Cascade adaptive control of uncertain unified chaotic systems*

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The chaos control of uncertain unified chaotic systems is considered. Cascade adaptive control approach with only one control input is presented to stabilize states of the uncertain unified chaotic system at the zero equilibrium point. Since an adaptive controller based on dynamic compensation mechanism is employed, the exact model of the unified chaotic system is not necessarily required. By choosing appropriate controller parameters, chaotic phenomenon can be suppressed and the response speed is tunable. Sufficient condition for the asymptotic stability of the approach is derived. Numerical simulation results confirm that the cascade adaptive control approach with only one control signal is valid in chaos control of uncertain unified chaotic systems.

Keywords: unified chaotic systems, cascade adaptive control approach, response speed, chaos control

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1. Introduction

Chaos, generated from deterministic dynamics, exhibits unpredictable dynamic behaviours and plays a significant part in nonlinear science. Since Lorenz found the first chaotic attractor in 1963,[1] chaotic systems have been increasingly receiving the attention of researchers in last few decades. Lots of chaotic systems, such as Chua's circuit,[2] Chen system,[3,4] Lü system,[5] Liu system,[6] etc. have been found in recent years. In 2002, Lü *et al.*[7] introduced a unified chaotic system, which contains the Lorenz system and Chen system as two extremes and the Lü system as a special case. Nowadays, chaos has a number of useful applications in secure communication, information processing, biological engineering, chemical processing, lasers and other areas.[8,9] However, chaotic behaviour may result in destructive effects as well,[9] therefore the undesired chaotic phenomenon needs to be suppressed.

In 1990, Ott *et al.*[10] gave out the Ott–Grebogi–Yorke (OGY) method (based on the linearization of the Poincaré map) to control the chaos, which shows that chaos is controllable. From then on, many methods have been proposed for chaos control, such as Pyragas method[11] (based on a time-delayed feedback), linear feedback,[12,13] nonlinear feedback,[14] backstepping design technology,[15,16] and so on. For the control and synchronization of unified chaotic system, there are also fruitful results that have been reported.[17−29] Impulsive control method[17] is utilized to control and synchronize the unified chaotic systems. Adaptive control,[18,19] fuzzy control,[20,21] passive control,[22,23] and sliding mode control,[24,25] etc. have been successfully applied to the control of unified chaotic systems. The pinning control strategy and a new linear control law[26] are combined to realize the synchronization of a network with Lorenz-type nodes, a controller based on control Lyapunov function (CLF)[27] is designed to synchronize two identical unified chaotic systems with different initial conditions. The finite-time synchronization of uncertain unified chaotic systems is designed in Ref. [28]. However, most of the control laws are obtained on condition that all or part of parameters of the system are known. Moreover, generally speaking, the number of the controllers equals the number of the system state variables. In fact, it is difficult to establish a faithful model for a chaotic system in real applications. The existing nonlinearities and uncertainties may result in a control failure. In addition, in most cases, the response speed of the closed-loop system is not directly

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related to the controller parameters. Chaos can be suppressed but the transient time is undesirable, i.e., the response speed is untunable. Therefore, a model-free chaos control approach, whose response speed is tunable, is of great theoretical and practical value.

In this paper, an adaptive control law based on the dynamic compensation mechanism is adopted for the control of unified chaotic systems. Only one control input is needed to suppress chaos. The approach is capable of tolerating the uncertainties of dynamic systems. Numeric results are presented to confirm the validity of the cascade adaptive control approach.

The rest of this paper is organized as follows. In Section 2, unified chaotic system and the control objective are stated. Cascade adaptive control approach with only one control signal is presented in Section 3. Finally, conclusions are given in Section 4.

2. Problem statement

The uncontrolled unified chaotic system[7] is described below

$$\begin{cases} \dot{x}_1 = (25\alpha + 10)(x_2 - x_1), \\ \dot{x}_2 = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1x_3, \\ \dot{x}_3 = x_1x_2 - (8 + \alpha)x_3/3, \end{cases} \quad (1)$$

where x_1, x_2 and x_3 are state variables and $\alpha \in [0, 1]$ is the system parameter. When $\alpha \in [0, 0.8)$, system (1) is called the generalized Lorenz chaotic system. When $\alpha = 0.8$, system (1) is named Lü chaotic system and when $\alpha \in (0.8, 1]$, system (1) is referred to as the generalized Chen chaotic system. As α varies continuously from 0 to 1, system (1) continuously to be chaotic. Taking parameters $\alpha = 0$, 0.8, 1 separately, the chaotic attractors of the unified chaotic system for different values of parameter α are shown in Fig. 1.

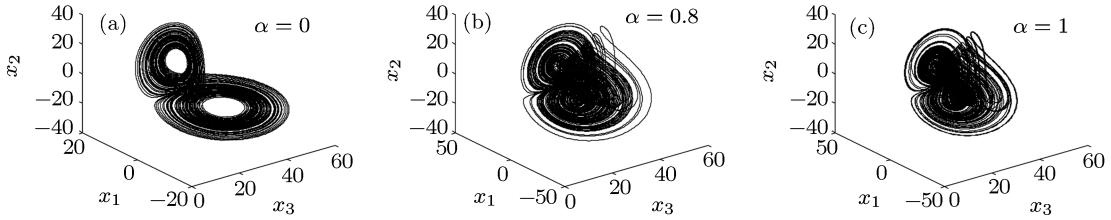


Fig. 1. Lorenz chaotic attractor (a), Lü chaotic attractor (b) and Chen chaotic attractor (c).

The control objective is to stabilize the unified chaotic systems at the equilibrium point $\boldsymbol{x} = 0$.

3. Cascade adaptive control of uncertain unified chaotic systems

3.1. Cascade adaptive control approach design

When the number of the controllers equals the number of the system state variables, the unified chaotic system can be stabilized at zero equilibrium point, but whether only one control signal can also work is a valuable problem. In other words, it is of significance to stabilize the unified chaotic systems with least control signals.

In this section, cascade adaptive control approach with only one control signal is employed to realize the control of unified chaotic system and the controlled unified chaotic system can be written as

$$\begin{cases} \dot{x}_1 = (25\alpha + 10)(x_2 - x_1), \\ \dot{x}_2 = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1x_3 + u, \\ \dot{x}_3 = x_1x_2 - (8 + \alpha)x_3/3. \end{cases} \quad (2)$$

Here, we define $y = x_1$, the control block diagram is given in Fig. 2.

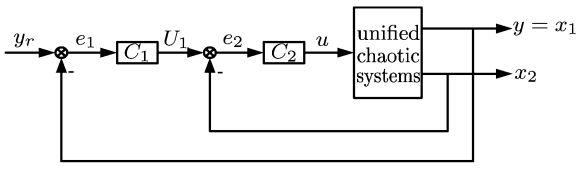


Fig. 2. Cascade adaptive control approach for uncertain unified chaotic systems.

In the diagram, C_1 and C_2 are both adaptive controllers, an adaptive controller based on the dynamic

compensation mechanism. One can refer to Ref. [30] for details. When the relative degree is one, the adaptive control law is

$$\begin{aligned} u &= h_0(y_r - y) - \hat{d}, \\ \hat{d} &= \xi - k_0(y_r - y), \\ \dot{\xi} &= -k_0\xi + k_0^2(y_r - y) - k_0 u, \end{aligned} \tag{3}$$

where $\hat{d}$ is the extended state observer, which estimates and compensates the uncertainties; y is the output of the system; y_r is the desired trajectory; h_0 determines the response speed of the system and it is chosen to be such a value that the root of $s + h_0$ is in the open left half-plane; ξ is the intermediate variable; k_0 is the tunable parameter, which determines the stability of the system.[31]

In terms of the difference between y_r and y, controller C_1 is used to compute U_1, which is the prescribed trajectory of x_2. Controller C_2 is used to compute real control input u, which guarantees x_2 tracking U_1. For controlled chaotic system (2), the dynamics of x_3 is stable, if x_1 and x_2 converge to zero.

For adaptive controller C_1, $y_r = 0$, $y = x_1$ and for controller C_2, $y_r = U_1$, $y = x_2$. Accordingly, we have

$$\begin{aligned} U_1 &= -h_{01}x_1 - \hat{d}_1, \\ \hat{d}_1 &= \xi_1 + k_{01}x_1, \\ \dot{\xi}_1 &= -k_{01}\xi_1 - k_{01}^2 x_1 - k_{01}U_1; \end{aligned} \tag{4}$$

$$\begin{aligned} u &= h_{02}(U_1 - x_2) - \hat{d}_2, \\ \hat{d}_2 &= \xi_2 - k_{02}(U_1 - x_2), \\ \dot{\xi}_2 &= -k_{02}\xi_2 + k_{02}^2(U_1 - x_2) - k_{02}u. \end{aligned} \tag{5}$$

After identical transformation, we have

$$\begin{aligned} U_1 &= c_{11}x_1 - \xi_1, \\ \dot{\xi}_1 &= c_{12}x_1; \end{aligned} \tag{6}$$

$$\begin{aligned} u &= c_{21}(x_2 - U_1) - \xi_2, \\ \dot{\xi}_2 &= c_{22}(x_2 - U_1). \end{aligned} \tag{7}$$

Substituting Eq. (6) into Eq. (7), we have

$$\begin{aligned} u &= -c_{21}c_{11}x_1 + c_{21}x_2 + c_{21}\xi_1 - \xi_2, \\ \dot{\xi}_2 &= -c_{22}c_{11}x_1 + c_{22}x_2 + c_{22}\xi_1. \end{aligned} \tag{8}$$

Substituting Eqs. (6)–(8) into system (2), we have closed-loop system (9) as follows:

$$\begin{cases} \dot{x}_1 = (25\alpha + 10)(x_2 - x_1), \\ \dot{x}_2 = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - c_{21}c_{11}x_1 \\ \qquad + c_{21}x_2 + c_{21}\xi_1 - \xi_2 - x_1x_3, \\ \dot{x}_3 = -(8 + \alpha)x_3/3 + x_1x_2, \\ \dot{\xi}_1 = c_{12}x_1, \\ \dot{\xi}_2 = -c_{22}c_{11}x_1 + c_{22}x_2 + c_{22}\xi_1. \end{cases} \tag{9}$$

Closed-loop system (9) can be written as the following compact form:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{A}\eta + \boldsymbol{O}(\boldsymbol{\eta}, t), \tag{10}$$

where $\boldsymbol{\eta} = (x_1, x_2, x_3, \xi_1, \xi_2)^{\mathrm{T}}$ and

$$\boldsymbol{O}(\boldsymbol{\eta}, t) = (0, -x_1x_3, x_1x_2, 0, 0)^{\mathrm{T}},$$

$$\boldsymbol{A} = \begin{pmatrix} -(25\alpha + 10) & 25\alpha + 10 & 0 & 0 & 0 \\ 28 - 35\alpha - c_{21}c_{11} & 29\alpha - 1 + c_{21} & 0 & c_{21} & -1 \\ 0 & 0 & -(\alpha + 8)/3 & 0 & 0 \\ c_{12} & 0 & 0 & 0 & 0 \\ -c_{22}c_{11} & c_{22} & 0 & c_{22} & 0 \end{pmatrix}_{5\times 5}. \tag{11}$$

Lemma 1[32] Consider the nonlinear system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{O}(\boldsymbol{x}, t), \tag{12}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$, and $\boldsymbol{O}(\boldsymbol{x}, t)$ denotes a nonlinear term, which satisfies $\boldsymbol{O}(\mathbf{0}, \mathbf{t}) = \mathbf{0}$ for any t. The zero solution of system (11) is uniformly asymptotically stable, if the following conditions hold:

1) $\forall t \geq 0$, $\lim\limits_{\|\boldsymbol{x}\|\to 0} \|\boldsymbol{O}(\boldsymbol{x}, t)\|/\|\boldsymbol{x}\| = 0$,

2) $\forall t \geq 0$, $\boldsymbol{A}(t)$ is bounded,

3) The zero solution of linear system $\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x}$ is uniformly asymptotically stable.

According to Lemma 1, we have the following theorem.

Theorem 1 For closed-loop system (10), controller parameters are chosen such that linear system $\dot{\boldsymbol{\eta}} = \boldsymbol{A}\boldsymbol{\eta}$ is uniformly asymptotically stable, i.e., conditions given in expression (13) are satisfied, and thus closed-loop system (10) is uniformly asymptotically stable.

Proof For the closed-loop system Eq. (10), $\boldsymbol{O}(\boldsymbol{\eta},t)\,|_{\boldsymbol{\eta}=0}=\boldsymbol{0}$ is satisfied for any t. Since

$$\frac{\|\boldsymbol{O}(\boldsymbol{\eta},t)\|}{\|\boldsymbol{\eta}\|}=\frac{\sqrt{(-x_1x_3)^2+(x_1x_2)^2}}{\sqrt{x_1^2+x_2^2+x_3^2+\xi_1^2+\xi_2^2+\xi_3^2}}\leq\frac{\sqrt{x_1^2x_3^2+x_1^2x_2^2}}{\sqrt{x_1^2}}=\sqrt{x_3^2+x_2^2},$$

hence $0\leq\|\boldsymbol{O}(\boldsymbol{\eta},t)\|/\|\boldsymbol{\eta}\|\leq\sqrt{x_3^2+x_2^2}$.

For $\lim\limits_{\|\boldsymbol{\eta}\|\to0}\sqrt{x_3^2+x_2^2}=0$, we have

$$\lim_{\|\boldsymbol{\eta}\|\to0}\|\boldsymbol{O}(\boldsymbol{\eta},t)\|/\|\boldsymbol{\eta}\|=0.$$

Therefore condition 1) in Lemma 1 is satisfied. As matrix $\boldsymbol{A}$ is a constant matrix, condition 2) is also satisfied. The linear part of system (10) is

$$\dot{\boldsymbol{\eta}}=\boldsymbol{A}\boldsymbol{\eta}. \tag{13}$$

The characteristic polynomial of system matrix $\boldsymbol{A}$ in system (12) is

$$\begin{aligned}f(\lambda)&=\det(\lambda\boldsymbol{I}-\boldsymbol{A})=\det\begin{pmatrix}\lambda+(25\alpha+10) & -(25\alpha+10) & 0 & 0 & 0\\ -28+35\alpha+c_{21}c_{11} & \lambda-(29\alpha-1+c_{21}) & 0 & -c_{21} & 1\\ 0 & 0 & \lambda+(\alpha+8)/3 & 0 & 0\\ -c_{12} & 0 & 0 & \lambda & 0\\ c_{22}c_{11} & -c_{22} & 0 & -c_{22} & \lambda\end{pmatrix}_{5\times5}\\&=\lambda^5+p_4\lambda^4+p_3\lambda^3+p_2\lambda^2+p_1\lambda+p_0,\end{aligned} \tag{14}$$

where

$$\begin{aligned}p_4&=-3.7\alpha-c_{21}+13.7,\\ p_3&=10c_{21}c_{11}+25\alpha c_{21}c_{11}-12.7c_{21}-25.3\alpha c_{21}-622\alpha+148.7\alpha^2+c_{22}-240.7,\\ p_2&=-70\alpha c_{21}+8.3\alpha^2c_{21}c_{11}+195\alpha^2-8.3\alpha^2c_{21}-25\alpha c_{12}c_{21}+26.7c_{21}c_{11}+12.7c_{22}+25.3\alpha c_{22}-25\alpha c_{22}c_{11}\\&\quad-10c_{12}c_{21}+70\alpha c_{21}c_{11}+50\alpha^3-1730\alpha-10c_{22}c_{11}-26.7c_{21}-720,\\ p_1&=-26.7c_{22}c_{11}+26.7c_{22}+70\alpha c_{22}+25\alpha c_{12}c_{22}-8.3\alpha^2c_{12}c_{21}-70\alpha c_{12}c_{21}-26.7c_{12}c_{21}+8.3\alpha^2c_{22}\\&\quad-70\alpha c_{22}c_{11}+10c_{12}c_{22}-8.3\alpha^2c_{22}c_{11},\\ p_0&=8.3\alpha^2c_{12}c_{22}+70\alpha c_{12}c_{22}+26.7c_{12}c_{22}.\end{aligned}$$

If $p_i>0$ ($i=0$–4), according to Hurwitz criterion, $f(\lambda)$ is of Hurwitz type if and only if the following conditions hold (where $p_s=0$, when $s<0$ or $s>5$):

$$\rho_j=\begin{vmatrix}p_1 & p_0 & 0 & & \cdots & & 0\\ p_3 & p_2 & p_1 & p_0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ p_{2j-3} & p_{2j-4} & p_{2j-5} & p_{2j-6} & p_{2j-7} & \cdots & p_{j-2}\\ p_{2j-1} & p_{2j-2} & p_{2j-3} & p_{2j-4} & p_{2j-5} & \cdots & p_j\end{vmatrix}>0,\quad j=1,2,3,4,5. \tag{15}$$

Hence, by choosing appropriate controller parameters condition 3) is satisfied.

For the closed-loop system (10), conditions 1), 2) and 3) hold, according to Lemma 1, zero solution of system (10) is uniformly asymptotically stable.

With the above analyses, we conclude the proof of Theorem 1.

3.2. Numerical simulations

The numerical simulation results are given to confirm the effectiveness of the cascade adaptive control approach. In the simulations, the 4-th order Runge–Kutta method is used to solve the systems in time steps of 0.001. The initial conditions are chosen to be $(x_1(0),\,x_2(0),\,x_3(0))^{\mathrm{T}}=(1,3,2)^{\mathrm{T}}$. Simulation time is 10 s, control inputs are added at the beginning of the

simulations. Controller parameters are chosen to be $k_{01} = 0.06$, $h_{01} = 0.001$; $k_{02} = 66$ and $h_{02} = 0.01$. For this selection, we have Table 1.

From Table 1, we can see that $p_i > 0$ ($i = 0$–4) and $\rho_j > 0$ ($j = 1$–5), i.e., conditions given in Eq. (13) are satisfied. According to Theorem 1, closed-loop system (10) is uniformly asymptotically stable. The state responses of the controlled unified chaotic systems and the corresponding control input are given in Figs. 3–5. As we can see from the figures, the cascade adaptive control approach with only one control input can also regulate the unified chaotic system to zero equilibrium point effectively.

Table 1. Characteristic parameters for different α values.

	$\alpha = 0$	$\alpha = 0.8$	$\alpha = 1$
p_4	79.7	76.7	76
p_3	636.4	1652.3	1936
p_2	1156.4	4232.6	5175.4
p_1	18.8	62	74
p_0	0.0011	0.0037	0.0044
ρ_5	1.08×10^{9}	3.21×10^{10}	5.43×10^{10}
ρ_4	1.08×10^{9}	3.21×10^{10}	5.43×10^{10}
ρ_3	1.38×10^{7}	4.33×10^{8}	7.41×10^{8}
ρ_2	2.17×10^{4}	2.62×10^{5}	3.83×10^{5}
ρ_1	18.8	62	74

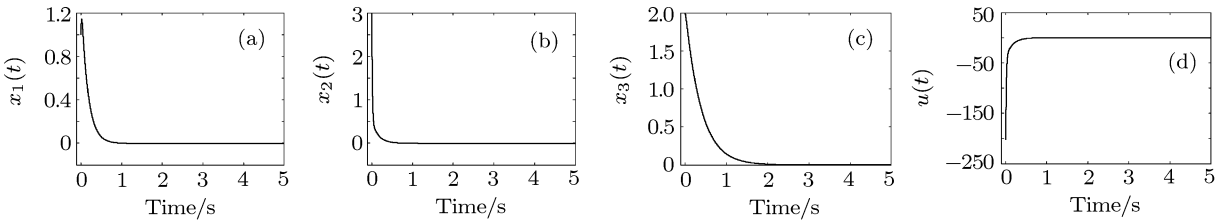


Fig. 3. Controlled states and control input by cascade adaptive control approach ($\alpha = 0$).

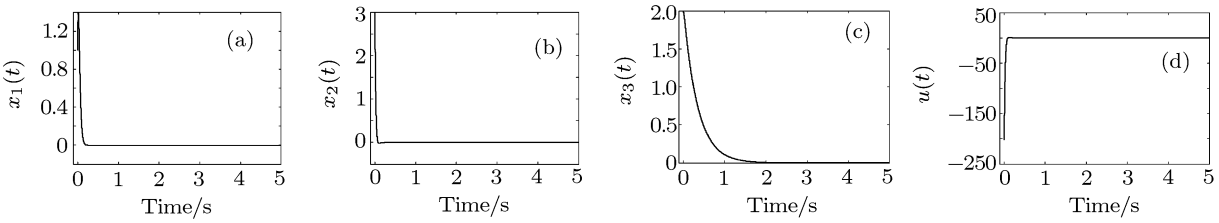


Fig. 4. Controlled states and control input by cascade adaptive control approach ($\alpha = 0.8$).

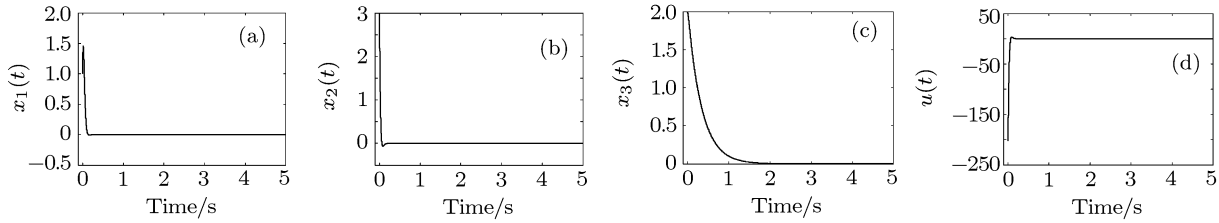


Fig. 5. Controlled states and control input by cascade adaptive control approach ($\alpha = 1$).

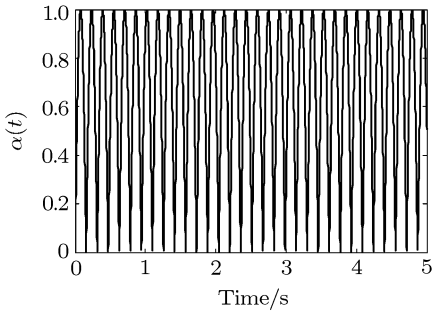


Fig. 6. Varying α in interval [0, 1].

In order to test the performance robustness of closed-loop system (10), we suppose parameter $\alpha = |1\sin(20t)|$ and thus $\alpha \in [0, 1]$, system (1) is still chaotic. The varying parameter α is shown in Fig. 6.

Choose the same controller parameters, the controlled state trajectories of the unified chaotic system and the control inputs are shown in Fig. 7.

From Fig. 7, we can see that the cascade adaptive control approach will also stabilize the unified chaotic systems, even if α varies in interval [0, 1]. To exhibit the time responses of the controlled states and the control signals more clearly, the y coordinates of all

figures have been shifted backward by 0.1.

For the cascade adaptive control approach, the response speed of inner loop should be faster than the one of the outer loop. Generally, h_{02} is 10 times greater than h_{01}, which guarantees the excellent control performance.

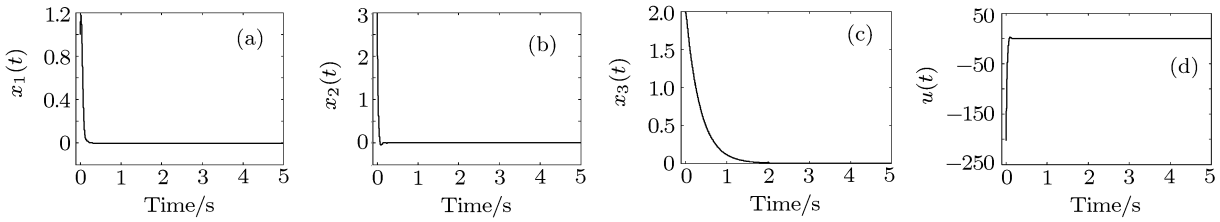


Fig. 7. Controlled states and control input by cascade adaptive control approach ($\alpha = |1\sin(20t)|$).

4. Conclusion

In this paper, the control of uncertain unified chaotic systems has been investigated. Cascade adaptive control approach with only one control signal has been presented to regulate the uncertain unified chaotic system to zero equilibrium point. The advantages of the approach can be summarized as follows.

(i) Synchronization is achieved with only one control input.

(ii) The exact model of the unified chaotic systems is not required. Adaptive controller adopted in cascade approach is capable of estimating and compensating the uncertainties effectively. By choosing appropriate values of h_0 and k_0, we can suppress the chaotic behaviour as desired.

(iii) The response speed of the closed-loop is tunable. Parameter h_0 determines the response speed. To change the value of h_0, the response speed is tunable.

Numeric results confirm that the cascade approach can stabilize the uncertain unified chaotic systems successfully.

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