Multirate diversity strategy of fractal modulation*

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Previous analyses of fractal modulation were carried out mostly from a signle perspective or a subband, but the analyses from the perspective of multiscale synthesis have not been found yet; while multiscale synthesis is just the essence of the mutlirate diversity which is the most important characteristic of fractal modulation. As for the mutlirate diversity of fractal modulation, previous studies only dealt with the general outspread of its concept, lacked the thorough and intensive quantitative comparison and analysis. In light of the above fact, from the perspective of multiscale synthesis, in this paper we provide a comprehensive analysis of the multirate diversity of fractal modulation and corresponding quantitative analysis. The results show that mutlirate diversity, which is a fusion of frequency diversity and time diversity, pays an acceptable price in spectral efficiency in exchange for a significant improvement in bit error rate. It makes fractal modulation particularly suitable for the channels whose bandwidth and duration parameters are unknown or cannot be predicted to the transmitter. Surely it is clearly of great significance for reliable communications. Moreover, we also attain the ability to flexibly make various rate-bandwidth tradeoffs between the transmitter and the receiver, to freely select the reception time and to expediently control the total bandwidth. Furthermore, the acquisitions or improvements of these fine features could provide support of the technical feasibility for the electromagnetic spectrum control technology in a complex electromagnetic environment.

Keywords: multirate diversity, fractal modulation, homogeneous signals, fractal signals

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1. Introduction

There is a wide range of contexts in which there is a need to be able to synthesize, analyse and process fractal signals. Indeed, fractal geometry[1] exists in nature, even everywhere. Fractal structure[2−4] can be found, for example, in the distribution of earthquakes, in natural landscapes, in turbulent flows, in ocean waves, in the patterns of error on communication channels, in the bronchi of the human lung and even in fluctuations of the stock market. In many applications, we are interested in modeling the inherent fractal behaviour in order that we might perform some form of signal processing. For instance, there are many problems of detection, classification, smoothing and prediction that involve fractal signals. Likewise, fractal behaviour is prevalent, suggesting that fractal geometry is somehow optimal or efficient. Consequently, there is increasing interest in the design of communication, telemetry and other engineering systems based on the use of fractal signals.

This paper is about the development and the exploitation of a framework for representing and characterizing fractal signals. Nevertheless, what is a fractal signal? Most generally, a fractal signal is a function possessing structure at every scale of detail. However, the fractals of most interest, and those to which we restrict our attention in this paper, are those with which the detail at each scale is similar. In this case we say that the fractal is self-similar or, scale-invariant, reflecting the fact that the signal has no absolute scale of reference. Fractal signals may be classified into two broad categories: one is the category in which the self-similarity of fractal signals is statistical and the other is the category in which the self-similarity is deterministic. For statistically self-similar signals, the details at all scales have the same statistics, which for deterministically self-similar signals the details at all scales are identical.

In this paper, we focus our attention on homogeneous signals,[5] which are characterized in terms of a novel deterministic self-similarity relation. Homogeneous signals have highly efficient wavelet-based representations and are potentially useful in a wide

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range of engineering applications. As an example of one promising direction for applications, Wornell and Oppenheim[5] first considered the use of homogeneous signal sets in a communications-based context. Specifically, they developed an approach to embedding information into homogeneous waveforms, which is termed "fractal modulation." As an interesting potential application of homogeneous signals, aside from a scale factor, this class of signals remains invariant under scaling of the time axis. As Wornell and Oppenheim have shown, these signals can carry information distributed over multiple time scales and frequency bands at multiple rates simultaneously. Such signals appear well-suited for transmission over noisy channels of simultaneously unknown duration and bandwidth. Not only is this a reasonable model for many physical channels, but it is also of the receiver constraints inherent in many point-to-point and broadcast communication scenarios.

The greatest advantage of fractal modulation emerges from its multirate diversity strategy: the message can be distributed over multiple time scales and frequency bands at multiple rates simultaneously. Previous analyses of fractal modulation, such as those in Refs. [5]–[11], were carried out mostly from a single perspective or a subband, with a lack of comprehensive analysis of multiscale synthesis, while the multiscale synthesis is exactly the special, unique place of fractal modulation compared with other modulations. In fact, multiscale synthesis is just the essence of the multirate diversity. Therefore, from the perspective of multiscale synthesis, this paper provides a comprehensive analysis of the multirate diversity of fractal modulation.

However, as to the multirate diversity of fractal modulation, previous papers basically dealt just with the general outspread of its concept, but the analyses from the perspective of multiscale synthesis have not been found yet. Because of the important value and significance of the multirate diversity, in this paper we attempt to fill in this deficiency in order that the efficacy contained in this technology and method could be brought into play effectively in communications, especially in the secret communication field.

Therefore, the main contents of this paper, from the perspective of multiscale synthesis, is to quantitatively analyse and discuss multirate diversity strategy in four areas of performance of limited band-scale modulation, data periodically transmitting, diversity strategy and spectral efficiency.

In this paper, we will address ourselves to the multirate diversity strategy thoroughly and accurately. For the sake of convenience, in the second part we will describe some basic knowledge of wavelet, which will be used in fractal modulation, including discrete wavelet transform, homogeneous signals and their wavelet representation and then introduce the concept of fractal modulation. The third part is the backbone of this article. In this part, from the perspective of multiscale synthesis, we will give a comprehensive analysis of multirate diversity — the most important characteristic of fractal modulation. The multirate diversity strategy will be discussed in four areas. First, we will demonstrate that the limited band-scale modulation not only makes fractal modulation transmit and receive signals at different rate-bandwidth combinations, but also controls the entire frequency bandwidth. Therefore, it can provide the feasibility support for the electromagnetic spectrum control technology in the complex electromagnetic environment. Second, the data periodically transmitting provides a great deal of flexibility to select the start of the reception time at the receiver, thus the receiver can dynamically select the appropriate rate-bandwidth working point upon request. Moreover, it also fundamentally provides the technical feasibility to flexibly construct and distribute the receiver and its reception time based on actual situations. Third, as a new diversity, compared with a simple frequency diversity and time diversity, multirate diversity has unique advantages and characteristics. We will give its qualitative demonstration. Finally, the detailed quantitative analysis and argumentation of integrated spectral efficiency of the multiscale situation will be presented.

2. Fractal modulation

2.1. Discrete wavelet transform

In this subsection, we establish the notational conventions and terminology for the aspects of wavelet theory we will exploit in this paper. For a more general review of wavelet theory, see, e.g., the classic references [12] , [13] and [14].

The discrete wavelet transform[14] (DWT) of a signal $x(t)$ is given by

$$x_{m,k} = \int_{-\infty}^{+\infty} x(t)\psi_{m,k}(t)\mathrm{d}t, \tag{1}$$

$$x(t) = \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} x_{m,k}\psi_{m,k}(t), \tag{2}$$

where $\psi(t) \in L^2(R)$ is the wavelet function.[14] Equation (1) represents the DWT and the $x_{m,k}$ are called the wavelet coefficients. And equation (1) is also called the resolution of $x(t)$. Equation (2) is the inverse discrete wavelet transform (IDWT) and is also called the reconstruction of $x(t)$. Associated with $\psi(t)$ are a corresponding scaling function, $\phi(t)$ and the scaling coefficients, $a_{m,k}$.

Mallet's fast wavelet transform (FWT) provides a computationally efficient, practical, discrete time algorithm for computing the DWT.[14] The scaling and the wavelet coefficients at scale m can be computed from the scaling coefficients at the next finer scale $m+1$ using

$$a_{m,k} = \sum_l h[l-2k]a_{m+1,l}, \tag{3}$$

$$x_{m,k} = \sum_l g[l-2k]a_{m+1,l}, \tag{4}$$

where $h[n]$ and $g[n]$ are the lowpass and the highpass filters in the associated 2-channel analysis filter band. Equations (3) and (4) represent the FWT for computing the DWT (1). Figure 1 shows this analysis operation.

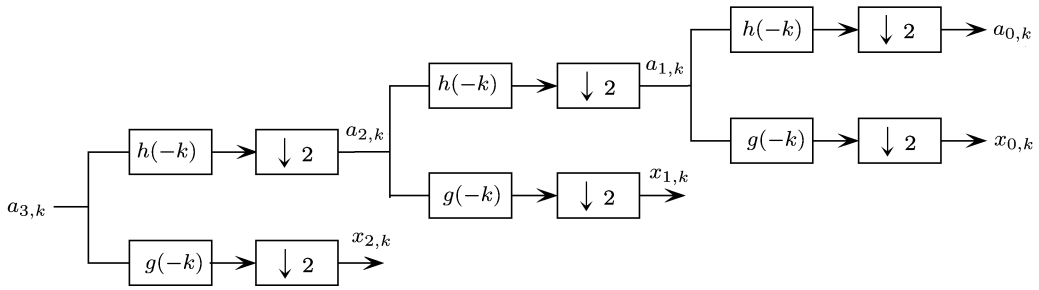


Fig. 1. A 2-channel analysis filter bank used to compute the DWT.

Conversely, it is possible to reconstruct the scaling coefficients $a_{m+1,k}$ by

$$a_{m+1,k} = \sum_l h[k-2l]a_{m,l} + g[k-2l]x_{m,l}. \tag{5}$$

Equation (5) represents the inverse fast wavelet transform (IFWT) for computing the IDWT; it corresponds to the 2-channel synthesis filter bank. Figure 2 shows the synthesis operation.

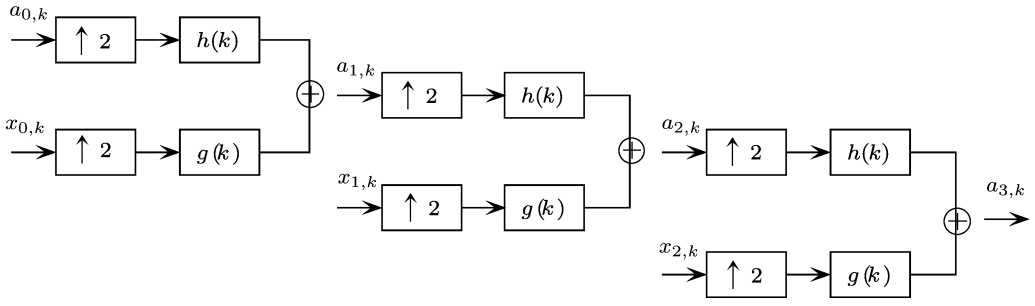


Fig. 2. A 2-channel synthesis filter bank used to compute the IDWT.

2.2. Homogeneous signals and wavelet representation

The concept of fractal modulation is based on the properties and characteristics of homogeneous signals, so next we will briefly introduce homogeneous signals and their wavelet representation.

Wornell and Oppenheim[5] first started to observe that in physical processes signals with self-similar properties can often be found; self-similar signals retain many of their essential characteristics under time scaling and for example, are an important class of signals in communications and remote sensing.

A class of self-similar signals that have been extensively investigated is related to random processes which possess statistical self-similarity; the autocorrelation functions relevant to those processes keep invariant to within an amplitude factor under an arbitrary scaling of the time axis, as, for instance, $1/f$ processes. The self-similar signals of another class are deterministic self-similar signals, which are invariant to within an amplitude factor under an arbitrary scaling of the time axis, as considered in Ref. [5].

A homogeneous time-varying signal $x(t)$ is a dyadic self-similar signal satisfying the deterministic scale-invariance property

$$x(t) = 2^{-nH} x(2^n t), \tag{6}$$

for any integer n; the Hurst parameter H is the scaling parameter of self-similarity. This is just the class of signals concerned in fractal modulation schemes.

Homogeneous signals are inherently well suited for modulating waveforms on the channels of simultaneously unknown duration and bandwidth.[5] In fact, as a consequence of their intrinsic self-similarity, these waveforms have the property that an arbitrarily short duration time-segment is sufficient to recover the entire waveform and hence the embedded information, given adequate bandwidth. Likewise, an arbitrarily low-bandwidth approximation of the waveform is sufficient to recover the undistorted waveform and again the embedded information, given an adequate time duration.[5] Additionally, homogeneous signals can be efficiently represented through a wavelet basis, which can be exploited in the development of practical systems due to its computationally efficient implementation.

An orthonormal wavelet transformation of a signal $x(t)$ is described in terms of the synthesis equations as

$$x(t) = \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} x_{m,k} \psi_{m,k}(t), \tag{7a}$$

$$x_{m,k} = \int_{-\infty}^{+\infty} x(t) \psi_{m,k}(t) \mathrm{d}t. \tag{7b}$$

Orthonormal wavelet basis functions are related to each other as usual, according to

$$\psi_{m,k}(t) = 2^{m/2} \psi(2^m t - k), \tag{7c}$$

where $\psi(t)$ represents the basic wavelet function and m and k are the dilation and translation indexes, respectively.[15−19]

When $x(t)$ is a homogeneous signal, it follows from Eq. (7b) that the wavelet coefficients take the form

$$x_{m,k} = \beta^{-m/2} x_{0,k}\,, \tag{8}$$

where $\beta = 2^{2H+1}$. By denoting $x_{0,k}$ with $q[k]$, i.e., the pattern sequence to transmit, then equation (7a) becomes

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \beta^{-m/2} q[k] \psi_{m,k}(t). \tag{9}$$

From Eq. (9) it follows that $x(t)$ is completely specified in terms of $q[k]$, which is referred to as the generating sequence for a homogeneous signal $x(t)$. The associated time-frequency portrait of a homogeneous signal, expressed in terms of generating sequence, is depicted in Fig. 3 (for the sake of illustration, H has been set to $-1/2$). The partitioning in such time frequency portraits is of course idealized; in general, there are both spectral and temporal overlaps between cells. As figure 3 shows, the synthesis of homogeneous signals can be accomplished by replicating a generating sequence $q[k]$ at each scale in representation (9) via an expansion in terms of an orthonormal wavelet basis.[12,20]

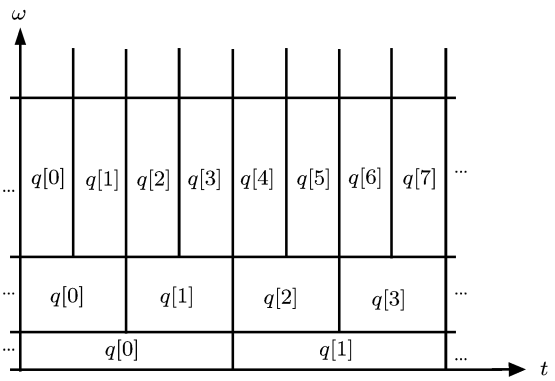


Fig. 3. Time-frequency portrait of a homogeneous signal ($H = -1/2$).

2.3. Fractal modulation

To embed a finite-power sequence $q[k]$ into a homogeneous waveform $x(t)$ of degree H, it suffices to use $q[k]$ as the coefficients of an expansion in terms of a wavelet-based orthonormal self-similar basis of degree H, i.e.,

$$x(t) = \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} \beta^{-m/2} q[k] \psi_{m,k}(t). \tag{10}$$

It can be seen from Eq. (10) that multiplying the data $q[k]$, which is to be transmitted, by $\beta^{-m/2}$ and then modulating it onto a group of orthogonal wavelet

bases, a deterministic self-similar signal $x(t)$ can be constructed. With deterministic self-similarity, $x(t)$ is a fractal signal. So this modulation is called "fractal modulation." In Fig. 4, panel (a) shows the sample path of a $1/f$ process which exhibits statistical self-similarity, while panel (b) exhibits the sample path of $x(t)$ — the transmitted signal for fractal modulation, which is deterministic self-similar.

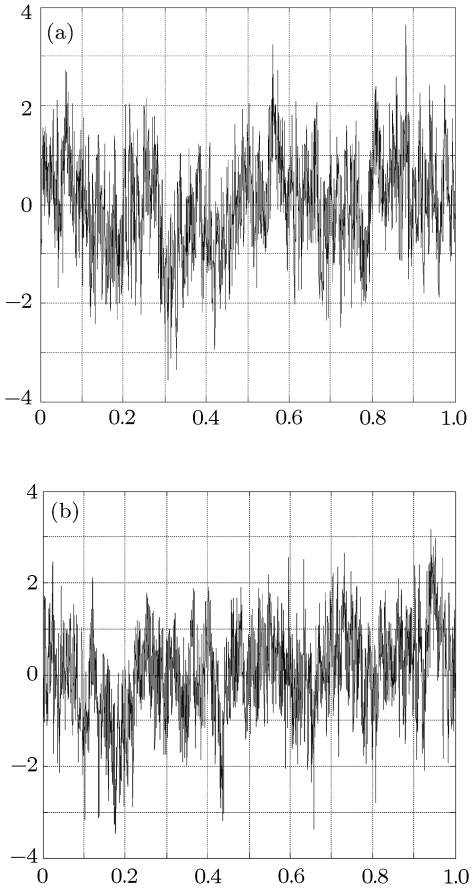


Fig. 4. Comparison between the sample path of a $1/f$ process (a) and the transmitted signal for fractal modulation (b) ($H = 1/2$).

Results illustrated in the preceding subsection suggest an efficient way to embed a symbol stream $q[k]$ into a homogeneous waveform $x(t)$ by only using such a sequence as the generating one for the signal $x(t)$, as shown in Eq. (10) . This synthesis is the essence of the fractal modulation, which yields $q[k]$ to be modulated into numbers of contiguous octave-width frequency bands. Figure 5 illustrates the time-frequency portrait of fractal modulation. We can see from Fig. 5 that the same message is modulated into frequency bands at multiple rates simultaneously. The characteristic, determined by the multi-resolution analysis[14] (MRA) of wavelet theory, is known as multirate diversity.

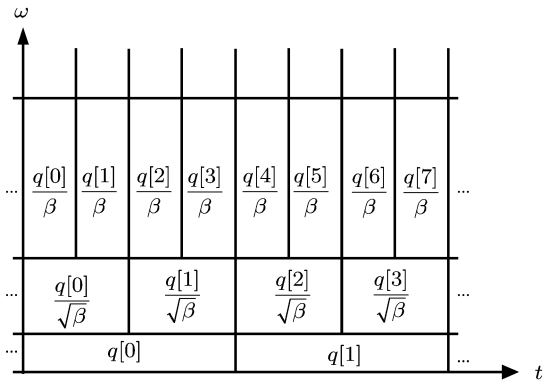


Fig. 5. Time-frequency portrait of the transmitted signal for fractal modulation ($H = -1/2$).

3. Multirate diversity strategy

As the most important characteristic of fractal modulation, the multirate diversity strategy is to transmit the same message over multiple time scales and frequency bands at multiple rates simultaneously. However, previous analyses were obtained mostly from a single scale, with lack of a comprehensive analysis of multiscale synthesis, while the multiscale synthesis is exactly the special, unique place of fractal modulation compared with other modulation. As to the multirate diversity of fractal modulation, previous papers basically dealt just with the general outspread of its concept, with lack of the thorough and intensive quantitative comparison and analyses. Our work is to make up for these deficiencies. Next, from the perspective of multiscale synthesis, we will discuss multirate diversity strategy in four areas of performance of limited band-scale modulation, data periodically transmitting, diversity strategy and spectral efficiency.

3.1. Limited band-scale modulation

In Eq. (10), the dilation index m values are taken to be infinite. This practice cannot be realized. As a matter of fact, only the limited band-scale modulation is feasible. In this paper, $\mu = \{M_L, M_L + 1, \ldots, M_U\}$, the range of m, is a finite set. Moreover, the limited band-scale modulation can actually bring two advantages to us.

First, from the spectrum point of view, fractal modulation divides the total available transmit spectrum into multiple, adjacent, octave-spaced bands so that the same information can be transmitted at different rates in different frequency bands. Thus, fractal

modulation can transmit and receive signals through any of the rate-bandwidth combinations, which is suitable particularly for the channels whose bandwidth and duration parameters are neither known nor available to the transmitter, for example, fading channels, multiple access channels, low probability of intercept (LPI) communication, military secure communication and broadcast communication to disparate receivers and so on.

Second, by selecting the valued numbers of the scale factor, i.e., m, we can make the bandwidth of fractal modulation meet the requirements of system. It brings a certain degree of convenience for communication system design and more flexibility for the concrete realization. Moreover, it can provide support for the electromagnetic spectrum control technology in the complex electromagnetic environment.

3.2. Data periodically transmitting

In Eq. (10), the generating sequence takes an infinite length. This cannot be achieved in practice, as the length can be limited only in actual modulation. In this paper, the length of the generating sequence is L, e.g. $q[k] = 0$, $k < 0$, $k > L - 1$.

At the same time, in order to take full advantage of frequency band, we will periodically launch the generating sequence. Consequently, equation (10) becomes

$$x(t) = \sum_{m\in\mu}\sum_{k}\beta^{-m/2}q[k \bmod L]\psi_{m,k}(t). \quad (11)$$

If we let

$$q = \{q[0]q[1]\dots q[L-1]\}$$

denote the data vector, then the time-frequency portrait associated with this signal is shown in Fig. 6.

Using this enhancement of fractal modulation, we not only maintain the ability to make various rate-bandwidths trade off at the receiver, but also acquire great flexibility in our choice of the start of the reception time. The receiver can dynamically select the appropriate rate-bandwidth working point based on actual demand. This method of signal transmission (data periodically transmitting) takes full advantage of the good time-frequency localization of wavelet analysis, namely, the low-frequency part of the signal being analysed with a high frequency resolution and the high-frequency part with a high time resolution. Thus, one can use receivers with different frequency bands to obtain information at different periods of time. For the receivers of lower frequency, it needs a narrower bandwidth, but takes a longer time; while for the receiver of higher frequency, it needs a wider frequency band, but the reception time could be shortened.

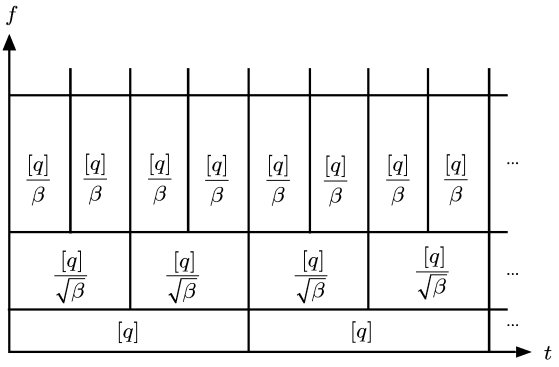


Fig. 6. Time-frequency portrait of the transmitted signal for fractal modulation of a finite-length data vector q and finite-length frequency bands.

Moreover, such a signal transmission method also gives us a fundamental provision of technical feasibility for variously and flexibly building and distributing the receivers and their reception time according to the actual time and environmental requirements.

3.3. Diversity strategy

Equation (11) can be rewritten as

$$\begin{aligned} x(t) &= \sum_{m\in\mu}\sum_{k}\beta^{-m/2}q[k \bmod L]\psi_{m,k}(t) \\ &= \sum_{k}\sum_{m\in\mu}\beta^{-m/2}q[k \bmod L]\psi_{m,k}(t) \\ &= \sum_{k}q[k \bmod L]\beta^{-M_L/2}\psi_{M_L,k}(t) + \cdots \\ &\quad + \sum_{k}q[k \bmod L]\beta^{-M_U/2}\psi_{M_U,k}(t). \end{aligned} \quad (12)$$

Since $\psi(t)$ is an orthonormal wavelet,

$$\langle\psi_{j,k}, \psi_{l,m}\rangle = 2^{(j+l)/2}\delta_{j,l}\delta_{k,m}. \quad (13)$$

For any m_i, $m_j \in \mu$ and $m_i \neq m_j$, we can obtain

$$\left\langle \sum_{k}q[k \bmod L]\beta^{-m_i/2}\psi_{m_i,k}(t), \sum_{k}q[k \bmod L]\beta^{-m_j/2}\psi_{m_j,k}(t)\right\rangle = 0. \quad (14)$$

Therefore, in principle, it ensures complete orthogonality between $\sum_{k}q[k \bmod L]\beta^{-m_i/2}\psi_{m_i,k}(t)$ and $\sum_{k}q[k \bmod L]\beta^{-m_j/2}\psi_{m_j,k}(t)$. So it does constitute

frequency diversity among the various bands (such as between scale m_i and scale m_j).

On the other hand, for any i, $j \in \{0\} \cup Z^+$ and $i \neq j$, we have

$$\left\langle \sum_{k=iL}^{(i+1)L-1} q[k \bmod L]\beta^{-m/2}\psi_{m,k}(t), \sum_{k=jL}^{(j+1)L-1} q[k \bmod L]\beta^{-m/2}\psi_{m,k}(t) \right\rangle = 0. \quad (15)$$

As a result, $\sum_{k=iL}^{(i+1)L-1} q[k \bmod L]\beta^{-m/2}\psi_{m,k}(t)$ and $\sum_{k=jL}^{(j+1)L-1} q[k \bmod L]\beta^{-m/2}\psi_{m,k}(t)$ are also completely orthogonal. Thus the time diversity is assuredly formed among the various periods of time at the same frequency band, for example, between the $(i+1)$-th period of time and the $(j+1)$-th period of time.

Obviously, from the above analysis, we can see that multirate diversity does have a unique characteristic: it is not a simple frequency diversity or time diversity alone, but simultaneously has two kinds of excellent performances from both full frequency diversity and full time diversity. Even more exactly, the multirate diversity mixes frequency diversity and time diversity together sufficiently as a hypostatic union.

Of course, due to practical technical requirements and technical conditions, the absolute, complete orthogonality from the principle level usually needs to make some changes in terms of technical implementation, such as adding frequency interval, time interval and other similar protective measures. However, it is the complete orthogonality discussed above which provides the fundamental guarantee for achieving technical performance for the simultaneous use of frequency diversity and time diversity. Its quantitative analysis will be concretely given in our following papers.

As the above, we have discussed the situation at the transmitter. Actually the case at the receiver is the same as that at the transmitter. Ignoring the multi-path interference and co-channel interference, with the ideal filter, signals at the receiver among adjacent frequency bands are also complete orthogonal and so are signals among adjacent periods of time. As for the ideal case above, with the largest ratio merging at the receiver, the gain of multirate diversity which has synthetically applied the information in time-domain and frequency domain can reach $K = \sum_{m=M_L}^{M_U} 2^{m-M_L} = 2^{M_U-M_L+1} - 1$. It shows that the gain of full time-frequency diversity of fractal modulation is indeed excellent compared with that of a simple frequency diversity, or a simple time diversity,.

What is more, in fact, multirate diversity, which is the fusion of frequency diversity and time diversity, is very different from the existing frequency diversity, time diversity and space diversity. The existing diversities are essentially the non-delivery division of the corresponding physical resources, such as frequency, time and space, while the multirate diversity is the multiplexing of certain frequency and time resources. Consequently, the multirate diversity has a much higher utilization of resources. In other words, the time diversity and the frequency diversity in multirate diversity depend on the complete division based on its inherent logic organization, rather than the existing diversities' complete division caused by external physical form.

In the last part of this subsection, we will give a simulation demonstration as follows. The outstanding merits of multirate diversity of fractal modulation in the receive diversity can be used to give three demodulation methods. The first method is to just take the first repeat cycle of the received signal in each scale, the second method, to make a joint estimation of the received signal in each scale, the third method, to make a joint estimation of the received signal among different scales. Simulation parameters are setting as follows: DB4 wavelet function, the length of $[q]$, i.e. $L = 1024$, the energy per bit, i.e. $E_{\rm b} = 1$, dilation index, i.e. $m \in \mu = \{10, 11, 12, 13\}$, the symbol period of the signal $T_{\rm s} = 1$ s, the longest symbol period at the scale 10, $T_{\rm S_{10}} = T_{\rm s}/L = 0.977$ ms and the Doppler shift, i.e. $f_{\rm d} = 60$ Hz. Assuming uniformly distributed data, we derive the theoretical performance for binary fractal modulation in the presence of additive white Gaussian noise and Rayleigh flat fading channels.

Figure 7 shows the comparison of performance between three demodulation methods of fractal modulation and theoretical BPSK modulation in an AWGN channel. This figure indicates that in order to achieve the same level (10^{-5}) of bit error rate (BER), compared with the first method, the second method achieves an 8-dB improvement and the BER of the third method is almost zero. The fractal modulation performance is depicted for only one scale (scale 13) since the performance does not vary across the scale.

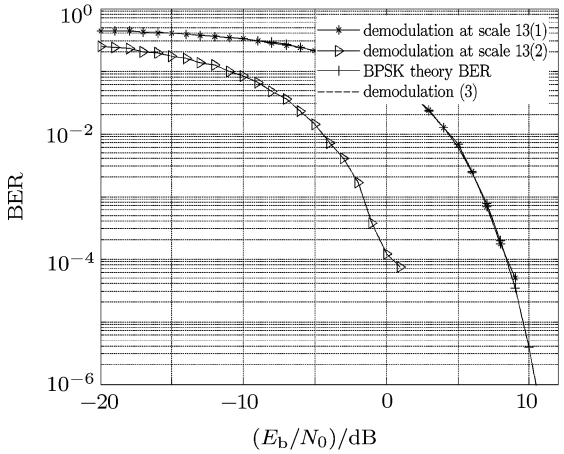


Fig. 7. BERs versus E_b/N_0 of three demodulation methods of fractal modulation in an AWGN channel.

Figure 8 shows the comparison of BER curve among the three demodulation methods of fractal modulation at scale 13 and $f_d = 60$ Hz in a flat fading channel. This figure shows that at the same cost of SNR, the second method can obtain better BER performance than the first method and the third method can achieves the best performance.

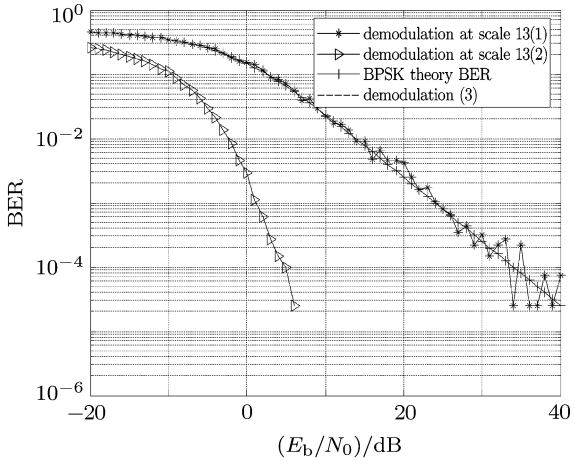


Fig. 8. BERs versus E_b/N_0 of three demodulation methods of fractal modulation in a flat fading channel.

Experimental results show that the use of the characteristics of multirate diversity of fractal modulation in the receive diversity could indeed improve the BER performance. More specific analysis can be found in Ref. [21].

3.4. Spectral efficiency

As for a certain single scale m, reference [5] holds that: "we view this as a multirate modulation of $q[k]$ where in the m-th frequency band $q[k]$ is modulated at rate 2^m using a double-sided bandwidth of 2^m Hz. Using a suitably designed receiver, $q[k]$ can, in principle, be recovered from $x(t)$ at an arbitrary rate 2^m using a baseband bandwidth of 2^{m+1} Hz. So, this modulation has a spectral efficiency of $\eta_{\mathrm{F_O}} = 1/2$ symbol/s/Hz in any a certain scale."

Note that Wornell and Oppenheim's conclusion refer to only the spectral efficiency of a single scale, while the multirate diversity strategy is the integrated application of multiple scale. Therefore, logically speaking, the discussion of the spectral efficiency should be carried out based on multiscale synthesis. Here we will carry on the quantitative analysis of the multiscale situation in view of Wornell and Oppenheim's result for a single scale.

The message is transmitted at each scale in different rates, however, for all scales, the whole data stream is transmitted in the rate of the maximum scale, while the bandwidth is the total sum of those from all scales. Assume that in the minimum scale M_L, the bit rate of data will be 2^{M_L} b/s, so its bandwidth is 2^{M_L+1} Hz.; at the maximum scale M_U, the bit rate is 2^{M_U} b/s and its bandwidth is 2^{M_U+1} Hz. Total bandwidth is

$$\begin{aligned} &2^{M_L+1} + 2^{M_L+2} + \cdots + 2^{M_U+1} = 2^{M_L+1} \\ &(1 + 2 + \cdots + 2^{M_U-M_L}) = 2^{M_L+1}K. \end{aligned} \tag{16}$$

The bit rate of the whole data stream is 2^{M_U} b/s, so the spectrum efficiency of fractal modulation is

$$\eta_{\mathrm{F_A}} = \frac{2^{M_U}}{2^{M_L+1}K} = \frac{2^{M_U-M_L}}{K}\eta_{\mathrm{F}}, \tag{17}$$

while $K = \sum_{m=M_L}^{M_U} 2^{m-M_L} = 2^{M_U-M_L+1} - 1$, so

$$\begin{aligned} \eta_{\mathrm{F_A}} &= \frac{2^{M_U-M_L}}{2^{M_U-M_L+1}-1}\eta_{\mathrm{F}} > \frac{2^{M_U-M_L}}{2^{M_U-M_L+1}}\eta_{\mathrm{F}} \\ &= \eta_{\mathrm{F_O}}/2 = 1/4. \end{aligned} \tag{18}$$

Thus, in the worst case, due to the multirate diversity, the spectral efficiency of fractal modulation is greater than 1/4.

4. Discussion

From the perspective of multiscale synthesis, in this paper a comprehensive thorough and intensive quantitative comparison and an analysis of the multirate diversity of fractal modulation are presented. Through this analysis, we also come to expand the scope, methods and approaches to the application of fractal modulation. In the view of the transmission

of single information it may sacrifice the spectral efficiency, however, it makes fractal modulation capable of using a variety of rate-bandwidth combinations to transmit and receive signals, significantly reducing the bit error rate, and improving the reliability of information transmission. From this point of view, the cost of bandwidth efficiency may return for benefits in these areas, not pointless to pay. Moreover, the multirate diversity also allows us to flexibly choose the scale numbers in the transmitter, to control the transmitted bandwidth and to conveniently select the reception time. Consequently, it provides a broad room for technology implements and technology operations of the electromagnetic spectrum control technology in the complex electromagnetic environment. In essence, multirate diversity integratively and simultaneously makes use of frequency diversity and time diversity, has much higher utilization of frequency and time, as compared with other existing diversities and provides multiple redundant information, thereby greatly enhancing the error performance.

Multirate diversity strategy provides diversities both in the transmitter and in the receiver, so it is a natural consideration to combine multirate diversity with MIMO, while MIMO is the key technology in the fourth-generation of communication. This is our next study that will be addressed.

As early as the 1950s, Cha and Xiong[22] had already theoretically proved that to overcome the interference effects of multi-path fading, the best transmitted signals over additive white noise channels should be the signals with white noise statistical properties. The statistical properties of spread-spectrum functions (pseudo-codes) are very close to those of white noise and thus spread spectrum communication inherently has anti-multi-path interference capability. On the other hand, under the actual circumstances, the relationship between the signals and the fractal noises has increasingly become an important issue, therefore, the theoretical analysis of whether the signal with fractal noise characteristics is the best signal to be transmitted over the channels with fractal noises is already obviously necessary. In the future work, we will go on giving performance analysis of fractal modulation under fractal noise environments.

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