Double-distance propagation of Gaussian beams passing through a tilted cat-eye optical lens in a turbulent atmosphere*

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By using the extended Huygens–Fresnel diffraction integral and the method of expanding the aperture function into a finite sum of complex Gaussian functions, an approximate analytical formula of the double-distance propagation for Gaussian beam passing through a tilted cat-eye optical lens and going back along the entrance way in a turbulent atmosphere has been derived. Through numerical calculation, the effects of incidence angle, propagation distance, and structure constant on the propagation properties of a Gaussian beam in a turbulent atmosphere are studied. It is found that the incidence angle creates an unsymmetrical average intensity distribution pattern, while the propagation distance and the structure constant can each create a smooth and symmetrical average intensity distribution pattern. The average intensity peak gradually deviates from the centre, and the central average intensity value decreases quickly with the increase in incidence angle, while a larger structure constant can bring the average intensity peak back to the centre.

Keywords: cat-eye optical lens, turbulent atmosphere, Gaussian beams, double-distance propagation

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1. Introduction

Most of the optical lenses used in photoelectric equivalents have detectors fixed on their focal plane, so the laser beam that irradiates the optical window of the photoelectric equivalent can be reflected back in the direction of the source. Such a phenomenon is defined as the "cat-eye effect", and the optical lens is defined as a "cat-eye optical lens".[1,2] This theory is widely used in active laser detection,[1−5] space communication,[6,7] laser tracking and measurement,[8] and so on. Because the design of laser systems requires an appropriate evaluation of beam characteristics, studies have been carried out to acquire analytical propagation formulas of a Gaussian beam passing through a cat-eye optical lens.[1,2] But these studies for laser beam propagating through a cat-eye optical lens under various conditions have not dealt with the propagation in a turbulent atmosphere. In actual applications, most cat-eye optical targets are in the far-field. When the laser beam propagates through the turbulent atmosphere and arrives at the cat-eye optical lens, and then reflects back through the turbulent atmosphere and arrives at the return place along the entrance way, the refractive index will produce random variations in the amplitude and phase of the laser beam and cause intensity fluctuation and pattern spread in the return place. Therefore, the double-distance propagation of the laser beam passing through the cat-eye optical lens and returning back in a turbulent atmosphere needs to be studied.

In recent years, propagations of various types of laser beams in a turbulent atmosphere have been investigated, such as dark hollow or annular beams,[9,10] Gauss–Bessel beams,[11] Gaussian–Schell-model beams.[12] Hermite Gaussian beams,[13,14] flattened Gaussian beams,[15,16] cylindrical vector beams,[17] and array beams.[18−20] More recently, studies of laser beams propagating in a turbulent atmosphere with misaligned apertures or optical systems have been performed.[21−25] But under most application conditions of cat-eye effect theory, the reflective cat-eye model contains at least two hard apertures and they are usually misaligned. What is more, the propagation of a laser beam through a cat-eye optical lens and return back contains two propagation events: come and back, they cannot be simply equivalent to a propagation for a double propagation distance but

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should be calculated one by one. However, up to now, there seems to be little work that is similar to this condition. In the rest of the present paper, we study the double-distance propagation properties for a Gaussian beam passing through a tilted cat-eye optical lens and going back along the entrance way in a turbulent atmosphere. An approximate analytical formula is derived and some numerical examples are given. Effects of incidence angle, propagation distance, and structure constant on the propagation properties are obtained.

2. Reflected model of cat-eye effect and window functions of the equivalent hard apertures

The reflected model of a cat-eye effect can be simplified into a pair of confocal lenses and a reflector in the confocal plane, so it can be regarded as a spatial filter with two equivalent apertures,[2] which can be seen in Fig. 1. We assume that the z axis is along the direction of the incident beam. Under most conditions, there is an angle between the z axis and the optical axis of the cat-eye optical lens, which is defined as the incident angle θ. Because cat-eye optical lenses are usually circularly symmetrical, we assume that the incline of the cat-eye optical lens exists just in the direction of the y axis, which is shown in Fig. 1, and the x axis is perpendicular to the y–z surface. In Fig. 1, R is the radius of the two lenses, f is the focus of the two lenses, O_y is the y coordinate value of the centre of the second equivalent aperture and it can be given by

$$O_y = 2f \tan\theta. \tag{1}$$

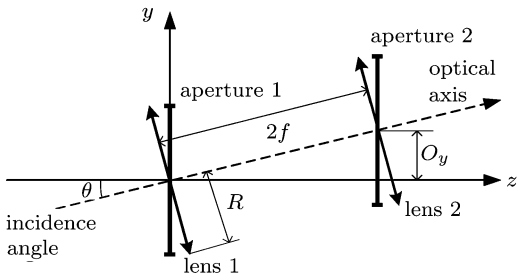


Fig. 1. Reflected model of cat-eye effect with two equivalent apertures.

Because of the incline of the cat-eye optical lens, the shapes of the two equivalent apertures are elliptical. In the x–y coordinate system, the window functions of the two hard-edge elliptical apertures can be written as

$$T_1(x,y) = \begin{cases} 1, & x^2 + \dfrac{y^2}{\cos^2\theta} \le R^2, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

$$T_2(x,y) = \begin{cases} 1, & x^2 + \dfrac{(y-O_y)^2}{\cos^2\theta} \le R^2, \\ 0, & \text{otherwise}. \end{cases} \tag{3}$$

The two hard-edge elliptical aperture functions can be expanded into the sum of complex Gaussian functions with finite numbers and they are given by[26−30]

$$A_1(x,y) = \sum_{j=1}^{J} F_j \times \exp\left\{-\frac{G_j}{R^2}\left[x^2 + \frac{y^2}{\cos^2\theta}\right]\right\}, \tag{4}$$

$$A_2(x,y) = \sum_{m=1}^{M} F_m \times \exp\left\{-\frac{G_m}{R^2}\left[x^2 + \frac{(y-O_y)^2}{\cos^2\theta}\right]\right\}, \tag{5}$$

where F_j and G_j are the expand modulus and compound Gaussian modulus, which can be obtained from Ref. [26], and $J = M = 10$.

3. Analytical expression of the average intensity at the return place

In this paper, the optical field of the incident Gaussian beam at the position of the waist can be written as

$$E_0(x_0, y_0, 0) = \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right), \tag{6}$$

where w_0 is the waist width, and x_0 and y_0 are the transverse coordinates of the beam.

Assume that the distance from the position of the waist of the Gaussian beam to the input pupil of the cat-eye optical lens is L. Within the framework of the paraxial approximation, the propagation of a laser beam in a turbulent atmosphere can be treated as the following extended Huygens–Fresnel integral formula,[9−25]

$$I_1(x_1,y_1,L) = \frac{k^2}{4\pi^2L^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_0(x,y,L)E_0^*(\varepsilon,\eta,L) \times \exp\left\{\frac{\mathrm{i}k}{2L}\left[(x_1-x)^2+(y_1-y)^2-(x_1-\varepsilon)^2-(y_1-\eta)^2\right]\right\} \times \langle \exp\{\psi(x,y,x_1,y_1) + \psi^*(\varepsilon,\eta,x_1,y_1)\rangle \,\mathrm{d}x\mathrm{d}y\mathrm{d}\varepsilon\mathrm{d}\eta, \quad (7)$$

where $I_1(x_1,\ y_1,\ L)$ is the average intensity of the Gaussian beam propagating through the turbulent atmosphere and arriving at the input pupil of the cat-eye optical lens, namely the front of the first aperture; $k = 2\pi/\lambda$ is the wave number, with λ representing the wavelength; the asterisk $(*)$ denotes the complex conjugation; the symbol $\langle\ \rangle$ indicates the ensemble average over the medium statistics covering the log-amplitude fluctuation and the phase fluctuation due to the turbulent atmosphere. The ensemble average term is

$$\begin{aligned}&\langle \exp\{\psi(x,y,x_1,y_1) + \psi^*(\varepsilon,\eta,x_1,y_1)\rangle\\ &= \exp\left[-0.5D\psi\left(x-\varepsilon,y-\eta\right)\right]\\ &= \exp\left\{-\frac{1}{\rho_0^2}\left[(x-\varepsilon)^2+(y-\eta)^2\right]\right\},\end{aligned} \quad (8)$$

where $\psi(x,\ y,\ x_1,\ y_1)$ denotes the random part of the complex phase of a spherical wave that propagates from the source point to the receiver point, $D_\psi(x-\varepsilon,\ y-\eta)$ is the wave structure function approximated by the phase structure function in Rytov's representation, ρ_0 is the coherence length of a spherical wave and can be expressed as

$$\rho_0 = (0.545C_n^2k^2z)^{-3/5}, \quad (9)$$

with C_n^2 being the structure constant.

After troublesome integration, we obtain the average intensity at the position of the input pupil of the cat-eye optical lens as

$$\begin{aligned}&I_1(x_1,y_1,L)\\ &= \frac{\rho_0^4k^2}{4L^2\left(1+\rho_0^4P_1^2P_1^{*2}\right)}\\ &\quad\times\exp\left[\frac{k^2\left(2+2\rho_0^2P_1^2+\rho_0^4P_1^4\right)}{4L^2P_1^2\left(1+\rho_0^4P_1^2P_1^{*2}\right)}\left(x_1^2+y_1^2\right)\right],\end{aligned} \quad (10)$$

where

$$P_1^2 = \frac{1}{w_0^2}+\frac{1}{\rho_0^2}+\frac{\mathrm{i}k}{2L}. \quad (11)$$

In the far-field condition, the size of the beam spot is much larger than the aperture of the cat-eye optical lens, so the optical field in the aperture of the cat-eye optical lens can be simply replaced by the product of the square root of the average intensity and a phasic factor, which can be written as

$$\begin{aligned}E_1(x_1,y_1,L) &= \sqrt{I_1(x_1,y_1,L)}\exp\left[\frac{\mathrm{i}k}{2L}(x_1^2+y_1^2)\right]\\ &= \frac{k\rho_0^2}{2L\sqrt{\left(1+\rho_0^4P_1^2P_1^{*2}\right)}}\exp\left\{\left[\frac{\mathrm{i}k}{2L}+\frac{k^2\left(2+2\rho_0^2P_1^2+\rho_0^4P_1^4\right)}{8L^2P_1^2\left(1+\rho_0^4P_1^2P_1^{*2}\right)}\right]\left(x_1^2+y_1^2\right)\right\}.\end{aligned} \quad (12)$$

The effect of the refractive index on the propagation of the beam propagating from the first aperture to the second aperture of the cat-eye optical lens can be ignored. The propagation matrix from the first aperture to the second aperture is

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \begin{bmatrix}-1 & 2f\\ 0 & -1\end{bmatrix}. \quad (13)$$

So by using Eq. (13) and based on the Collins diffraction formula,[27] the optical field before the second aperture for this Gaussian beam passing through the cat-eye optical lens can be expressed as

$$\begin{aligned}E_2(x_2,y_2,L+2f) &= \frac{\mathrm{i}k}{4\pi f}\exp\left[\frac{\mathrm{i}k}{4f}(x_2^2+y_2^2)\right]\\ &\quad\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E_1(x_1,y_1,L)A_1(x_1,y_1)\exp\left\{-\frac{\mathrm{i}k}{4f}[-(x_1^2+y_1^2)-2(x_1x_2+y_1y_2)]\right\}\mathrm{d}x_1\mathrm{d}y_1.\end{aligned} \quad (14)$$

Through integration, we obtain

$$E_2(x_2,y_2,L+2f)=\frac{\mathrm{i}k^2\rho_0^2}{8fL\sqrt{(1+\rho_0^4P_1^2P_1^{*2})}}\exp\left[\frac{\mathrm{i}k}{4f}(x_2^2+y_2^2)\right]\sum_{j=1}^{J}\frac{F_j}{P_{2x}P_{2y}}\exp\left(-\frac{k^2x_2^2}{16f^2P_{2x}^2}-\frac{k^2y_2^2}{16f^2P_{2y}^2}\right), \quad (15)$$

where

$$P_{2x}^2=-\frac{\mathrm{i}k}{4f}-\frac{\mathrm{i}k}{2L}+\frac{G_j}{R^2}-\frac{k^2\left(2+2\rho_0^2P_1^2+\rho_0^4P_1^4\right)}{8L^2P_1^2\left(1+\rho_0^4P_1^2P_1^{*2}\right)}, \quad (16)$$

$$P_{2y}^2=-\frac{\mathrm{i}k}{4f}-\frac{\mathrm{i}k}{2L}+\frac{G_j}{R^2\cos^2\theta}-\frac{k^2\left(2+2\rho_0^2P_1^2+\rho_0^4P_1^4\right)}{8L^2P_1^2\left(1+\rho_0^4P_1^2P_1^{*2}\right)}. \quad (17)$$

Then, by using the extended Huygens–Fresnel principle once again, the average intensity of the beam propagating through the turbulent atmosphere and arriving at the return place can be given as

$$\begin{aligned}I_3(x_3,y_3,2L+2f)\ =\ &\frac{k^2}{4\pi^2L^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E_2(x,y,L+2f)E_2^*(\varepsilon,\eta,L+2f)A_2(x,y)A_2^*(\varepsilon,\eta)\\&\times\exp\left\{\frac{\mathrm{i}k}{2L}\left[(x_3-x)^2+(y_3-y)^2-(x_3-\varepsilon)^2-(y_3-\eta)^2\right]\right\}\\&\times\exp\left\{-\frac{1}{\rho_0^2}\left[(x-\varepsilon)^2+(y-\eta)^2\right]\right\}\mathrm{d}x\mathrm{d}y\mathrm{d}\varepsilon\mathrm{d}\eta. \quad (18)\end{aligned}$$

After troublesome integration, we obtain the final average intensity as

$$\begin{aligned}I_3(x_3,y_3,2L+2f)\ =\ &\frac{k^6\rho_0^4}{256f^2L^4\left(1+\rho_0^4P_1^2P_1^{*2}\right)}\sum_{j=1}^{J}\sum_{s=1}^{J}\sum_{m=1}^{M}\sum_{n=1}^{M}\frac{F_jF_s^*F_m^*F_n}{P_{2x}P_{2y}P_{3x}P_{3y}P_{4x}P_{4y}P_{5x}P_{5y}}\\&\times\exp\left[-\frac{\left(G_m^*+G_n\right)O_y^2}{R^2\cos^2\theta}\right]\exp\left\{-\frac{k^2}{4L^2}\left[\frac{x_3^2}{P_{4x}^2}+\frac{1}{P_{4y}^2}\left(y_3-\frac{2\mathrm{i}LO_y}{kR^2\cos^2\theta}G_m^*\right)^2\right]\right\}\\&\times\exp\left\{-\frac{k^2}{4L^2\rho_0^2}\left[\frac{\left(1+P_{4x}^2\rho_0\right)^2x_3^2}{P_{4x}^4P_{5x}^2}+\frac{\left(1+P_{4y}^2\rho_0\right)^2}{P_{4y}^4P_{5y}^2}\right.\right.\\&\times\left.\left.\left(y_3-\frac{2\mathrm{i}LO_y}{kR^2\cos^2\theta}\left(\frac{G_m^*-P_4^2\rho_0^2G_n}{1+P_4^2\rho_0^2}\right)\right)^2\right]\right\}, \quad (19)\end{aligned}$$

where

$$\begin{aligned}P_{3x}^2\ =\ &\frac{\mathrm{i}k}{4f}+\frac{\mathrm{i}k}{2L}+\frac{G_s^*}{R^2}\\&-\frac{k^2\left(2+2\rho_0^2P_1^2+\rho_0^4P_1^4\right)}{8L^2P_1^2\left(1+\rho_0^4P_1^2P_1^{*2}\right)}, \quad (20)\end{aligned}$$

$$\begin{aligned}P_{3y}^2\ =\ &\frac{\mathrm{i}k}{4f}+\frac{\mathrm{i}k}{2L}+\frac{G_s^*}{R^2\cos^2\theta}\\&-\frac{k^2\left(2+2\rho_0^2P_1^2+\rho_0^4P_1^4\right)}{8L^2P_1^2\left(1+\rho_0^4P_1^2P_1^{*2}\right)}, \quad (21)\end{aligned}$$

$$P_{4x}^2=\frac{\mathrm{i}k}{2L}+\frac{\mathrm{i}k}{4f}+\frac{k^2}{16f^2P_{3x}^2}+\frac{G_m^*}{R^2}+\frac{1}{\rho_0^2}, \quad (22)$$

$$P_{4y}^2=\frac{\mathrm{i}k}{2L}+\frac{\mathrm{i}k}{4f}+\frac{k^2}{16f^2P_{3y}^2}+\frac{G_m^*}{R^2\cos^2\theta}+\frac{1}{\rho_0^2}, \quad (23)$$

$$\begin{aligned}P_{5x}^2\ =\ &-\frac{\mathrm{i}k}{2L}-\frac{\mathrm{i}k}{4f}+\frac{k^2}{16f^2P_{2x}^2}\\&+\frac{G_n}{R^2}+\frac{1}{\rho_0^2}+\frac{1}{P_{4x}^2\rho_0^4}, \quad (24)\end{aligned}$$

$$\begin{aligned}P_{5y}^2\ =\ &-\frac{\mathrm{i}k}{2L}-\frac{\mathrm{i}k}{4f}+\frac{k^2}{16f^2P_{2y}^2}\\&+\frac{G_n}{R\cos^2\theta^2}+\frac{1}{\rho_0^2}+\frac{1}{P_{4y}^2\rho_0^4}. \quad (25)\end{aligned}$$

Under the condition of $C_n^2=0$, equations (19)–(25) reduce to the formulas for a Gaussian beam propagating through a cat-eye optical lens in free space, which are consistent with the formulas derived in Ref. [2]

4. Numerical calculation and discussion

By using Eq. (19), numerical calculations are performed. Here, $I_0(0,\,0,\,0)=1$, which denotes the maximal intensity at the position of the waist of the Gaussian beam, w_0=1 mm, λ = 1.06 μm, f = 500 mm,

and $R = 50$ mm. Figure 2 displays the relationships between the on-axis average intensity $I_3(0,0,2f + 2L)/I_0(0,0,0)$ in the return place and incidence angle θ for different values of propagation distance L. We can see that the on-axis intensity decreases quickly with the increases in incidence angle and propagation distance. When the incidence angle is larger than 3°, most light will be sheltered by the second hard aperture, namely the out pupil of the cat-eye optical lens. So the following numeration will be carried out under $\theta \leq 3°$ conditions.

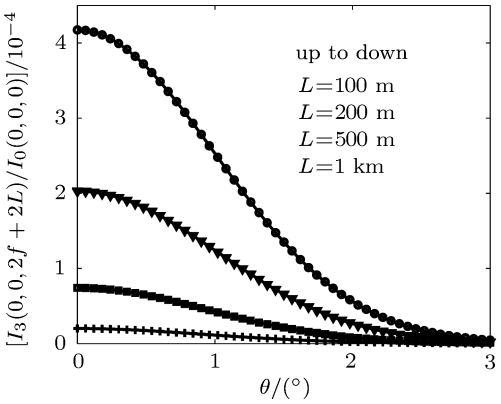


Fig. 2. Relationships between $I_3(0,0,2f+2L)/I_0(0,0,0)$ in the return place and θ for different L values.

In order to illustrate the influence of the incidence angle on the intensity profiles for a Gaussian beam propagating through a cat-eye optical lens in a turbulent atmosphere, the three-dimensional intensity distributions and their contour graphs in the return place by choosing the parameters as $L = 100$ m and $\theta = 0°$, 1°, 2°, 3° are shown in Fig. 3. From Fig. 3(a) we can see that the intensity profile is a Gaussian beam intensity profile when $\theta = 0°$, and the intensity distribution behaves as an approximate circular spot and has a central diffraction peak. In Fig. 3(b), when $\theta = 1°$, the intensity distribution behaves as an approximate elliptical spot, and the only diffraction peak begins to deviate from the centre along the direction of y. From Figs. 3(c) and 3(d), we find that another diffraction peak begins to appear besides the central diffraction peak when θ keeps on increasing, and it gradually deviates from the centre along the direction of the y axis and its peak value gradually increases with respect to the central intensity value. Meanwhile, the central intensity value decreases quickly with the increase in θ. So θ plays an important role in the unsymmetrical distribution pattern of the average intensity. The reason is that the increase in θ results in a lateral displacement of the barycentre of the second aperture, and the diffraction effects of the two apertures disagree in the direction of the y axis, especially when θ is quite large.

Figure 4 shows the three-dimensional intensity distributions and their contours in the return place for Gaussian beam propagating through a cat-eye optical lens in a turbulent atmosphere by choosing the parameters as $\theta = 3°$, $L = 500$ m, 2 km, 5 km, and 15 km. From Fig. 4, we can see that the propagation distance creates a smooth and symmetrical average intensity distribution pattern. With the increase in L, the maximal diffraction peak begins to shift to the centre along the direction of the y axis. The beam spot does almost not spread with the increase in L when L is smaller than 1 km, but spreads more rapidly when L is larger than 2 km. The central irradiance gradually becomes the maximum and the total beam profile becomes a circular solid beam spot, until in the far field when $L = 15$ km, the beam eventually becomes a Gaussian beam under the influence of the turbulent atmosphere. Through numerical calculation, we know that when θ is smaller, the propagation distance, which is necessary for the beam profile to become a Gaussian pattern, is smaller. For example, a distance of 3 km is enough for the beam profile to become a Gaussian pattern when $\theta = 2°$.

In order to evaluate the effect of the structure constant on the intensity profiles for a Gaussian beam propagating through a cat-eye optical lens in a turbulent atmosphere, four different turbulent conditions are considered, including the free space ($C_n^2 = 0$), weak refractive index fluctuation ($C_n^2 = 10^{-14}$ m$^{-2/3}$), medium refractive index fluctuation ($C_n^2 = 10^{-13}$ m$^{-2/3}$), and strong refractive index fluctuation ($C_n^2 = 10^{-12}$ m$^{-2/3}$). Figure 5 shows the three-dimensional intensity distributions and their contours in the return place by choosing the parameters as $\theta = 3°$, $L = 200$ m. From Fig. 5(a), we can see that when θ is a critical angle and the beam propagates in free space, the deviation in the diffraction peak from the centre is much larger than that in a turbulent atmosphere. From Figs. 5(b)–5(d), we can see that the increase in C_n^2 can bring the average intensity peak back to the centre, and can create a smooth and symmetrical average intensity distribution pattern. For example, in Fig. 5(d) under a strong refractive index fluctuation condition, the maximal intensity peak is just at the central position, and the beam profile becomes a Gaussian pattern. The effect of C_n^2 on the intensity profile seems to be similar to that of L, but it is worthwhile noticing that the intensity profiles are especially sensitive to C_n^2 specifically within the range (10^{-13} m$^{-2/3}$ $\leq C_n^2 \leq 10^{-11}$ m$^{-2/3}$), and C_n^2 has no spread effect on the intensity distribution.

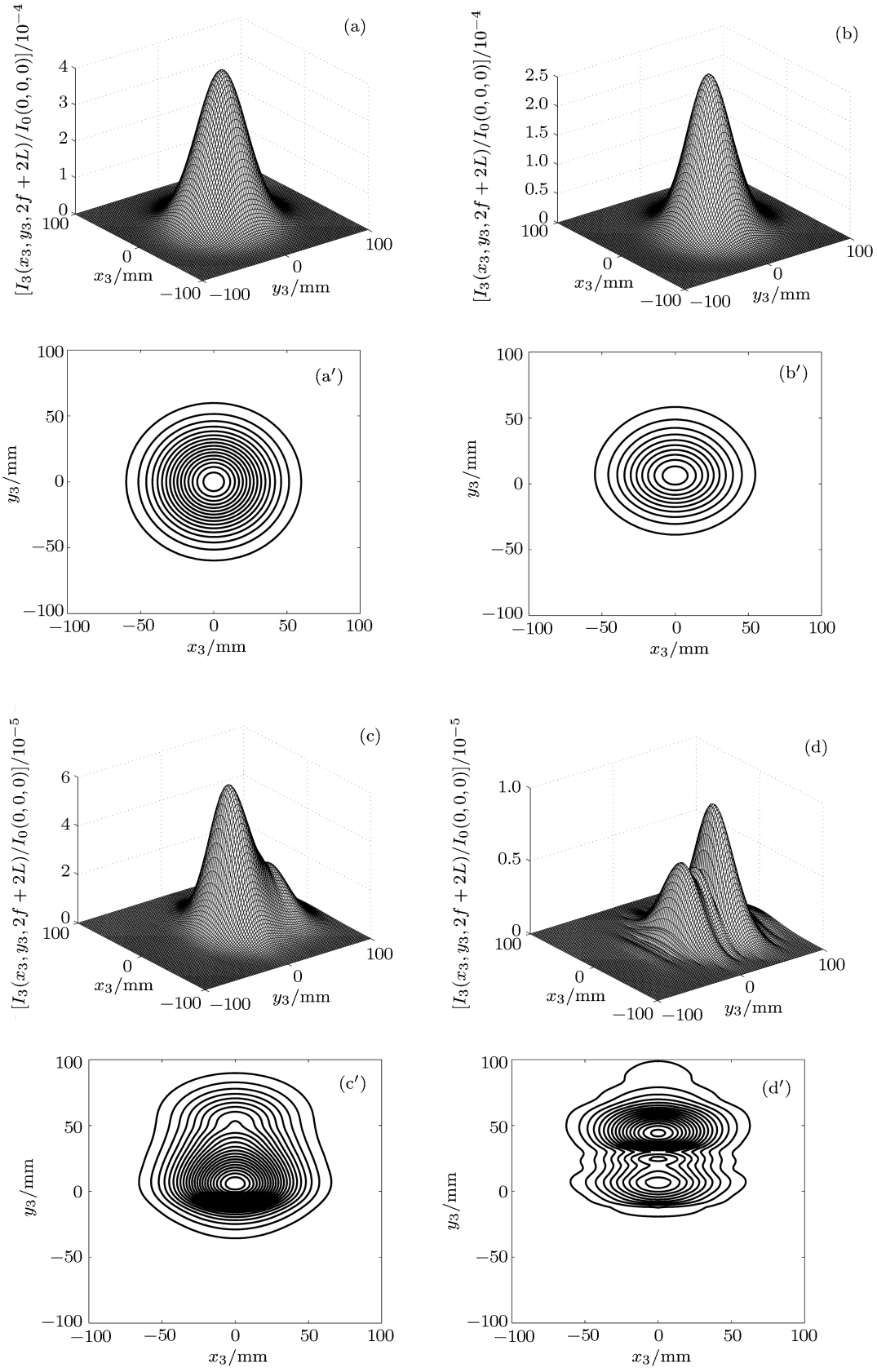


Fig. 3. Three-dimensional intensity distributions and their contours in the return place for a Gaussian beam propagating through a cat-eye optical lens in a turbulent atmosphere by choosing the parameters as $L = 100$ m and $\theta = 0°$ (a), 1° (b), 2° (c), and 3° (d).

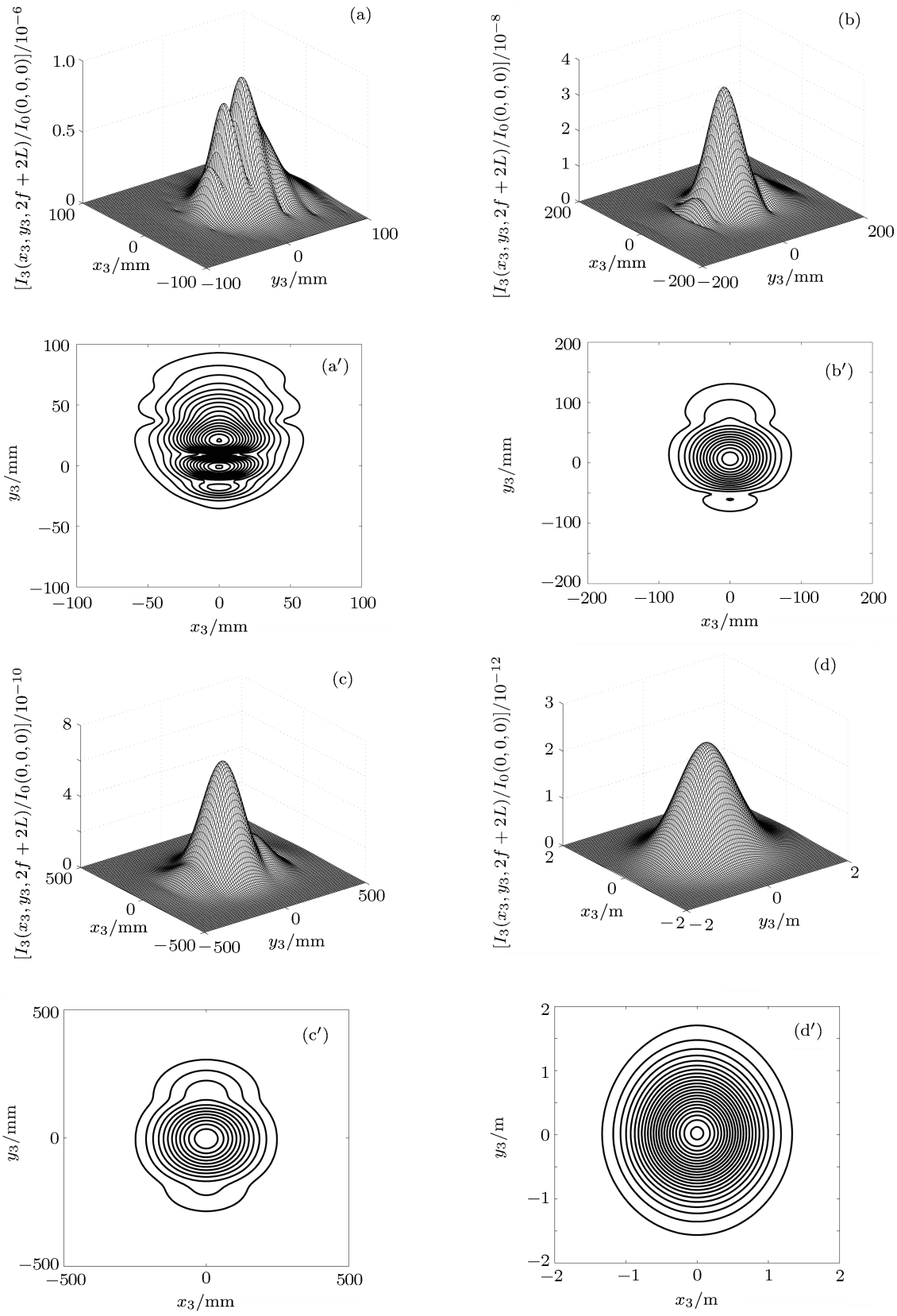


Fig. 4. Three-dimensional intensity distributions and their contours in the return place for a Gaussian beam propagating through a cat-eye optical lens in a turbulent atmosphere by choosing the parameters as $\theta = 3^\circ$ and $L = 500$ m (a), 2 km (b), 5 km (c), 15 km (d).

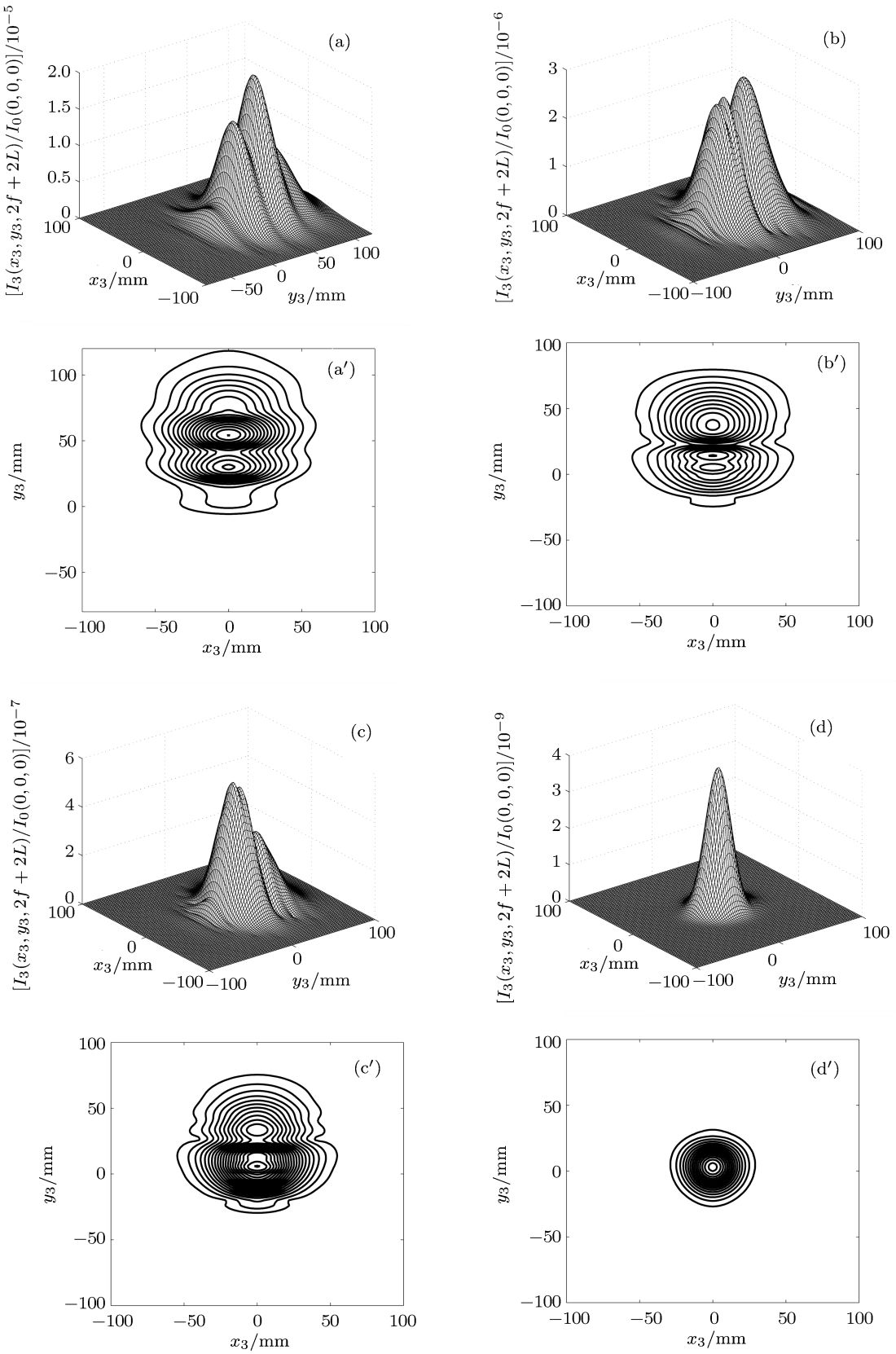


Fig. 5. Three-dimensional intensity distributions and their contours in the return place for a Gaussian beam propagating through a cat-eye optical lens in a turbulent atmosphere by choosing the parameters as $\theta = 3°$, $L = 200$ m, and $C_n^2 = 0$ m$^{-2/3}$ (a), 10^{-14} m$^{-2/3}$ (b), 10^{-13} m$^{-2/3}$ (c), and 10^{-12} m$^{-2/3}$ (d).

5. Conclusion

In conclusion, an approximate analytical formula of the average irradiance for a Gaussian beam passing through a tilted cat-eye optical lens and going back along the entrance way in a turbulent atmosphere based on the extended Huygens–Fresnel integral and the method of expanding the aperture function into a finite sum of complex Gaussian functions has been derived. Effects of the incidence angle, propagation distance, and structure constant on the propagation properties of a Gaussian beam in a turbulent atmosphere have been investigated numerically. We have found that the incidence angle creates an unsymmetrical average intensity distribution pattern, while the propagation distance and structure constant can each create a smooth and symmetrical average intensity distribution pattern. Similar to the results in former studies,[31] the present results show that the longer distance the laser beam propagates through a turbulent medium and the larger the structure constant, the more the laser energy is reduced. But in the application of laser detection system based on cat-eye effect theory, under incline conditions, a nearer propagation distance of a laser beam passing through free space will result in a larger deviation in the intensity peak from the centre and a larger unsymmetrical aberrance of the intensity distribution, then the detection difficulty will increase. However, from our studies, we find that a larger propagation distance and a larger structure constant each can bring the average intensity peak back to the centre more quickly and create a better Gaussian pattern of the intensity distribution. Our analytical formula provides an effective and convenient way to analyse the double-distance propagation of Gaussian beams passing through the cat-eye optical lens and returning back in a turbulent atmosphere. The methods and results are also usable for the double-distance propagations of other laser beams through a double-apertured misaligned optical system in a turbulent atmosphere.

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