Pinning impulsive synchronization of stochastic delayed coupled networks*

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In this paper, the pinning synchronization problem of stochastic delayed complex network (SDCN) is investigated by using a novel hybrid pinning controller. The proposed hybrid pinning controller is composed of adaptive controller and impulsive controller, where the two controllers are both added to a fraction of nodes in the network. Using the Lyapunov stability theory and the novel hybrid pinning controller, some sufficient conditions are derived for the exponential synchronization of such dynamical networks in mean square. Two numerical simulation examples are provided to verify the effectiveness of the proposed approach. The simulation results show that the proposed control scheme has a fast convergence rate compared with the conventional adaptive pinning method.

Keywords: time-varying delay, pinning impulsive synchronization, stochastic complex network, exponential synchronization

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1. Introduction

Synchronization is an important property of complex network, which can be seen in natural, social, physical and biological fields and has been found to have applications in many cases in the real world.[1−32] Recently, synchronization of stochastic delayed complex network has aroused much interest since time-delays and stochastic disturbances in complex network need to be taken into account for modeling a realistic network.[6,12,14,15,17,18,22,23]

Recently, different synchronization schemes have been proposed for achieving the synchronization of complex dynamical networks, see Refs. [6]–[10], [13], [16]–[24] for example. Among the existing publications, some papers focused on pinning control on complex dynamical networks by the idea of linear feedback control or adaptive control based on intuitive knowledge of network topologies, here the controlled synchronization state is a special solution of the homogenous system.[13,16−22,24]

On the other hand, impulsive control strategies have been widely used to synchronize coupled chaotic dynamical systems due to their potential advantages over general continuous control schemes. What is more, the impulsive controller usually has a relatively simple structure. In this sense, impulsive synchronization strategy is very useful in practical application. It is worthwhile pointing out that in most recent results appearing in the literature dealing with the stabilization or synchronization of complex network by impulsive control only the networks described by impulsive delayed differential equations (IDDE) are considered. However, due to the difficulty in investigating the stability of stochastic impulsive delayed differential equations (SIDDE), there are only a few publications dealing with stabilization or synchronization problems of stochastic impulsive delayed complex networks with coupling delays. Moreover, in comparison with the pinning adaptive approach,[17] the proposed hybrid control scheme including impulsive control and adaptive control has faster convergence rate

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to achieve the synchronization than the conventional pinning adaptive method.

Motivated by the above discussion, we investigate the pinning exponential synchronization of stochastic delayed complex networks (SDCNs) with coupling delays in mean square in the present paper. Via a Lyapunov functional and some analysis techniques, some new sufficient conditions for pinning synchronization are established for the addressed networks. In the end, the developed approach is applied to a scale-free network composed of delayed neural networks, and the corresponding simulation results are given to show the effectiveness of the proposed results.

2. Preliminaries

Notation Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all real matrices. The superscript 'T' denotes matrix transposition and the notation $X \geq Y$ ($X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (positive-definite). In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry. $\mathbb{Z}^+$ denotes the set including positive integers. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ denote the expectation of x and the expectation of x conditional on y. $\|\cdot\|$ represents the Euclidean vector norm in $\mathbb{R}^n$. The Kronecker product of matrices $Q \in \mathbb{R}^{m\times n}$ and $R \in \mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ and denoted as $Q\otimes R$. A^s denotes the matrix of $(A + A^{\mathrm{T}})/2$. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ represent the maximum and minimum eigenvalues of a matrix. The matrix G_l denotes the minor matrix of a matrix G by removing all the i_k-th $(1 \leq i_k \leq N, 1 \leq k \leq l, 1 \leq l \leq N)$ row-column pairs of G. I_N is the identity matrix with dimension N.

First, we consider an SDCN with coupling time-varying delay consisting of N coupled identical nodes, where each node of the network is an n-dimensional dynamical system with time-delay, given as

$$\begin{aligned}\mathrm{d}x_i(t) = &\Big[Cx_i(t) + A\tilde{f}(x_i(t)) + B\tilde{g}(x_i(t-\tau(t))) + u_i\\ &+ c\sum_{j=1}^{N} r_{ij}\Gamma x_j(t) + q\sum_{j=1}^{N} h_{ij}\Gamma x_j(t-d(t))\Big]\mathrm{d}t\\ &+\kappa(t,e_i(t))\mathrm{d}w(t), \quad i = 1,2,\ldots,N, \end{aligned} \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^{\mathrm{T}} \in \mathbb{R}^n$ ($i = 1, 2\ldots, N$) is the state vector of the i-th node; $u_i = [u_1, u_2, \ldots, u_N]^{\mathrm{T}}$ is the hybrid control input; $C = \mathrm{diag}(c_1, c_2, \ldots, c_n)$ is a known matrix; $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ represent the connection weight matrix and the discrete time-varying connection weight matrix, respectively; $\tau(t)$ denotes the time-varying delay and $d(t)$ refers to the coupling time-varying delay; $w(t)$ is one-dimensional Brownian motion; $\tilde{f}$ and $\tilde{g}$ are the nonlinear functions satisfying $\tilde{f}(x_i(t)) = (\tilde{f}_1(x_{i1}(t)), \ldots, \tilde{f}_n(x_{in}(t)))^{\mathrm{T}} \in \mathbb{R}^n$ and $\tilde{g}(x_i(t-\tau(t))) = (\tilde{g}_1(x_{i1}(t-\tau(t))), \ldots, \tilde{g}_n(x_{in}(t-\tau(t))))^{\mathrm{T}} \in \mathbb{R}^n$, respectively; c and q are the positive coupling strengths of the network; Matrix Γ describes inner coupling of the complex network which satisfies $\gamma_{ii} > \frac{1}{2}\sum_{j=1,j\neq i}^{n}(|\gamma_{ij}| + |\gamma_{ji}|)$; Matrices $R = (r_{ij})_{N\times N}$ and $H = (h_{ij})_{N\times N}$ are the coupling configuration matrices representing respectively the coupling strength and the topological structure of the network which satisfies

$$\begin{aligned} &r_{ii} = -\sum_{j=1,j\neq i}^{N} r_{ij},\\ &r_{ij} \geq 0 \quad (i \neq j), i, j = 1, 2, \ldots, N,\\ &h_{ii} = -\sum_{j=1,j\neq i}^{N} h_{ij},\\ &h_{ij} \geq 0 \quad (i \neq j), i, j = 1, 2, \ldots, N. \end{aligned} \tag{2}$$

Remark 1 Complex neural network model (1) is quite general, which contains the coupling time-varying delays and stochastic disturbances. It should be pointed out that the pinning synchronization problem of stochastic complex network with or without coupling delays has not received much attention in the literature. On the other hand, the coupling matrix and inner coupling matrix are not assumed to be symmetric in this paper.

The initial conditions associated with system (1) are given as follows:

$$x_i(s) = \xi_i(s), \quad -\tau^* \leq s \leq 0, \quad i = 1, 2\ldots, N, \tag{3}$$

where for an arbitrary $\xi_i \in L^2_{\mathscr{F}_0}([-\tau^*, 0], \mathbb{R}^n), \tau^* = \sup_{t\in\mathbb{R}} \in \{\tau(t), d(t)\}$, where $L^2_{\mathscr{F}_0}([-\tau^*, 0], \mathbb{R}^n)$ is the family of all $\mathscr{F}_0$-measurable $\mathscr{C}([-\tau^*, 0]; \mathbb{R}^n)$-valued random variables satisfying that

$$\sup_{-\tau^*\leq\varphi\leq 0}\mathbb{E}|\xi_i(\varphi)|^2 < \infty,$$

and $\mathscr{C}([-\tau^*, 0]; \mathbb{R}^n)$ denotes the family of all continuous $\mathbb{R}^n$-valued functions $\xi_i(\varphi)$ in $[-\tau^*, 0]$.

$s(t) \in \mathbb{R}^n$ is a solution of an isolated node

$$\mathrm{d}s(t) = [Cs(t) + A\tilde{f}(s(t)) + B\tilde{g}(s(t-\tau(t)))]\mathrm{d}t, \tag{4}$$

with $x_0 \in \mathbb{R}^n$.

Here, let $e_i(t) = x_i(t) - s(t)$ be the error state. It follows from Eqs. (1) and (4) and condition (2) that

$$\mathrm{d}e_i(t) = \Big[Ce_i(t) + Af(e_i(t)) + Bg(e_i(t-\tau(t))) + u_i$$

$$+c\sum_{j=1}^{N} r_{ij}\Gamma e_j(t)+q\sum_{j=1}^{N} h_{ij}\Gamma e_j(t-d(t))\Big]\mathrm{d}t$$
$$+\kappa(t,e_i(t))\mathrm{d}w(t),\quad i=1,2,\ldots,N, \tag{5}$$

where $f(e_i(t)) = \tilde{f}(x_i(t)) - \tilde{f}(s(t))$ and $g(e_i(t)) = \tilde{g}(x_i(t)) - \tilde{g}(s(t))$. Supposing that $\tilde{f}_i(0) = \tilde{g}_i(0) = 0$, it is easy to verify that $f_i(0) = g_i(0) = 0$. The activation functions $\tilde{f}_i(x)$ and $\tilde{g}_i(x)$ satisfy the Lipschitz condition, i.e., for all $i = 1, 2, \ldots, n$, there exist constants $\lambda_i > 0$ and $\phi_i > 0$ such that $|\tilde{f}_i(x) - \tilde{f}_i(y)| \leq \lambda_i|x-y|$ and $|\tilde{g}_i(x) - \tilde{g}_i(y)| \leq \phi_i|x-y|, \forall x, y \in \mathbb{R}^n$. Hence, error system (5) admits a trivial solution $e_i(0) \equiv 0, i = 1, 2, \ldots, N$. $\tau(t)$ and $d(t)$ are bounded and continuously differentiable functions, $0 \leq \tau(t) \leq \tau$, $0 \leq d(t) \leq d$, $\dot{\tau}(t) \leq \mu < 1$ and $\dot{d}(t) \leq \varpi < 1$. We assume that $\kappa(t, e_i(t)) \leq e_i^{\mathrm{T}}(t)\Sigma^{\mathrm{T}}\Sigma e_i(t)$. For complex network (5), a hybrid controller consisting of adaptive feedback controller, pinning controller and impulsive controller is designed for achieving synchronization. The hybrid controller is designed as follows:

$$u_i = u_{1i} + u_{2i}. \tag{6}$$

The u_{1i} is an impulsive controller which is designed as follows:

$$u_{1i} = \sum_{k=1}^{\infty} J_k e_i(t)\delta(t-t_k),\quad i=1,2,\ldots,l,$$
$$u_{1i} = 0,\quad i=l+1,l+2,\ldots,N, t=t_k, \tag{7}$$

where J_k is a control gain and $\delta(t-t_k)$ is the well-known Dirac impulse function with discontinuity points $t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$, with $t > t_0$.

The controller u_{2i} is a pinning adaptive controller described by

$$u_{2i} = -d_i(t)e_i(t),\quad i=1,2,\ldots,l, t\neq t_k,$$
$$u_{2i} = 0,\quad i=l+1,l+2,\ldots,N, t\neq t_k, \tag{8}$$

where $d_i(t)$ is the adaptive feedback gain updated by

$$\dot{d}_i(t) = \beta_i e_i^{\mathrm{T}}(t)e_i(t)\mathrm{e}^{\epsilon t},\quad i=1,2,\ldots,l, \tag{9}$$

where $\epsilon > 0$ is a positive constant. $\beta_i > 0$ $(i = 1, 2, \ldots, l)$ is arbitrary constant which can tune the convergence speed. Thus, we have the following hybrid controlled SDCNs:

$$\begin{aligned}\mathrm{d}e_i(t) = &\Big[Ce_i(t)+Af(e_i(t))\\ &+Bg(e_i(t-\tau(t)))+u_{2i}\\ &+c\sum_{j=1}^{N} r_{ij}\Gamma e_j(t)\\ &+q\sum_{j=1}^{N} h_{ij}\Gamma e_j(t-d(t))\Big]\mathrm{d}t\\ &+\kappa(t,e_i(t))\mathrm{d}w(t),\quad t\neq t_k,\\ \Delta e_i(t_k) = &\ e_i(t_k^+)-e_i(t_k^-) = J_k e_i(t_k^-),\quad t=t_k,\\ &\ k\in\mathbb{Z}^+,\quad i=1,2,\ldots,l,\\ \Delta e_i(t_k) = &\ e_i(t_k^+)-e_i(t_k^-)=0,\quad t=t_k,\\ &\ k\in\mathbb{Z}^+,\quad i=l+1,\ldots,N,\end{aligned} \tag{10}$$

where $e_i(t_k^+) = \lim_{t\to t_k^+} e_i(t)$ and $e_i(t_k^-) = \lim_{t\to t_k^-} e_i(t)$. Without loss of generality, we assume that $\lim_{t\to t_k^+} e_i(t) = e_i(t_k)$, which means the solution $e_i(t)$ is continuous from the right.

Remark 2 Note that the hybrid pinning controller is composed of adaptive pinning controller and impulsive pinning controller. The proposed controller can benefit the small synchronization cost from pinning adaptive controller, as well as the fast convergence of synchronization from introducing impulsive pinning controller into pinning adaptive scheme. In the simulation example, we will show the good performance of the proposed controller.

The following definition is needed in this paper: let $x_i(t; t_0, X_0)$ $(1 \leq i \leq N)$ be a solution of the array of stochastic complex network with coupling delays (1), where $X_0 = (x_1^0, x_2^0, \ldots, x_N^0)$. If there is a nonempty subset $\Omega \subseteq \mathbb{R}^n$, with $x_i^0 \in \Omega$ $(1 \leq i \leq N)$, such that $x_i(t; t_0, X_0) \in \mathbb{R}^n$ for all $t \geq t_0, 1 \leq i \leq N$ and

$$\begin{aligned}&\lim_{t\to\infty}\mathbb{E}\sum_{i=1}^{N}\|e_i(t,\varphi_i)\|^2\\ &\leq M\mathrm{e}^{-\epsilon(t-t_0)}\sum_{i=1}^{N}\sup_{-\tau^*\leq s\leq 0}\|\varphi_i(s)\|^2,\\ &\quad i=1,2,\ldots,N,\end{aligned} \tag{11}$$

where $\epsilon > 0$, $M > 0$ and φ_i is the initial condition of the error system (10), then stochastic delayed complex network (1) with coupling delay is said to achieve exponential synchronization in mean square and $\Omega \times \cdots \times \Omega$ is called the region of synchrony for networks (1).

3. Main results

Theorem 1 Let the controller be the hybrid controller described by Eqs. (6)–(9). If there exist positive constants $\epsilon > 0$ and $\varepsilon > 0$ such that

1)

$$\Phi = \begin{bmatrix} \Phi_1 & \frac{1}{2}q\xi\tilde{h}^s & 0 \\ \frac{1}{2}q\xi\tilde{h}^s & \frac{\varpi-1}{2}I_N & 0 \\ 0 & 0 & (\frac{\mu-1}{2}\mathrm{e}^{-\tau}+\frac{1}{2}\varepsilon^{-1})I_N \end{bmatrix} < 0, \quad (12)$$

where

$$\begin{aligned}\Phi_1 = &\Big[\lambda_{\max}(C)+\frac{1}{2}\lambda_{\max}(A^{\mathrm{T}}A) \\ &+\frac{1}{2}\varepsilon\lambda_{\max}(B^{\mathrm{T}}B)+\frac{1}{2}(1+\epsilon+\Lambda_1+\Lambda_2) \\ &+\frac{1}{2}\lambda_{\max}(\Sigma^{\mathrm{T}}\Sigma)\Big]I_N + c\xi\tilde{r}^s - \Delta,\end{aligned}$$

and

2) let $\sigma_k = \max\{1,(1+J_k)^2\}$, $\sup_{1\le k<+\infty}\{\sigma_k\} \le U < \mathrm{e}^{\epsilon\varrho\tau^*}$, U is a constant and $\varrho\tau^* < \inf\{t_k - t_{k-1}\}$, then system (1) is said to be exponentially synchronized in mean square, that is, the trivial solution of error system (10) is exponentially stable in mean square. The proof is given in the appendix.

We have the following results to show how many nodes should be pinned.

Theorem 2 Let the controller be the hybrid controller described by Eqs. (6)–(9). If there exist positive constants $\epsilon > 0$ and $\varepsilon > 0$ such that

i)

$$\frac{\mu-1}{2}\mathrm{e}^{-\tau}+\frac{1}{2}\varepsilon^{-1} < 0, \quad (13)$$

$$\lambda_{\max}\left[\left(c\xi\tilde{r}^s+\frac{1}{2(1-\varpi)}q^2\xi^2\tilde{h^s}^2\right)_l\right] < -\delta, \quad (14)$$

where

$$\begin{aligned}\delta = &\lambda_{\max}(C)+\frac{1}{2}\lambda_{\max}(A^{\mathrm{T}}A)+\frac{1}{2}\varepsilon\lambda_{\max}(B^{\mathrm{T}}B) \\ &+\frac{1}{2}(1+\epsilon+\Lambda_1+\Lambda_2)+\frac{1}{2}\lambda_{\max}(\Sigma^{\mathrm{T}}\Sigma),\end{aligned}$$

and

ii) let $\sigma_k = \max\{1,(1+J_k)^2\}$, $\sup_{1\le k<+\infty}\{\sigma_k\} \le U < \mathrm{e}^{\epsilon\varrho\tau^*}$, U is a constant and $\varrho\tau^* < \inf\{t_k - t_{k-1}\}$, then system (5) is said to be synchronized in mean square, that is, the trivial solution of error system (5) is exponentially stable in mean square.

Proof The proof is straightforward using Schur complement[33] and Lemma in Refs. [16] and [17] and hence omitted here.

Remark 3 It is very important to study how many nodes should be pinned to realize the impulsive synchronization of stochastic delayed coupled networks.[20,32] In this paper, Theorem 2 is used to determine the number of pinned nodes. The Theorem 2 is useful and the simulation examples will be provided to support the results.

Remark 4 Compared with the existing models, the proposed model is very general and the proposed hybrid control method is useful for achieving the synchronization with high convergence speed and simplicity. Note that one can easily extend our results to the case of delayed neural networks without stochastic disturbances,[7] delayed complex network with impulsive controller.[9] On the other hand, the pinning problem of the investigated model can cover the model of well-studied work in Refs. [17], [18] and [22] since we consider the stochastic disturbances and a novel hybrid control method.

4. Numerical simulation

In this section, numerical simulation examples are given to show the effectiveness and superiority of the proposed method.

Example 1

For the sake of pinning scheme, we introduce a quantity

$$E(t) = \sqrt{\frac{(x_i(t)-s(t))^2}{N}}, \quad (15)$$

which is used to measure the quality of the pinning process.

Here, we let the number of nodes be $N = 100$ and assume that the network structure in expression (1) obeys the scale-free distribution of the Barabási–Albert model.[34] Consider the following neural networks with delays:[35]

$$\begin{aligned}\mathrm{d}x(t) = &[Cx(t)+A\tilde{f}(x(t)) \\ &+B\tilde{g}(x(t-\tau(t)))]\mathrm{d}t, \quad (16)\end{aligned}$$

where $x(t) = (x_1(t), x_2(t))^{\mathrm{T}}$, $\tilde{f}(x(t)) = \tilde{g}(x(t)) = \tanh(x(t)) = (\tanh(x_1), \tanh(x_2))^{\mathrm{T}}, \tau(t) = 1$, and $C = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $A = \begin{pmatrix} 2 & -0.1 \\ -5 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{pmatrix}$.

Dynamical network (1) with hybrid controller can be described as

$$
\begin{aligned}
\mathrm{d}x_i(t) &= \Big[Cx_i(t) + A\tilde{f}(x_i(t)) + B\tilde{g}(x_i(t-\tau(t))) + u_{2i} + c\sum_{j=1}^{N} r_{ij}\Gamma x_j(t) \\
&\quad + q\sum_{j=1}^{N} h_{ij}\Gamma x_j(t-d(t))\Big]\mathrm{d}t + \kappa(t,e_i(t))\mathrm{d}w(t), \\
&\quad i = 1,2,\ldots,l, \quad t \neq t_k, t \geq t_0, \\
\mathrm{d}x_i(t) &= \Big[Cx_i(t) + A\tilde{f}(x_i(t)) + B\tilde{g}(x_i(t-\tau(t))) + c\sum_{j=1}^{N} r_{ij}\Gamma x_j(t) \\
&\quad + q\sum_{j=1}^{N} h_{ij}\Gamma x_j(t-d(t))\Big]\mathrm{d}t + \kappa(t,e_i(t))\mathrm{d}w(t), \\
&\quad i = l+1,2,\ldots,N, \quad t \neq t_k, t \geq t_0, \\
\Delta x_i(t) &= u_{1i}, t = t_k, i = 1,2,\ldots,N, k = 1,2,\ldots,
\end{aligned} \tag{17}
$$

where u_{2i} is the controller obtained by using the adaptive pinning method designed in Eqs. (8) and (9),

$$\Gamma = \mathrm{diag}(1,1), \quad d(t) = 1$$

and

$$c = 30, \quad q = 0.05, \quad \beta_i = 100 \quad (i = 1,2,\ldots,l).$$

Coupling matrix $R = H$ satisfies the scale-free distribution of the Barabási–Albert model. The noise intensity is chosen as

$$\kappa(t,e_i(t)) = 0.03e_i(t).$$

The other parameters are taken as $\epsilon = 0.2, \varrho = 1.001$ and $\varepsilon = 4$. The equidistant impulsive interval $t_k - t_{k-1} \equiv \Delta t = 1.1$ and the impulsive control gain $J_k = -2$. Therefore, when $l = 21$, we have

$$\frac{\mu-1}{2}\mathrm{e}^{-\tau} + \frac{1}{2}\varepsilon^{-1} = -0.05 < 0, \tag{18}$$

$$
\begin{aligned}
&\lambda_{\max}\left[\left(c\xi\tilde{r}^s + \frac{1}{2(1-\varpi)}q^2\xi^2\tilde{h^s}^2\right)_l\right] \\
&= -33.28 < -\delta = -31.92,
\end{aligned} \tag{19}
$$

and

$$\sigma_k = 1 < \mathrm{e}^{\epsilon\varrho\tau^*} = 1.22. \tag{20}$$

It is clear that the conditions of Theorem 2 are satisfied. Therefore, the exponential synchronization of complex network (1) can be achieved in mean square. For this example, all the impulsive synchronization stability criteria in Refs. [7]–[9] and [27] fail to draw a conclusion since the stochastic disturbances are considered, which leads to the difficulty in mathematical derivation. On the other hand, we further investigate the pinning synchronization of such stochastic delayed complex network. The pinning synchronization conditions in Refs. [17], [18] and [22] cannot generate a feasible solution since both delay in single node and coupling delay are considered, as well as the stochastic perturbations and impulsive effects are taken into account. However, using our criteria in Theorem 2, we conclude that the SDCN (1) can be uniformly synchronized exponentially in mean square with using hybrid controller since the conditions in Theorem 2 are feasible.

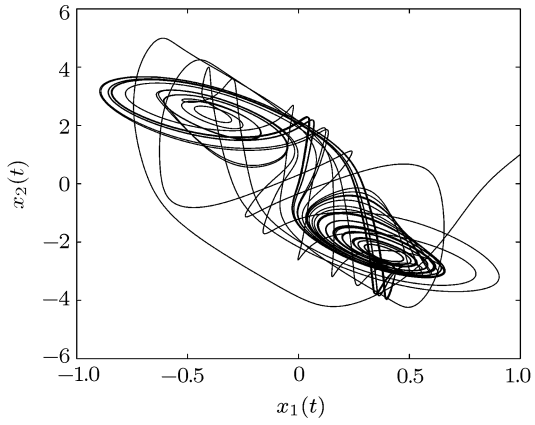


Fig. 1. Chaotic behaviours of model (16).

Figure 1 shows the chaotic behaviours of the chaotic neural networks (16). Figures 2(a) and 2(b) show the error states of synchronization errors when the first 21 nodes in the networks are pinned. Under the hybrid controller in expressions (6)–(9), the SDCN can be synchronized exponentially in mean square, as depicted in Figs. 2(a) and 2(b). From Figs. 2(c) and 2(d), it is seen that the adaptive gains tend to be constant. Figures 2(e) and 2(f) show that the states of the SDCN reach consensus after about 6 s.

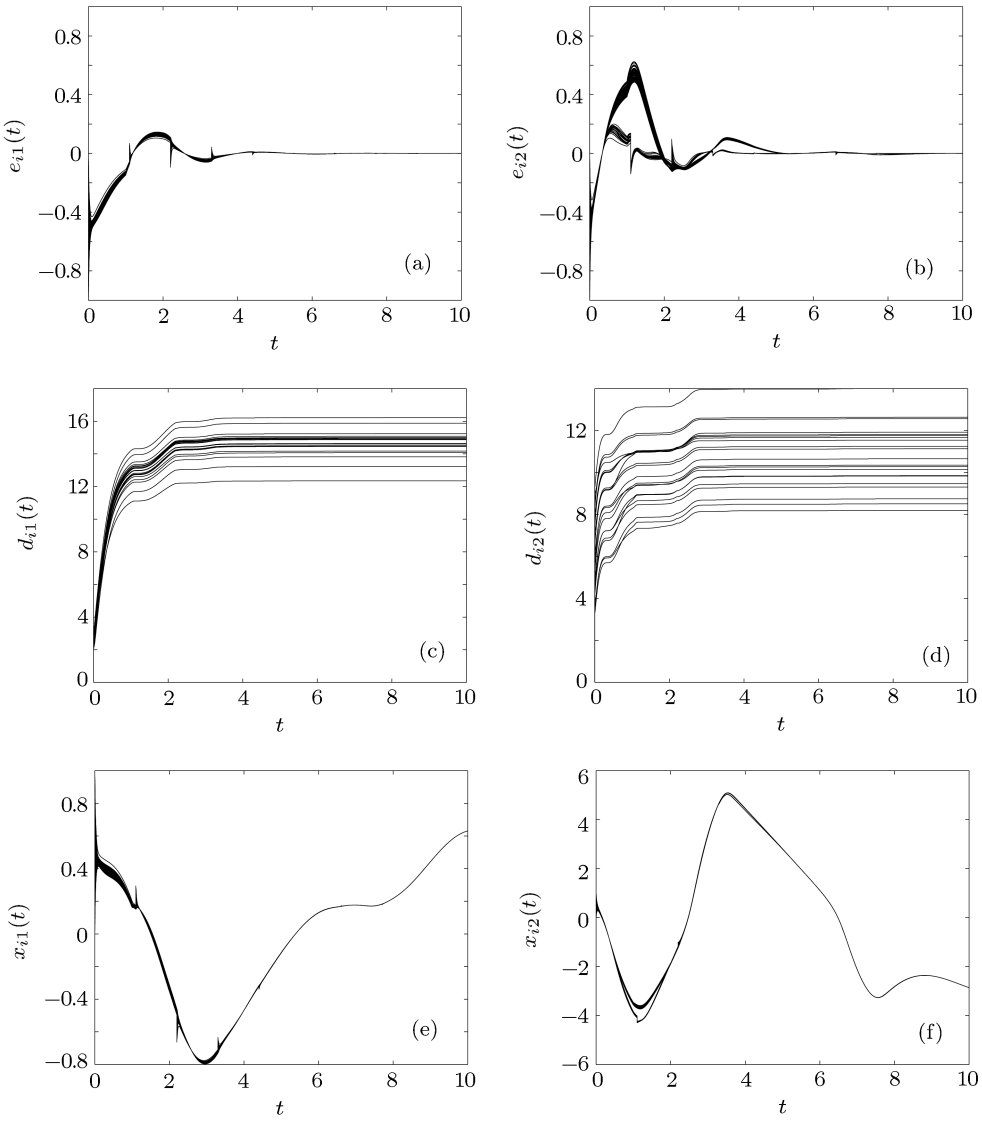


Fig. 2. (a) Synchronization errors of $t - e_{i1}(t)$, (b) $t - e_{i2}(t)$, (c) $t - d_{i1}(t)$, (d) $t - d_{i2}(t)$, (e) $t - x_{i1}(t)$ and (f) $t - x_{i2}(t)$.

Example 2 In order to show the superiority of our proposed hybrid controller, we consider $\tau(t) = 0.1$ and $d(t) = 0.1$. $t_k - t_{k-1} = \triangle t$ and it is set to be $\triangle t = 0.11$. The other parameters are chosen as those in Example 1. We can check that the conditions are satisfied in Theorem 2. Then, we compare the impulsive pinning adaptive method with pinning adaptive method. From Fig. 3, it can be seen that the synchronization is achieved after about 3 s using our proposed method. However, using the conventional pinning adaptive method, the synchronization is reached after around 9 s. Thus, the proposed novel hybrid pinning control scheme has faster convergence speed than the pinning adaptive method.

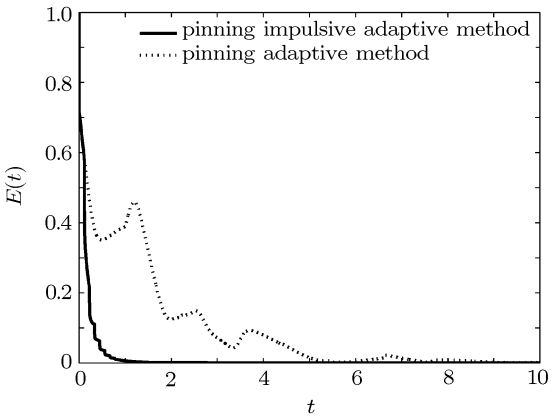


Fig. 3. Comparison between pinning adaptive method and pinning impulsive adaptive method.

5. Conclusion

The exponential pinning synchronization problem of stochastic delayed complex network by using a hybrid controller is investigated in this paper. The hybrid pinning controller consists of impulsive technique and adaptive feedback approach. Using the Lyapunov stability theory and pinning impulsive adaptive controller, several sufficient conditions are obtained for the exponential synchronization of SDCN in mean square. In the end, numerical simulation examples are provided to verify the effectiveness of the proposed approach and show the proposed control scheme outperforming the conventional control method.

Appendix

Error system (10) can be further rewritten as

$$\begin{cases} \mathrm{d}e_i(t) = \Big[Ce_i(t) + Af(e_i(t)) + Bg(e_i(t-\tau(t))) + u_{2i} + c\sum_{j=1}^{N} r_{ij}\Gamma e_j(t) \\ \qquad\qquad + q\sum_{j=1}^{N} h_{ij}\Gamma e_j(t-d(t))\Big]\mathrm{d}t + \kappa(t,e_i(t))\mathrm{d}w(t), & t \neq t_k, \\ e_i(t_k) = (1+J_k)e_i(t_k^-), \qquad i = 1,2,\ldots,l, & t = t_k, \\ e_i(t_k) = e_i(t_k^-), \qquad i = l+1,\ldots,N, & t = t_k. \end{cases} \tag{A1}$$

Define a Lyapunov-like function for Eq. (5) as

$$\begin{aligned} V(t) =& \sum_{i=1}^{N}\frac{1}{2}\mathrm{e}^{\epsilon t}e_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N}\frac{1}{2}\int_{t-\tau(t)}^{t}\mathrm{e}^{\epsilon s}\|g^{\mathrm{T}}(e_i(s))\|\|g(e_i(s))\|\mathrm{d}s \\ &+ \sum_{i=1}^{N}\frac{1}{2}\int_{t-d(t)}^{t}\mathrm{e}^{\epsilon s}\|e_i^{\mathrm{T}}(s)\|\|e_i(s)\|\mathrm{d}s + \frac{1}{2\beta_i}\sum_{i=1}^{l}(d_i(t)-d)^2, \end{aligned} \tag{A2}$$

where d is a sufficiently large positive constant.

Taking the derivative of $V(t)$ when $t \neq t_k$, we have by Itô's differential formula,[36] the stochastic derivative of $V(t)$ along Eq. (5) can be obtained as

$$\begin{aligned} \mathrm{d}V(t) =& \Big\{\epsilon\mathrm{e}^{\epsilon t}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N}\mathrm{e}^{\epsilon t}e_i^{\mathrm{T}}(t)\Big[Ce_i(t) + Af(e_i(t)) \\ &+ Bg(e_i(t-\tau(t))) + c\sum_{j=1}^{N}r_{ij}\Gamma e_j(t) + q\sum_{j=1}^{N}h_{ij}\Gamma e_j(t-d(t))\Big] \\ &- \sum_{i=1}^{l}\mathrm{e}^{\epsilon t}d_i e_i^{\mathrm{T}}(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\mathrm{e}^{\epsilon t}\|g^{\mathrm{T}}(e_i(t))\|\|g(e_i(t))\| + \frac{1}{2}\sum_{i=1}^{N}\mathrm{e}^{\epsilon t}\|e_i^{\mathrm{T}}(t)\|\|e_i(t)\| \\ &- \sum_{j=1}^{N}\frac{1-\dot{\tau}(t)}{2}\mathrm{e}^{\epsilon(t-\tau(t))}\|g^{\mathrm{T}}(e_i(t-\tau(t)))\|\|g(e_i(t-\tau(t)))\| \\ &- \sum_{j=1}^{N}\frac{1-\dot{d}(t)}{2}\mathrm{e}^{\epsilon(t-d(t))}\|e_i^{\mathrm{T}}(t-d(t))\|\|e_i(t-d(t))\| \\ &+ \sum_{i=1}^{l}\mathrm{e}^{\epsilon t}d_i e_i^{\mathrm{T}}(t)e_i(t) - \sum_{i=1}^{l}d\mathrm{e}^{\epsilon t}e_i^{\mathrm{T}}(t)e_i(t) \\ &+ \sum_{i=1}^{N}\frac{1}{2}\mathrm{Tr}[\kappa^{\mathrm{T}}(t,e_i(t))\mathrm{e}^{\epsilon t}\kappa(t,e_i(t))]\Big\}\mathrm{d}t + \mathrm{e}^{\epsilon t}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\kappa(t,e_i(t))\mathrm{d}w(t) \\ \leq& \Big\{\mathrm{e}^{\epsilon t}[\epsilon\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)e_i(t) + \sum_{i=1}^{N}e_i^{\mathrm{T}}(t)Ce_i(t) + \frac{1}{2}\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)A^{\mathrm{T}}Ae_i(t) \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\sum_{i=1}^{N} f^{\mathrm{T}}(e_i(t))f(e_i(t))+\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)Bg(e_i(t-\tau(t)))-\sum_{i=1}^{l}\mathrm{d}e_i^{\mathrm{T}}(t)e_i(t)\\
&+c\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi r_{ij}\|e_i^{\mathrm{T}}(t)\|\|e_j(t)\|+c\sum_{i=1}^{N} r_{ii}\lambda_{\min}(\Gamma^s)e_i^{\mathrm{T}}(t)e_i(t)\\
&+q\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi h_{ij}\|e_i^{\mathrm{T}}(t)\|\|e_j(t-d(t))\|+q\sum_{i=1}^{N}-\xi h_{ii}\|e_i^{\mathrm{T}}(t)\|\|e_i(t-d(t))\|\\
&+\frac{1}{2}\sum_{i=1}^{N}\|g^{\mathrm{T}}(e_i(t))\|\|g(e_i(t))\|-\frac{1-\mu}{2}\sum_{i=1}^{N}\mathrm{e}^{-\tau}\|g^{\mathrm{T}}(e_i(t-\tau(t)))\|\|g(e_i(t-\tau(t)))\|\\
&+\frac{1}{2}\sum_{i=1}^{N}\|e_i^{\mathrm{T}}(t)\|\|e_i(t)\|-\frac{1-\varpi}{2}\sum_{i=1}^{N}\mathrm{e}^{-d}\|e_i^{\mathrm{T}}(t-d(t))\|\|e_i(t-d(t))\|\\
&+\sum_{i=1}^{N}\frac{1}{2}\mathrm{Tr}[\kappa^{\mathrm{T}}(t,e_i(t))\kappa(t,e_i(t))]\Big\}\mathrm{d}t+\mathrm{e}^{\epsilon t}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\kappa(t,e_i(t))\mathrm{d}w(t), \qquad \text{(A3)}
\end{aligned}
$$

where $\xi=\sqrt{\lambda_{\max}(\Gamma^{\mathrm{T}}\Gamma)}$. One obtains

$$
f^{\mathrm{T}}(e_i(t))f(e_i(t))\leq \Lambda_1 e_i^{\mathrm{T}}(t)e_i(t),\quad g^{\mathrm{T}}(e_i(t))g(e_i(t))\leq \Lambda_2 e_i^{\mathrm{T}}(t)e_i(t), \qquad \text{(A4)}
$$

where $\Lambda_1=\max\{\lambda_j^2: j=1,2,\ldots,n\}$ and $\Lambda_2=\max\{\phi_j^2: j=1,2,\ldots,n\}$. Using Eqs. (A3) and (A4) and $2x^{\mathrm{T}}y\leq \varepsilon x^{\mathrm{T}}x+\varepsilon^{-1}y^{\mathrm{T}}y$,[37] we have

$$
\begin{aligned}
\mathrm{d}V(t)\leq&\Big\{\mathrm{e}^{\epsilon t}[\epsilon e^{\mathrm{T}}(t)e(t)+e^{\mathrm{T}}(t)(I_N\otimes C)e(t)+\frac{1}{2}e^{\mathrm{T}}(t)(I_N\otimes A^{\mathrm{T}}A)e(t)+\frac{1}{2}\Lambda_1 e^{\mathrm{T}}(t)e(t)\\
&+\frac{1}{2}\varepsilon e^{\mathrm{T}}(t)(I_N\otimes B^{\mathrm{T}}B)e(t)+\frac{1}{2}\varepsilon^{-1}g^{\mathrm{T}}(e(t-\tau(t)))g(e(t-\tau(t)))\\
&+c\xi e^{\mathrm{T}}(t)\tilde{r}^s e(t)+q\xi e^{\mathrm{T}}(t)\tilde{h}^s e(t-d(t))-e^{\mathrm{T}}(t)\Delta e(t)+\frac{1}{2}\Lambda_2 e^{\mathrm{T}}(t)e(t)\\
&-\frac{1-\mu}{2}\mathrm{e}^{-\tau}g^{\mathrm{T}}(e(t-\tau(t)))g(e(t-\tau(t)))+\frac{1}{2}e^{\mathrm{T}}(t)e(t)-\frac{1-\varpi}{2}\mathrm{e}^{-d}e^{\mathrm{T}}(t-d(t))e(t-d(t))\\
&+\frac{1}{2}e^{\mathrm{T}}(t)(I_N\otimes\Sigma^{\mathrm{T}}\Sigma)e(t)]\mathrm{d}t+\mathrm{e}^{\epsilon t}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\kappa(t,e_i(t))\mathrm{d}w(t)\Big\}, \qquad \text{(A5)}
\end{aligned}
$$

where

$$
\begin{aligned}
&e(t)=(\|e_1(t)\|,\|e_2(t)\|,\ldots,\|e_N(t)\|)^{\mathrm{T}},\\
&g(e(t))=(\|g(e_1(t))\|,\|g(e_2(t))\|,\ldots,\|g(e_N(t))\|)^{\mathrm{T}},
\end{aligned}
$$

$\tilde{r}^s=(\tilde{r}+\tilde{r}^{\mathrm{T}})/2$ ($\tilde{r}$ is a modified matrix of r via replacing the diagonal elements r_{ii} by $(\frac{\lambda_{\min}(\Gamma^s)}{\xi})r_{ii}$ and $\tilde{h}^s=(\tilde{h}+\tilde{h}^{\mathrm{T}})/2$ ($\tilde{h}$ is a modified matrix of h via replacing the diagonal elements h_{ii} by $-h_{ii}$. Taking mathematical expectation of Eq. (A5), we have

$$
\mathbb{E}\Big\{\frac{\mathrm{d}V(t)}{\mathrm{d}t}\Big\}=\mathrm{e}^{\epsilon t}\eta^{\mathrm{T}}(t)\Phi\eta(t)\leq 0, \qquad \text{(A6)}
$$

where

$$
\Phi=\begin{bmatrix}\Phi_1 & \frac{1}{2}q\xi\tilde{h}^s & 0\\ \frac{1}{2}q\xi\tilde{h}^s & \frac{\varpi-1}{2}I_N & 0\\ 0 & 0 & (\frac{\mu-1}{2}\mathrm{e}^{-\tau}+\frac{1}{2}\varepsilon^{-1})I_N\end{bmatrix},
$$

$$
\eta(t)=[e^{\mathrm{T}}(t),e^{\mathrm{T}}(t-d(t)),g^{\mathrm{T}}(e(t-\tau(t)))]^{\mathrm{T}}
$$

and

$$
\begin{aligned}
\Phi_1=&\Big[\lambda_{\max}(C)+\frac{1}{2}\lambda_{\max}(A^{\mathrm{T}}A)\\
&+\frac{1}{2}\varepsilon\lambda_{\max}(B^{\mathrm{T}}B)+\frac{1}{2}(\epsilon+1+\Lambda_1+\Lambda_2)\\
&+\frac{1}{2}\lambda_{\max}(\Sigma^{\mathrm{T}}\Sigma)\Big]I_N+c\xi\tilde{r}^s-\Delta.
\end{aligned}
$$

On the other hand, when $t=t_k$, we have

$$
\mathbb{E}\{V(t_k)\}=\mathbb{E}\Big\{\sum_{i=1}^{N}\frac{1}{2}\mathrm{e}^{\epsilon t_k}e_i^{\mathrm{T}}(t_k)e_i(t_k)+\sum_{i=1}^{N}\frac{1}{2}\int_{t_k-\tau(t_k)}^{t_k}\mathrm{e}^{\epsilon s}\|g^{\mathrm{T}}(e_i(s))\|\|g(e_i(s))\|\mathrm{d}s
$$

$$
\begin{aligned}
&+\sum_{i=1}^{N}\frac{1}{2}\int_{t_k-d(t_k)}^{t_k}\mathrm{e}^{\epsilon s}\|e^{\mathrm{T}}(s)\|\|e(s)\|\mathrm{d}s+\frac{1}{2\beta_i}\sum_{i=1}^{l}(d_i(t_k)-d)^2\Big\}\\
=&\mathbb{E}\Big\{\sum_{i=1}^{l}\frac{1}{2}\mathrm{e}^{\epsilon t_k^-}(1+J_k)^2e_i^{\mathrm{T}}(t_k^-)e_i(t_k^-)+\sum_{i=l+1}^{N}\frac{1}{2}\mathrm{e}^{\epsilon t_k^-}e_i^{\mathrm{T}}(t_k^-)e_i(t_k^-)\\
&+\sum_{i=1}^{N}\frac{1}{2}\int_{t_k^--\tau(t_k^-)}^{t_k^-}\mathrm{e}^{\epsilon s}\|g^{\mathrm{T}}(e_i(s))\|\|g(e_i(s))\|\mathrm{d}s\\
&+\sum_{i=1}^{N}\frac{1}{2}\int_{t_k^--d(t_k^-)}^{t_k^-}\mathrm{e}^{\epsilon s}\|e_i^{\mathrm{T}}(s)\|\|e_i(s)\|\mathrm{d}s+\frac{1}{2\beta_i}\sum_{i=1}^{l}(d_i(t_k^-)-d)^2\Big\}\\
\le&\mathbb{E}\{\sigma_kV(t_k^-)\}\\
\le&\mathbb{E}\{\sigma_kV(t_{k-1})\}.
\end{aligned}
\tag{A7}
$$

By making use of Eqs. (A6) and (A7) for $t\in[t_{k-1},t_k)$, we have

$$
\begin{aligned}
\mathbb{E}\{V(t)\}&\le\mathbb{E}\{V(t_{k-1})\}\le\mathbb{E}\{\sigma_{k-1}V(t_{k-2})\}\\
&\le\mathbb{E}\{\sigma_1\sigma_2\ldots\sigma_{k-1}V(t_0)\}\\
&\le\mathbb{E}\{U^{k-1}V(t_0)\}\\
&=\mathbb{E}\{\mathrm{e}^{(k-1)\ln U}V(t_0)\}.
\end{aligned}
\tag{A8}
$$

Since $\varrho\tau^*<\inf\{t_k-t_{k-1}\}$, one has $(k-1)\varrho\tau^*<t_{k-1}-t_0$. Thus, it is easy to check that

$$
\begin{aligned}
\mathbb{E}\{V(t)\}&\le\mathbb{E}\{\mathrm{e}^{(k-1)\ln U}V(t_0)\}\\
&\le\mathbb{E}\Big\{\exp\Big[\frac{\ln U}{\varrho\tau^*}(t_{k-1}-t_0)\Big]V(t_0)\Big\}\\
&\le\mathbb{E}\Big\{\exp\Big[\frac{\ln U}{\varrho\tau^*}(t-t_0)\Big]V(t_0)\Big\}.
\end{aligned}
\tag{A9}
$$

Note that there exists a constant M such that

$$
\begin{aligned}
&\mathbb{E}\{V(t_0)\}\\
=&\mathbb{E}\Big\{\sum_{i=1}^{N}\frac{1}{2}\mathrm{e}^{\epsilon t_0}e_i^{\mathrm{T}}(t_0)e_i(t_0)\\
&+\sum_{i=1}^{N}\frac{1}{2}\int_{t_0-\tau(t_0)}^{t_0}\mathrm{e}^{\epsilon s}\|g^{\mathrm{T}}(e_i(s))\|\|g(e_i(s))\|\mathrm{d}s\\
&+\sum_{i=1}^{N}\frac{1}{2}\int_{t_0-\tau(t_0)}^{t_0}\mathrm{e}^{\epsilon s}\|e^{\mathrm{T}}(s)\|\|e(s)\|\mathrm{d}s\\
&+\frac{1}{2\beta_i}\sum_{i=1}^{l}(d_i(t_0)-d)^2\Big\}\\
\le&\sum_{i=1}^{N}M\sup_{s\in[-\tau^*,0]}\mathbb{E}\|\varphi_i(s)\|^2.
\end{aligned}
\tag{A10}
$$

From the construction of $V(t)$, we have

$$
\mathbb{E}\{V(t)\}\ge\mathrm{e}^{\epsilon t}\sum_{i=1}^{N}\mathbb{E}\|e_i(t)\|^2,
\tag{A11}
$$

By making use of Eqs. (A10) and (A11), we have

$$
\begin{aligned}
&\mathbb{E}\sum_{i=1}^{N}\|e_i(t)\|^2\\
\le&M\sum_{i=1}^{N}\exp\Big[-\frac{\epsilon\varrho\tau^*-\ln U}{\varrho\tau^*}(t-t_0)\Big]\\
&\times\sup_{s\in[-\tau^*,0]}\mathbb{E}\|\varphi_i(s)\|^2.
\end{aligned}
\tag{A12}
$$

Thus, this completes the proof.

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