An improved cellular automaton model considering the effect of traffic lights and driving behaviour*

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This paper proposes an improved cellular automaton model to describe the urban traffic flow with the consideration of traffic light and driving behaviour effects. Based on the model, the characteristics of the urban traffic flow on a single-lane road are investigated under three different control strategies, i.e., the synchronized, the green wave and the random strategies. The fundamental diagrams and time-space patterns of the traffic flows are provided for these strategies respectively. It finds that the dynamical transition to the congested flow appears when the vehicle density is higher than a critical level. The saturated flow is less dependent on the cycle time and the strategies of the traffic light control, while the critical vehicle density varies with the cycle time and the strategies. Simulated results indicate that the green wave strategy is proven to be the most effective one among the above three control strategies.

Keywords: traffic flow, cellular automaton, control strategy, vehicle density

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1. Introduction

Recently, much attention has been paid to the urban traffic situation and the relevant adverse effect on surrounding environment, especially the traffic flow at road junction controlled by traffic lights. A variety of approaches have been developed to describe the collective behaviour of traffic flow. Among these, the cellular automaton (CA) models have been extensively used to investigate traffic flow in many aspects, which provide an effective technique for exploring a large-scale traffic system. Nagel and Schreckenberg[1] proposed a CA model, i.e., the NaSch model, to mimic single-lane highway traffic. Biham *et al.*[2] presented a simple CA model (BML model) for city traffic, in which vehicles only hop from an intersection to another adjacent one. Chowdhury and Schadschneider[3] reported an extended BML model incorporating with the NaSch model, i.e., the ChSch model, in which a road section is inserted between two successive intersections. The ChSch model gives a more detailed description of city traffic flows. Based on the ChSch model, Brockfeld *et al.*[4] have investigated the optimization of traffic lights. They pointed out that the flow is improved by traffic light control strategies, the derivation of the optimal cycle time in the homogeneous network can be reduced to the case of a single street. Sasaki and Nagatani[5] used an optimal velocity model to study the traffic flow with three control strategies. They found that the traffic saturates in a certain range of vehicle density and the value of the saturated flow is independent of the cycle time and the control strategies. However, the traffic lights in previous studies[6−12] are considered to affect only the leading vehicle in front of intersections. The following vehicles will not be directly influenced by the traffic light until it becomes the leading one to the intersection. The influence of anticipation from traffic lights is seldom considered in most traffic flow studies at the moment.

In this study, we investigate the effect of three traffic light strategies on a single-lane traffic flow by a CA model. Unlike the routine description on the evolution of traffic flow controlled by traffic lights in previous models,[13−16] the anticipation effect is taken into account in our model, which reflects drivers' be-

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haviour facing the change of traffic lights in detail.

In our model, an anticipation model is proposed to describe the urban traffic with the consideration of traffic lights effects and driving behaviour. The simulation is carried out and the results are analysed by comparing the three control strategies, i.e., the synchronized, green wave and random strategies. The relevant qualitative analyses for the saturated flow and critical vehicle density are also preceded and concluded.

2. The model

In common sense, a driver will accelerate when his/her vehicle is far from an intersection and decelerate when his/her vehicle gets close to it during the red light period. From this aspect, the most important factor is his/her particular behaviour as traffic lights turn to switch. The model presented here is to describe the drivers' behaviour based on their anticipation to the change of traffic lights.

Suppose that N vehicles move in one direction along a lane with L cells, which is divided into M sections by intersections. Under a periodic boundary condition, the number of vehicles is conserved for a certain average vehicle density. Each cell is either empty or occupied by just one vehicle moving at the velocity v ranging from 0 to $v_{\max}$. For simplicity, the yellow light periods are ignored (see Fig. 1) and the green light period $T_{\rm G}$ is assumed equal to the red light period $T_{\rm R}$. The position, velocity and headway (distance to its former) of the n-th vehicle at the discrete time t are denoted by x_n, v_n and d_n respectively. In addition, we define D_n as its distance to the intersection in front and τ as the remained time that the traffic light tends to switch. Based on the above considerations, the following updating rule is designed for each vehicle in parallel.[17]

Step 0 Anticipation of traffic lights and determination of the acceleration a_n of the n-th vehicle.

Case 1 When the traffic light is red in front of the n-th vehicle:

$$a_n = \begin{cases} 1, & D_n > v_{\max} \times \tau, \quad \text{with probability } p_1, \\ -1, & v \times \tau \le D_n \le v_{\max} \times \tau, \quad \text{with probability } p_2, \\ -1, & D_n < v \times \tau, \quad \text{with probability } p_1. \end{cases} \tag{1}$$

Case 2 When the traffic light is green in front of the n-th vehicle:

$$a_n = \begin{cases} 1, & D_n > v_{\max} \times \tau, \quad \text{with probability } p_1, \\ 1, & v \times \tau \le D_n \le v_{\max} \times \tau, \quad \text{with probability } p_2, \\ 1, & D_n < v \times \tau, \quad \text{with probability } p_1. \end{cases} \tag{2}$$

For simplicity, we set $p_2 = 1 - p_1$.

Step 1 $v_n \to v_n + a_n$.

Step 2 $v_n \to \max(1, \min(v_n, v_{\max}))$.

This step indicates that a driver intends to move forward faster, if possible.

Step 3 $v_n \to \min(v, d_n^{(\rm eff)})$.

Here $d_n^{(\rm eff)}$ is the effective headway. It is determined by setting $d_n^{(\rm eff)} = \min(d_n, D_n)$ during the red light period while by $d_n^{(\rm eff)} = d_n$ during the green light period.

After the above velocity update, the vehicle moves forward with its actual velocity.

Step 4 $x_n \to x_n + v_n$.

This model is quite different from the NaSch model, in which the influence of traffic lights on vehicles is not considered and the ChSch model, in which the traffic lights only affect the leading vehicle and have no direct influence on others, while we consider the influence of varying traffic lights on all vehicles in the proposed model.

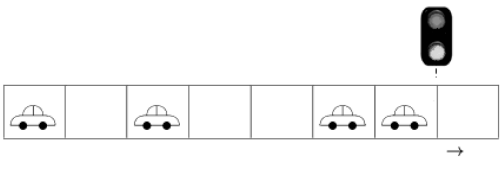


Fig. 1. Sketch of model.

3. Simulation results

The characteristics of urban traffic flows under three control strategies, i.e., the synchronized, green wave and random strategies, are investigated by using the proposed model and presented here. The related parameters are given as: a) the total length of the lane ($L = 200$) is equally divided (for simplicity, assuming

$M = 2$ here) so that the cell of length is $D = 100$; b) The traffic lights without occupying any cells are located at the 100-th and 200-th cells respectively; c) suppose that the maximum velocity $v_{\max} = 4$, then we obtain a characteristic time scale for $T_{\rm G} \sim 25$ s; d) the probability p_1 is set to be $p_1 = 0.9$. The flow q is obtained by counting the number of vehicles passing through every intersection.

Each run of simulation takes 7.2×10^3 time steps, which represents a real time period of 2 hours. Results produced during the first 3.6×10^3 time steps are discarded in order to eliminate the transient effects. The total runs of simulation are 20. The final results are averaged over all 20 runs of simulation. The effects of above three traffic light strategies can be obtained and analysed below. All simulations are carried out on a Matlab platform by a self-written programme.

3.1. Strategy of synchronized traffic lights

In the synchronized traffic light strategy, all traffic lights switch from red (green) to green (red) simultaneously. The time period for switch between red and green lights is defined as the cycle time. Figure 2 presents the profiles of the traffic flow against vehicle density at different values of period T obtained from the model. It is found that there exists a plateau with the saturated flow ($q \approx 0.35$) within a certain range of vehicle density. The values of saturated flow are independent of the cycle time but the critical vehicle density changes with the cycle time. Besides, the results from the NaSch model with the consideration of traffic lights are also presented here. It is obvious that the model with anticipation effect generates higher traffic flow than the NaSch model directly combined with traffic lights, especially at high vehicle densities (> 0.6).

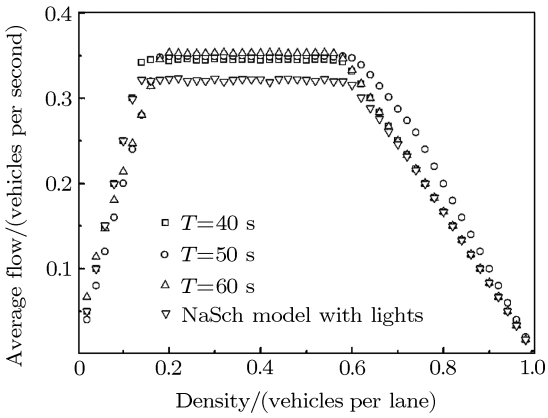


Fig. 2. Traffic flow against vehicle density in synchronized strategy for cycle times $T = 40$, 50 and 60 s.

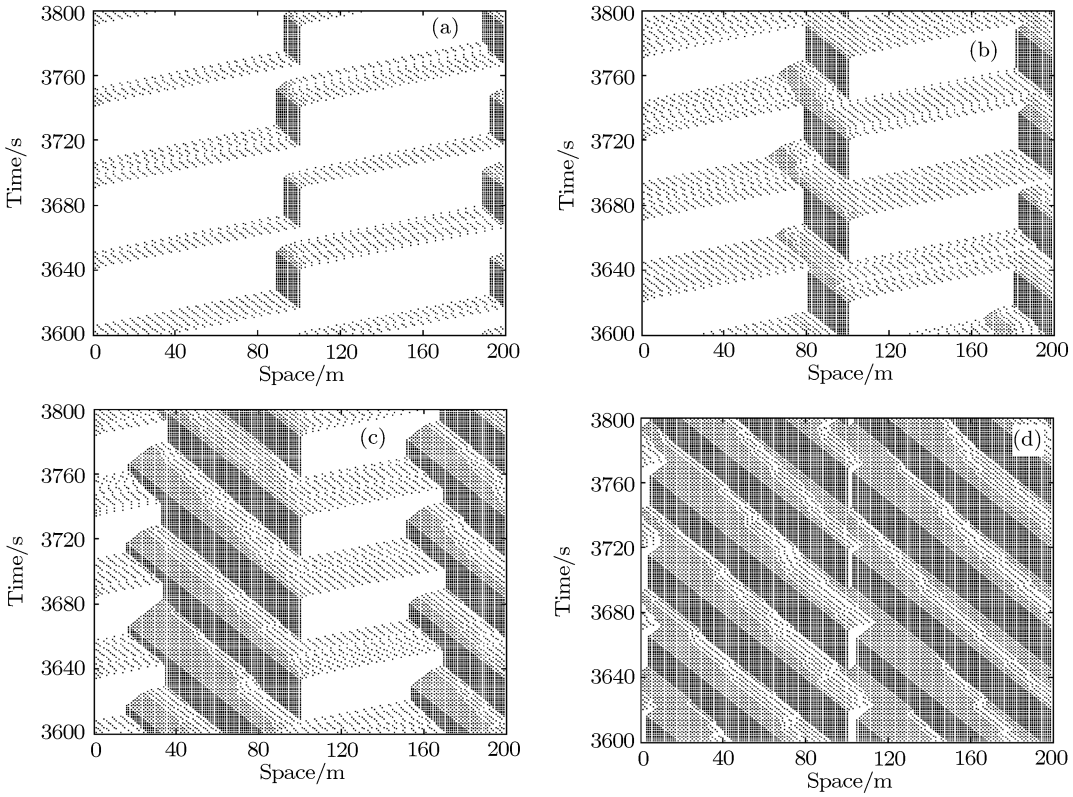


Fig. 3. Time–space evolution patterns of vehicles for cycle $T = 50$ under synchronized strategy (a) $\rho = 0.1$, (b) $\rho = 0.2$, (c) $\rho = 0.4$ and (d) $\rho = 0.65$.

Moreover, the time-space evolution patterns of traffic flow induced by the traffic lights at different vehicle densities are also investigated. Figure 3 describes the time-space evolution patterns of vehicle at the cycle time $T = 50$ under the synchronized strategy. The time-space evolutions (a) and (b) are obtained for vehicle density with $\rho = 0.1$ and $\rho = 0.2$ respectively. Vehicles move freely during the green light period and do not stop until the traffic light switches to red. Due to the low vehicle density, the traffic lights become the main hindrance along the journey. The queue starts to accumulate when the traffic light switches to red and dissolve completely in the green periods. The time-space evolution (c) corresponds to vehicle density of $\rho = 0.4$ and the flow is saturated at this level. As soon as the traffic light switches to green, the vehicle closest to the traffic light restarts. The density wave propagates backward from the traffic light. The time-space evolution (d) is obtained for $\rho = 0.65$. Due to the long length in high density, the queue cannot be dissolved even in green periods. Then the queue accumulates in the later periods and extends backwards.

Besides, the effect of the number of traffic lights on the traffic flow under the synchronized strategy is inspected as well. Due to the periodic boundary conditions, the traffic flow is independent of the number of traffic lights and can be reduced to the simple one considered here under the synchronized strategy.

3.2. Strategy of green-wave traffic lights

In the green wave traffic lights strategy, the lights switch with a certain time delay $T_{\rm delay}$ between two successive lights. The purpose of using green wave strategy is to let the vehicle move uninterruptedly. The first important thing to note is how long the delay time is the best in the green wave strategy. As soon as a vehicle comes to halt, it contributes to the total delay once. The total delay finally is the sum of every standing vehicle in an hour. Figure 4(a) presents the total delay via the vehicle density with different delay times. It is obvious that the curve of $T_{\rm delay} = 25 = T/2$ produces the least total delay at lower vehicle density (i.e., $\rho < 0.16$) compared with other two delay times (i.e., $T = 5$ and $T = 15$). It is proved that $T_{\rm delay} = D/V_{\max} = T/2$ is the best value in this case. Figure 4(b) shows the profile of average traffic flow via vehicle density with different delay times under the green-wave strategy. It is found that the delay time $T_{\rm delay} = 25$ leads to a higher flow than other strategies at lower vehicle density and this parameter will be used in later study.

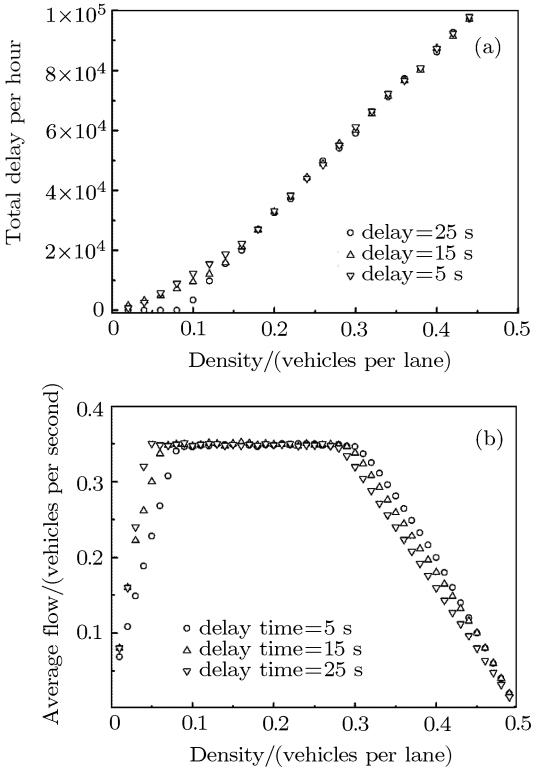


Fig. 4. Total delays (a) and plot of the flow (b) against density at average road section under the green wave strategy with delay time $T_{\rm delay} = 25$, 10, 5.

As it can be seen in Fig. 5, the variations of the traffic flow against vehicle density at different cycle times T can be inspected and assessed by the green wave strategy. The situation is similar to that of the synchronized one. The traffic flow increases linearly with the vehicle density first. When the density is higher than the critical one (i.e., $\rho > 0.1$), the flow achieves saturation at a constant value.

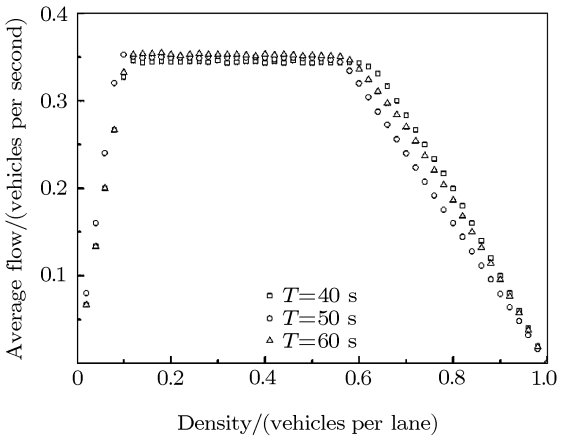


Fig. 5. Curves of the flow against density in green wave strategy for cycle times $T = 40$, 50, 60 s.

When the vehicle density increases further and is higher than the second critical one (i.e., $\rho > 0.57$),

the flow decreases as the density increases. The saturated flow again is independent of the cycle time but the critical vehicle density changes with it. Comparing with Fig. 2 in synchronized strategy (i.e., $\rho > 0.16$ and $\rho > 0.6$), we find that the critical densities are lower than that in green wave strategy, which indicates that the green wave strategy is more effective than the synchronized one.

Figure 6 depicts the time-space evolution of vehicle at cycle time $T = 50$ and delay time $T_{\rm delay} = T/2 = 25$ s under the green wave strategy. The time-space evolutions in Figs. 6(a) and 6(b) correspond to the vehicle densities $\rho = 0.1$ and $\rho = 0.2$ respectively. Under the green wave strategy, vehicles move successfully over the traffic lights without stopping. Comparing with the synchronized strategy, we find that it allows the most vehicles keep moving and the least traffic jam. However, when the vehicle density increases, traffic jams become inevitable no matter how to adjust the traffic lights. The time-space evolution patterns for the densities at $\rho = 0.4$ and $\rho = 0.65$ in green wave strategy are similar to the corresponding situations in the synchronized one.

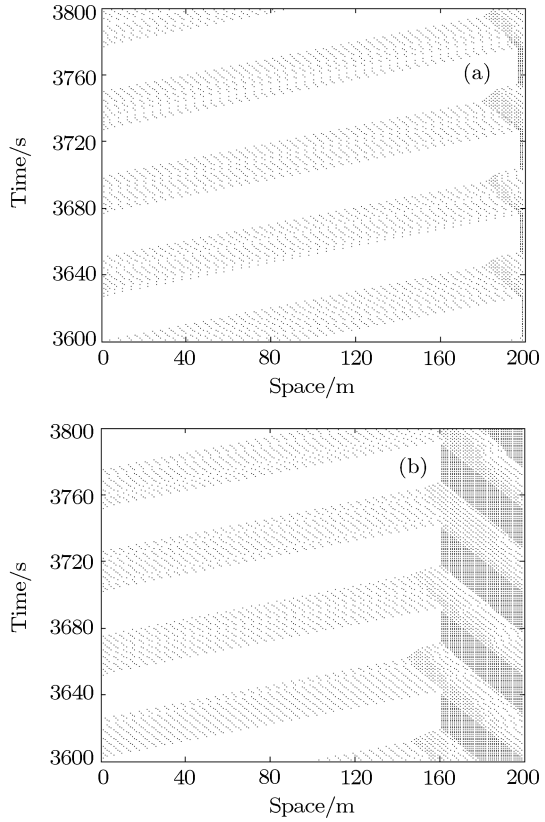


Fig. 6. Time–space evolution patterns of vehicles for cycle $T = 50$ and delay time $T_{\rm delay} = 25$ s in green wave strategy (a) $\rho = 0.1$ and (b) $\rho = 0.2$.

Furthermore, we study the effect of the number of traffic lights on the traffic flow which is shown in Fig. 7. It shows the plot of the flow and velocity against density for cycle time $T = 50$ with delay time $T_{\rm delay} = 25$ in different numbers of traffic lights. Due to the periodic boundary condition, there is no difference between the traffic with two lights and four lights. However, the average flow and average velocity vary with different traffic lights at low density.

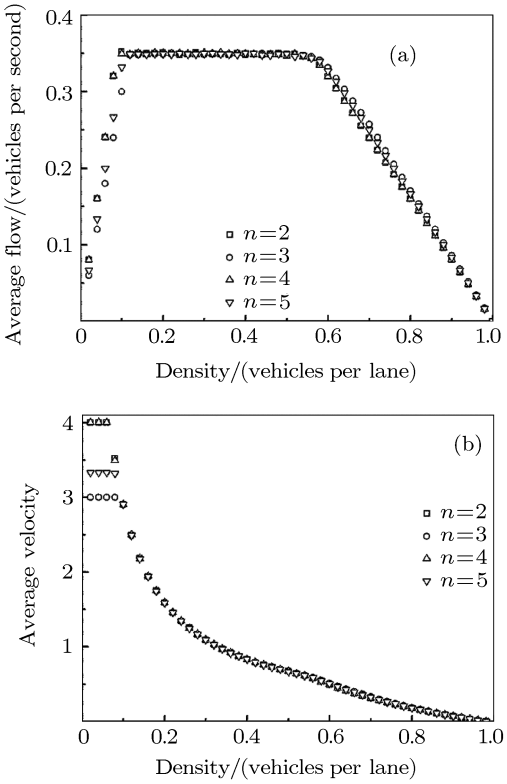


Fig. 7. Plot of flow (a) and velocity (b) against density under the green wave strategy for different traffic lights $n = 2, 3, 4, 5$.

3.3. Strategy of random switching traffic lights

In the strategy of random switching traffic lights, all traffic lights change independently within an allowed range. The efficiency of the random switching strategy lies between the synchronized and the green-wave strategies. The characteristics of urban traffic flow under random strategy are analogous to the situations in the above two strategies.

Furthermore, we studied the velocity distribution in the three different strategies (shown in Fig. 8). At lower vehicle density, most vehicles move with the maximum velocity in green-wave strategy. The green-wave strategy is proven to be the best one among the three strategies, which can allow a cluster of vehicles passing through the intersections without interruption and further reduce the traffic pollution.[18] Therefore, we suggest that the green wave strategy should be ap-

plied in real urban traffic controls.

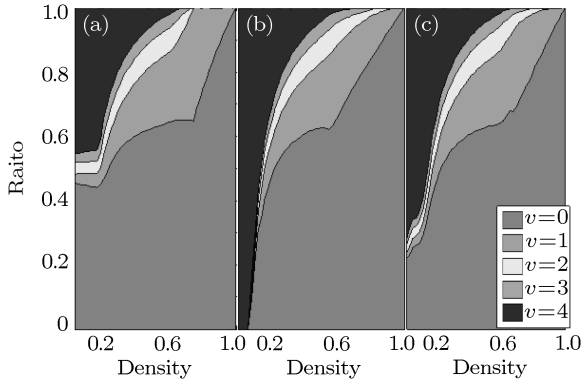


Fig. 8. Velocity distributions against density under three different strategies: (a) synchronized, (b) green wave and (c) random.

4. Qualitative analysis

As we know, the flow of the system closely depends on the vehicle density, the traffic light period and the maximum velocity. To obtain a better understanding of the present model, it is necessary to carry on a qualitative analysis detailed below.

It is assumed that, during the red phase, the compact queues of length N are formed in front of each traffic light. The aim is to estimate the time period t during which the queue completely leave an intersection. There are two different contributions to t.

First, the last vehicle in the compact queue of N vehicles starts moving after $t_1 = N/(1-p)$ time steps,[3] since the leading vehicle in the remainder of the queue moves with the probability $1-p$.

Second, the last vehicle completely passes the intersection after $t_2 = N/v$ time steps, during which the average velocity is specified as

$$\bar{v} = (v_{\max} - 1) \times p + v_{\max} \times (1-p) = v_{\max} - p. \quad (3)$$

Thus, the queue of N vehicles completely leaves the intersection after time period at least

$$t = t_1 + t_2 = \frac{N}{1-p} + \frac{N}{v_{\max} - p}.$$

Suppose that the queue of N vehicle completely leaves the intersection just in the green periods, i.e.

$$t = t_1 + t_2 = \frac{N}{1-p} + \frac{N}{v_{\max} - p} \approx T_{\mathrm{G}}.$$

According to the definition of flow rate, the maximum flow rate is calculated as follows:

$$q_{\max} = \frac{N}{T} = \frac{1}{2}\frac{N}{T_{\mathrm{G}}} \approx \frac{1}{2} \times \left(\frac{1}{1-p} + \frac{1}{v_{\max} - p}\right)^{-1}$$
$$= \frac{1}{2} \times \frac{(1-p)(v_{\max} - p)}{v_{\max} - 2 \times p + 1}. \quad (4)$$

Furthermore, the qualitative analyses of critical densities are also conducted. There exists a relation at low vehicle density as follows:

$$L \approx N + N \times v_{\max}. \quad (5)$$

Hence, we can obtain the first critical vehicle density in terms of the definition of vehicle density:

$$\rho_1 = \frac{N}{L} \approx \frac{1}{v_{\max} + 1}. \quad (6)$$

Concerning the periodic boundary condition, the flow is determined by the movement of the whole queue at high vehicle density.[19] The flow is given by $q = (1-p) \times (1-\rho)$ which corresponds to that given by the NaSch model with the randomization probability p. Then the second vehicle critical density can be calculated by $\rho_2 = 1 - q/(1-p)$.

According to the parameter used in the study, the saturated vehicle flow is calculated as $q_{\max} = 0.3655$ while the critical vehicle densities are obtained as $\rho_1 = 0.2$ and $\rho_2 = 0.56$ respectively. The results from qualitative analyses are close to those from the simulations (see Fig. 2), which indicate the rationality of the anticipation model proposed here.

Furthermore, we also present the qualitative analysis on the existence of the green wave phenomena.[20] As can be seen in Fig. 9, there appears a queue of length d stopped at the intersection C. We assume that the system is under the synchronized strategy and the results can be extended to the other strategies. The vehicles at the positions A and C begin to move synchronously when the traffic lights just switch from red to green. A starting wave, which moves towards the end of the queue with velocity v_1, reaches the end of queue after the time period $t_1 = d/v_1$; while the vehicle at the position A arrives at the end of the queue after the time period $t_2 = (L-d)/v_2$.

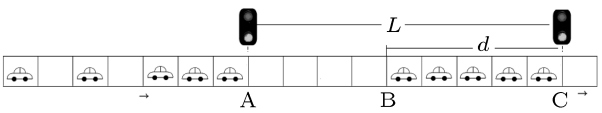


Fig. 9. Sketch of the model with queue.

If $t_1 > t_2$, it means that the starting wave has not arrived at the end of the queue when the vehicle at the position A reaches there. That vehicle has to stop at the position B and restarts when the starting wave approaches. From this, we can obtain

$$d > \frac{v_1 L}{v_1 + v_2}$$

and define

$$\mu = \frac{d}{L} > \frac{v_1}{v_1 + v_2}$$

as a criterion of the existence of the green wave. The vehicle cannot move smoothly to the next road section if μ satisfies the above condition. In other words, if the queue in successive road sections satisfies the above condition, the system would not possess any green wave phenomenon no matter how the strategy is adjusted. The adjustment of strategy is only effective in a low-density regime and is consistent with the simulation results presented in Section 3.

5. Conclusions

The traffic flow controlled by traffic lights on a single-lane roadway is studied via an anticipation model and reported in this paper. The strategies of synchronized, green wave and random switching lights have been simulated and compared with each other. The variations of the traffic flow against the vehicle density and the space–time evolution are analysed under these three different strategies. It is found that the saturation of traffic flow exists in a density range. The critical density depends on the cycle time of traffic lights and the control strategy. The green-wave strategy is proved to be the best one in the above three strategies, which can allow a cluster of vehicles across the intersections smoothly. In order to generate a green wave effect, it is suggested that the adjacent traffic lights should possess appropriate delays in urban traffic. The related qualitative analyses of the saturated flow and the critical vehicle density are also performed and the results agree with the simulation ones.

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