** 4125

[29] Zhang Y, Wang N, Tian H, Wang H, Qiu W, Wang J and Yuan P 2008 *Phys. Lett.* A **372** 5848

[30] Smith D, Chang H and Fuller K A 2003 *J. Opt. Soc. Am.* B **20** 1967