Feasible schemes for quantum swap gates of optical qubits via cavity QED*

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Feasible schemes for implementing quantum swap gates of both coherent-state qubits and photonic qubits are proposed using a Λ-type atomic ensemble trapped in a bimodal optical cavity. In both protocols, the decoherence from atomic spontaneous emission is negligible due to the fact that the excited states of the atoms are adiabatically eliminated under large detuning condition and the swap gates can be created in a single step. In our schemes, the required atoms–cavity interaction time decreases with the increase of the number of atoms, which is very important in view of decoherence. The experimental feasibilities of the schemes are also discussed.

Keywords: swap gate, coherent-state qubits, photonic qubits, fidelity

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1. Introduction

Quantum computers implement computational tasks based on the fundamental principle of quantum mechanics. This new type of quantum information processor takes the advantage of superposition principle and quantum entanglement to solve many problems, such as factoring a large number and searching data in an array more efficiently than the present conventional computers.[1,2] Over the last decade, extensive research interest has been stimulated in both theoretical and practical aspects of quantum computers. It has been shown that the quantum logical gates are the essential building blocks of a quantum computer, and any quantum computational network can be decomposed into a series of basic quantum logic gates. Various different physical systems have been proposed for implementing quantum logic gates in recent years, such as cavity quantum electrodynamics (CQED),[3−10] nuclear magnetic resonance (NMR),[11] quantum dots,[12,13] and superconducting quantum interference devices (SQUID).[14−16]

Among these systems being explored for hardware implementations of quantum computers, CQED is almost an alternative qualified candidate since the atoms trapped in cavity own relatively long decoherence time and the strong coupling between atoms and photons guarantees quantum state coherent manipulation and control.[17] In the realm of CQED according to the difference in interaction between the atoms and cavity fields, there are two kinds of schemes for designing quantum logic gates, i.e., single atom quantum gate and atomic ensembles quantum gate. In the second one, atomic ensembles can collectively enhance the coupling strength due to the many-atom interference effects. In recent years, some theoretical models about implementing a quantum logic gate with an N-type,[18] M-type,[19] or V-type[20,21] configuration of atomic ensembles have been reported. Moreover, the atomic ensemble can be used to realize frequency down (or up)-conversions of bimodal cavity field.[22−25] On the other hand, the swap gate, one of the basic two-qubit quantum gates, plays an important role in quantum information processing, particularly in information transmission and entanglement swapping. However, for constructing such a gate with elementary gates from the universal set, one needs three Control-Not gates or three conditional phase gates plus six Hadamard gates.[26,27] This complicated procedure will decrease the efficiency of quantum computation.

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Recently, two schemes have been reported for realizing quantum swap gates of qubits, the Fock photonic states[20] and coherent states,[21] with V-type atomic ensembles in cavities. In the present paper, we show that the quantum swap gates of both coherent-state qubits and photonic qubits can be implemented only by one step with Λ-type atomic ensemble trapped in a bimodal optical cavity. The purpose for the direct construction of swap gates is to save a great many of quantum resources in practical quantum information processing and reduce the number of required unitary operations. Our schemes have the following advantages. (i) It requires only commonly addressing the atoms inside the optical cavity, which provides a convenience in experiment. (ii) During the operation, the atoms are always populated in ground states. Thus the decoherence arising from the atomic spontaneous emission is effectively suppressed. (iii) The required atoms–cavity interaction time is inversely proportional to the number of atoms. This can produce any atom–field coupling in principle as long as one can let enough atoms interact with the cavity modes, which can help to choose a desired reasonable time to avoid decoherence.

2. A swap gate for coherent-state qubits

The system under consideration is composed of N identical three-level atoms in a Λ-configuration placed inside a bimodal cavity. The concrete atomic level structure and corresponding transitions are shown in Fig. 1, including one excited states $|e\rangle$, and two stable ground states $|g\rangle$ and $|s\rangle$. The transitions $|g\rangle \rightarrow |e\rangle$ and $|s\rangle \rightarrow |e\rangle$ couple to the cavity modes with coupling constants g_h and g_v, respectively.

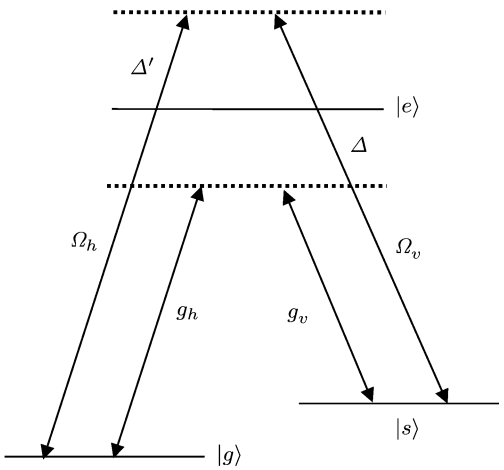


Fig. 1. Involved level structure of the Λ-type three-level atoms and the corresponding atomic transitions.

The classical fields (relevant frequencies are ω_{Ω_h} and ω_{Ω_v}) drive dispersively the atomic transitions $|g\rangle \rightarrow |e\rangle$ and $|s\rangle \rightarrow |e\rangle$ with Rabi frequencies Ω_h and Ω_v, respectively. The Hamiltonian of the system can be written as[24] (assuming $\hbar = 1$)

$$\begin{aligned} H &= H_0 + H_{\rm int}^1 + H_{\rm int}^2, \\ H_0 &= \omega_h a_h^\dagger a_h + \omega_v a_v^\dagger a_v + \sum_{k=1}^{N} (\omega_e |e\rangle_{kk}\langle e| \\ &\quad + \omega_g |g\rangle_{kk}\langle g| + \omega_s |s\rangle_{kk}\langle s|), \\ H_{\rm int}^1 &= \sum_{k=1}^{N} (g_h a_h |e\rangle_{kk}\langle g| + g_v a_v |e\rangle_{kk}\langle s| + {\rm H.c}), \\ H_{\rm int}^2 &= \sum_{k=1}^{N} (\Omega_h {\rm e}^{-{\rm i}\,\omega_{\Omega_h} t} |e\rangle_{kk}\langle g| \\ &\quad + \Omega_v {\rm e}^{-{\rm i}\,\omega_{\Omega_v} t} |e\rangle_{kk}\langle s| + {\rm H.c}), \end{aligned} \tag{1}$$

where a_h and a_v are the annihilation operators associated with two cavity modes, respectively. In the following, we assume that g_h, g_v, Ω_h and Ω_v are real numbers for simplicity. Then the Hamiltonians describing the atom–field interaction in the interaction picture are

$$\begin{aligned} H_{\rm eff}^1 &= \sum_{k=1}^{N} (g_h a_h {\rm e}^{{\rm i}\,\Delta t} |e\rangle_{kk}\langle g| \\ &\quad + g_v a_v {\rm e}^{{\rm i}\,\Delta t} |e\rangle_{kk}\langle s| + {\rm H.c}), \\ H_{\rm eff}^2 &= \sum_{k=1}^{N} (\Omega_h {\rm e}^{-{\rm i}\,\Delta' t} |e\rangle_{kk}\langle g| \\ &\quad + \Omega_v {\rm e}^{-{\rm i}\,\Delta' t} |e\rangle_{kk}\langle s| + {\rm H.c}). \end{aligned} \tag{2}$$

The detunings for associated atomic transitions are $\Delta = \omega_e - \omega_h - \omega_g = \omega_e - \omega_v - \omega_s$ and $\Delta' = \omega_{\Omega_h} - \omega_e - \omega_g = \omega_{\Omega_v} - \omega_v - \omega_s$. Considering the large detuning conditions that $\Delta \gg \sqrt{N} g_h$, $\sqrt{N} g_v$ and $\Delta' \gg \Omega_h$, Ω_v, the excited level $|e\rangle$ can be adiabatically eliminated. Then we can derive the corresponding Hamiltonian $H_{\rm eff}^1$ as follows:

$$\begin{aligned} H_{\rm eff}^1 &= -\sum_{k=1}^{N} \Big[\frac{g_h^2}{\Delta} a_h^\dagger a_h |g\rangle_{kk}\langle g| \\ &\quad + \frac{g_v^2}{\Delta} a_v^\dagger a_v |s\rangle_{kk}\langle s| \\ &\quad + \frac{g_h g_v}{\Delta} (a_h^\dagger a_v |g\rangle_{kk}\langle s| + {\rm H.c}) \Big]. \end{aligned} \tag{3}$$

The first two terms on the right-hand side in the above equation correspond to the photon-number-dependent Stark shifts, while the third and fourth terms describe the Raman transitions between $|g\rangle$ and $|s\rangle$. At the same time, $H_{\rm eff}^2$ can be replaced by the effective Hamiltonian

$$H_{\rm eff}^2 = \sum_{k=1}^{N}\Big[\frac{\Omega_h^2}{\Delta'}|g\rangle_{kk}\langle g| + \frac{\Omega_v^2}{\Delta'}|s\rangle_{kk}\langle s| + \frac{\Omega_h\Omega_v}{\Delta'}(|g\rangle_{kk}\langle s| + |s\rangle_{kk}\langle g|)\Big]. \quad (4)$$

Using the two dressed states, that is, $|\pm\rangle_k = (|g\rangle_k \pm |s\rangle_k)/\sqrt{2}$, the effective Hamiltonians $H_{\rm eff}^1$ and $H_{\rm eff}^2$ can be conveniently rewritten as

$$\begin{aligned} H_{\rm eff}^1 = & -\frac{1}{2}\sum_{k=1}^{N}[\chi_h a_h^\dagger a_h(I_k + \sigma_k^+ + \sigma_k^-) \\ & + \chi_v a_v^\dagger a_v(I_k - \sigma_k^+ - \sigma_k^-) \\ & + \chi(a_h^\dagger a_v(\sigma_k^z - \sigma_k^+ + \sigma_k^-) \\ & + a_h a_v^\dagger(\sigma_k^z + \sigma_k^+ - \sigma_k^-))], \end{aligned} \quad (5)$$

and

$$H_{\rm eff}^2 = \sum_{k=1}^{N}\mu(I_k + \sigma_k^z), \quad (6)$$

where $\chi_h = g_h^2/\Delta$, $\chi_v = g_v^2/\Delta$, $\chi = g_h g_v/\Delta$, $\mu_h = \Omega_h^2/\Delta'$, $\mu_v = \Omega_v^2/\Delta'$, and $\mu = \Omega_h\Omega_v/\Delta'$. Here we have adopted the form of Pauli's matrices in the new dressed basis, i.e.,

$$\begin{aligned} I_k &= |+\rangle_{kk}\langle +| + |-\rangle_{kk}\langle -|, \\ \sigma_k^z &= |+\rangle_{kk}\langle +| - |-\rangle_{kk}\langle -|, \\ \sigma_k^\pm &= |\pm\rangle_{kk}\langle \mp|. \end{aligned} \quad (7)$$

Then, the time evolution of the whole system is governed by the Schrödinger equation

$$\mathrm{i}\,\frac{\partial|\Psi(t)\rangle}{\partial t} = (H_{\rm eff}^1 + H_{\rm eff}^2)|\Psi(t)\rangle. \quad (8)$$

For the sake of further simplifying the dynamics of the system, we perform the transformation $\mathrm{e}^{\mathrm{i}\,H_{\rm eff}^2 t}$,[28–31] thus the effective Hamiltonian of this system takes the form

$$\begin{aligned} H_{\rm eff} &= \mathrm{e}^{\mathrm{i}\,H_{\rm eff}^2 t}H_{\rm eff}^1\,\mathrm{e}^{-\mathrm{i}\,H_{\rm eff}^2 t} \\ &= -\frac{1}{2}\sum_{k=1}^{N}[(\chi_h a_h^\dagger a_h + \chi_v a_v^\dagger a_v)I_k \\ &\quad + \chi(a_h^\dagger a_v + a_h a_v^\dagger)\sigma_k^z]. \end{aligned} \quad (9)$$

We assume all the atoms are initially in the state $|-\rangle_k$, then the effective Hamiltonian becomes

$$H_{\rm eff} = \frac{1}{2}\sum_{k=1}^{N}[\chi(a_h^\dagger a_v + a_h a_v^\dagger)|-\rangle_{kk}\langle -| - (\chi_h a_h^\dagger a_h + \chi_v a_v^\dagger a_v)|-\rangle_{kk}\langle -|]. \quad (10)$$

Thus, the time evolution operator of Eq. (8) is $U(t) = \mathrm{e}^{-\mathrm{i}\,H_{\rm eff}t} = \mathrm{e}^{-\mathrm{i}\,\theta(a_h^\dagger a_v + a_h a_v^\dagger)}\mathrm{e}^{\mathrm{i}\,\phi_1 a_h^\dagger a_h}\mathrm{e}^{\mathrm{i}\,\phi_2 a_v^\dagger a_v}$, where $\theta = N\chi t/2$, $\phi_1 = N\chi_h t/2$, $\phi_2 = N\chi_v t/2$. For simplicity, we assume that the coupling strengths of the two cavity modes are equal, so that $\theta = \phi_1 = \phi_2$ throughout the present paper. When $U(\theta,\phi_1,\phi_2)$ acts on the four initial two-qubit coherent states, the exact evolution of this system is obtained by choosing the interaction time $t = \pi/N\chi$,

$$\begin{aligned} & U(\theta,\phi_1,\phi_2)|-\rangle_k(|\alpha\rangle+|-\alpha\rangle)_a(|\alpha\rangle+|-\alpha\rangle)_b \\ \to\ & |-\rangle_k(|\alpha\rangle+|-\alpha\rangle)_a(|\alpha\rangle+|-\alpha\rangle)_b, \\ & U(\theta,\phi_1,\phi_2)|-\rangle_k(|\alpha\rangle+|-\alpha\rangle)_a(|\alpha\rangle-|-\alpha\rangle)_b \\ \to\ & |-\rangle_k(|\alpha\rangle-|-\alpha\rangle)_a(|\alpha\rangle+|-\alpha\rangle)_b, \\ & U(\theta,\phi_1,\phi_2)|-\rangle_k(|\alpha\rangle-|-\alpha\rangle)_a(|\alpha\rangle+|-\alpha\rangle)_b \\ \to\ & |-\rangle_k(|\alpha\rangle+|-\alpha\rangle)_a(|\alpha\rangle-|-\alpha\rangle)_b, \\ & U(\theta,\phi_1,\phi_2)|-\rangle_k(|\alpha\rangle-|-\alpha\rangle)_a(|\alpha\rangle-|-\alpha\rangle)_b \\ \to\ & |-\rangle_k(|\alpha\rangle-|-\alpha\rangle)_a(|\alpha\rangle-|-\alpha\rangle)_b. \end{aligned} \quad (11)$$

We encode the diagonal states $|0_L\rangle$ and $|1_L\rangle$ of two-coherent cavity–fields as logic zero and one states, where $|0_L\rangle = \mathscr{N}_+(\alpha)(|\alpha\rangle+|-\alpha\rangle)$ and $|1_L\rangle = \mathscr{N}_-(\alpha)(|\alpha\rangle-|-\alpha\rangle)$, with the normalized factors being $\mathscr{N}_\pm(\alpha)=(2\pm 2\mathrm{e}^{-2\alpha^2})^{-1/2}$.[21] Thus equation (11) describes a two-coherent states qubit swap gate operation based on the new basis of coherent cavity–fields

$$\begin{aligned} &|-\rangle_k|0_L\rangle_a|0_L\rangle_b \to |-\rangle_k|0_L\rangle_a|0_L\rangle_b, \\ &|-\rangle_k|0_L\rangle_a|1_L\rangle_b \to |-\rangle_k|1_L\rangle_a|0_L\rangle_b, \\ &|-\rangle_k|1_L\rangle_a|0_L\rangle_b \to |-\rangle_k|0_L\rangle_a|1_L\rangle_b, \\ &|-\rangle_k|1_L\rangle_a|1_L\rangle_b \to |-\rangle_k|1_L\rangle_a|1_L\rangle_b. \end{aligned} \quad (12)$$

If all the atoms are initially in the state $|+\rangle_k$, then the time evolution becomes $U(t) = \mathrm{e}^{-\mathrm{i}\,H_{\rm eff}t} = \mathrm{e}^{\mathrm{i}\,\theta(a_h^\dagger a_v + a_h a_v^\dagger)}\mathrm{e}^{\mathrm{i}\,\phi_1 a_h^\dagger a_h}\mathrm{e}^{\mathrm{i}\,\phi_2 a_v^\dagger a_v}$. Choosing the experimental parameter $t = 4\pi/N\chi$, we can obtain the two-qubit swap gate:

$$\begin{aligned} &|+\rangle_k|0_L\rangle_a|0_L\rangle_b \to |+\rangle_k|0_L\rangle_a|0_L\rangle_b, \\ &|+\rangle_k|0_L\rangle_a|1_L\rangle_b \to |+\rangle_k|1_L\rangle_a|0_L\rangle_b, \\ &|+\rangle_k|1_L\rangle_a|0_L\rangle_b \to |+\rangle_k|0_L\rangle_a|1_L\rangle_b, \\ &|+\rangle_k|1_L\rangle_a|1_L\rangle_b \to |+\rangle_k|1_L\rangle_a|1_L\rangle_b. \end{aligned} \quad (13)$$

In the above analyses we have not take into account the influence of decoherence on the scheme. The main decoherence channels in cavity QED commonly arise from atomic spontaneous emission and cavity decay.[32] In our scheme, the excited atomic level $|e\rangle$ does not participate in the atom–cavity interaction due to the large detuning. Therefore, the spontaneous emission from atomic excited states can be ignored. However, in order to realize the swap gate, the two coherent states in two modes must exchange their information, thus the cavity decay should become a dominant noise source in the gate operation. In order to confirm this point, we discuss the time evolution

of the system under the cavity loss and perform a direct numerical calculation for the fidelity of the swap gate. The conditional Hamiltonian under the dissipation dynamics can be well suitable for evaluating the effects on the gate fidelity, which is governed by

$$H_{\rm con} = H_{\rm eff} - \frac{\rm i}{2}\sum_{m=h,v}\kappa_m a_m^\dagger a_m,$$

where κ_m $(m \in h, v)$ denotes the decay rate of the cavity mode. For simplicity, we assume $\kappa_h = \kappa_v$ in the numerical simulation. The calculated relation of the fidelity can be determined by $F = \langle\Psi_0|U^+\rho(t)U|\Psi_0\rangle$,[33] where $\rho(t)$ represents the temporal density matrix reduced by tracing out the cavity mode part, $|\Psi_0\rangle$ denotes the initial atomic state, and U describes the swap gate. Without loss of generality, we suppose that the two coherent states are initially in the general form of

$$\begin{aligned}|\Psi_0\rangle =\ & [\cos\psi\mathscr{N}_+(\alpha)(|\alpha\rangle_a + |-\alpha\rangle_a)\\ & + \sin\psi\mathscr{N}_-(\alpha)(|\alpha\rangle_a - |-\alpha\rangle_a)]\\ & \otimes [\cos\varphi\mathscr{N}_+(\alpha)(|\alpha\rangle_b + |-\alpha\rangle_b)\\ & + \sin\varphi\mathscr{N}_-(\alpha)(|\alpha\rangle_b - |-\alpha\rangle_b)],\end{aligned} \quad (14)$$

where ψ, $\varphi \in [0, 2\pi)$. We plot the fidelity of the two-qubit swap gate versus the magnitude of α with different κ values in Fig. 2.

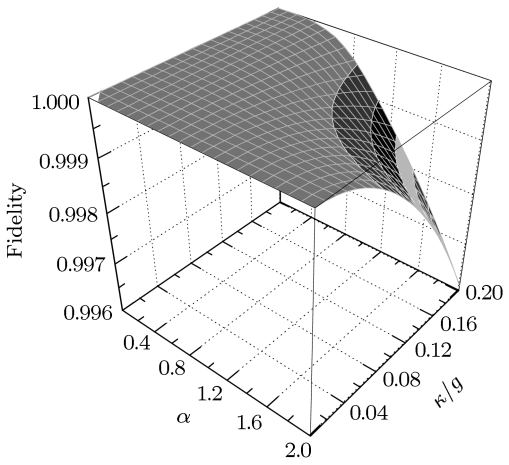


Fig. 2. Fidelity of the swap gate of the quantum qubits encoded in the coherent states versus both κ/g and α. Other common parameters are $\kappa_h = \kappa_v = \kappa$, $g_h = g_v = g$, $N = 10^4$, $\Delta = 10^3 g$.

It can be seen that the gate fidelity is not changed much as the value of α increases, but it monotonically decreases when κ/g grows, which is in good agreement with the above-mentioned analysis. It should be noted that figure 2 well demonstrates the feasibility of the present scheme, as the fidelity keeps an ultrahigh level even when κ/g reaches 0.2.

3. Swap gate for photonic qubits

In this section, we will show that this model can be used to realize a swap gate for photonic qubits, if the bimodal cavity–fields are initially prepared in the Fock states. The difference is that the two cavity modes have different detunings Δ_h and Δ_v (Δ_h and Δ_v denote h mode and v mode, respectively), i.e., $\Delta_h \neq \Delta_v$. We choose the detunings $\Delta_v - \Delta_h = \delta$, and Δ_h, $\Delta_v \gg \delta$, then the Hamiltonian in Eq. (3) can be replaced by

$$\begin{aligned}H_{\rm eff}^{1'} =\ & -\sum_{k=1}^{N}\Big[\frac{g_h^2}{\Delta}a_h^\dagger a_h|g\rangle_{kk}\langle g| + \frac{g_v^2}{\Delta}a_v^\dagger a_v|s\rangle_{kk}\langle s|\\ & + \frac{g_h g_v}{\Delta}(a_h^\dagger a_v {\rm e}^{{\rm i}\,\delta t}|g\rangle_{kk}\langle s| + {\rm H.c})\Big].\end{aligned} \quad (15)$$

On the other hand, the Hamiltonian in Eq. (4) keeps the same form. Implementing a similar procedure to that in Eqs. (5)–(9), we perform the transformation[28–31] ${\rm e}^{{\rm i}\,H_{\rm eff}^{2'}t}$ with

$$H_{\rm eff}^{2'} = \sum_{k=1}^{N}\mu(I_k + \sigma_k^z)$$

in dressed basis, the effective Hamiltonian of the whole system reduces to

$$\begin{aligned}H'_{\rm eff} =\ & -\frac{1}{2}\sum_{k=1}^{N}[\chi_h a_h^\dagger a_h(I_k + {\rm e}^{{\rm i}\,\mu t}\sigma_k^+ + {\rm e}^{-{\rm i}\,\mu t}\sigma_k^-)\\ & + \chi_v a_v^\dagger a_v(I_k - {\rm e}^{{\rm i}\,\mu t}\sigma_k^+ - {\rm e}^{-{\rm i}\,\mu t}\sigma_k^-)\\ & + \chi(a_h^\dagger a_v {\rm e}^{{\rm i}\,\delta t}(\sigma_k^z - {\rm e}^{{\rm i}\,\mu t}\sigma_k^+ + {\rm e}^{-{\rm i}\,\mu t}\sigma_k^-)\\ & + a_h a_v^\dagger {\rm e}^{-{\rm i}\,\delta t}(\sigma_k^z + {\rm e}^{{\rm i}\,\mu t}\sigma_k^+ - {\rm e}^{-{\rm i}\,\mu t}\sigma_k^-))].\end{aligned} \quad (16)$$

Under the condition $\mu \gg \chi_h$, χ_v, χ, δ, the high rapid oscillating terms in Eq. (16) can be neglected. This leads to the following effective Hamiltonian:

$$\begin{aligned}H'_{\rm eff} =\ & -\frac{1}{2}\sum_{k=1}^{N}[(\chi_h a_h^\dagger a_h + \chi_v a_v^\dagger a_v)I_k\\ & + \chi(a_h^\dagger a_v {\rm e}^{{\rm i}\,\delta t} + a_h a_v^\dagger {\rm e}^{-{\rm i}\,\delta t})\sigma_k^z].\end{aligned} \quad (17)$$

In order to further simplify the dynamics of the system, we need to take further approximations. Performing the unitary transformation $U = {\rm e}^{-{\rm i}\,At}$ with

$$A = -\frac{1}{2}\sum_{k=1}^{N}(\chi_h a_h^\dagger a_h + \chi_v a_v^\dagger a_v)I_k$$

on the second part of the above-mentioned effective Hamiltonian, i.e., $\chi(a_h^\dagger a_v {\rm e}^{{\rm i}\,\delta t} + a_h a_v^\dagger {\rm e}^{-{\rm i}\,\delta t})\sigma_k^z$, and choosing $\eta = N(\chi_h - \chi_v)/2$, we can obtain a new Hamiltonian to be

$$H'_{\rm eff} = -\frac{1}{2}\sum_{k=1}^{N}[\chi(a_h^\dagger a_v {\rm e}^{{\rm i}(\delta-\eta)t}$$

$$+ a_h a_v^{\dagger} \mathrm{e}^{-\mathrm{i}(\delta-\eta)t})\sigma_k^z]. \quad (18)$$

With the choice of parameter $\eta = \delta$, the Hamiltonian is finally simplified into

$$H'_{\rm eff} = -\frac{1}{2}\sum_{k=1}^{N}\chi(a_h^{\dagger}a_v + a_h a_v^{\dagger})\sigma_k^z. \quad (19)$$

Using such a Hamiltonian (Eq. (19)) we can realize a swap gate operation very conveniently.[25,34] For this purpose we take our initial field of bimodal cavity to be in Fock state, i.e., $|\psi(0)\rangle_{\rm total} = |+\rangle_k \otimes (|h\rangle_a|v\rangle_b)$, and the time evolution of the whole system can be easily expressed as

$$\begin{aligned}|\psi(t)\rangle_{\rm total} &= \mathrm{e}^{\mathrm{i}\,\lambda(a_h^{\dagger}a_v + a_h a_v^{\dagger})\sigma_k^z t}|\psi(0)\rangle_{\rm total} \\ &= P(t)|\psi(0)\rangle_{\rm total}, \end{aligned} \quad (20)$$

where we have defined $\lambda = N\chi/2$. The operator $P(t)$ directly acts on the two-mode Fock state of the cavity-field, this can lead to[25,34]

$$\begin{aligned}P(t)|h\rangle_a|v\rangle_b = &\frac{1}{\sqrt{h!v!}}(a_h^{\dagger}\cos\lambda t + \mathrm{i}\,a_v^{\dagger}\sin\lambda t)^h \\ &\times (a_v^{\dagger}\cos\lambda t + \mathrm{i}\,a_h^{\dagger}\sin\lambda t)^v|0\rangle_a|0\rangle_b. \end{aligned} \quad (21)$$

Supposing that $h,\ v = 0$ or 1 initially, after the interaction time $t_{\rm int} = \pi/2\lambda$ we can realize the swap gate for photonic qubits using the operator of $P(\pi/2\lambda)$ as follows:

$$\begin{aligned}&|+\rangle_k|0\rangle_a|0\rangle_b \to |+\rangle_k|0\rangle_a|0\rangle_b, \\ &|+\rangle_k|0\rangle_a|1\rangle_b \to \mathrm{i}\,|+\rangle_k|1\rangle_a|0\rangle_b, \\ &|+\rangle_k|1\rangle_a|0\rangle_b \to \mathrm{i}\,|+\rangle_k|0\rangle_a|1\rangle_b, \\ &|+\rangle_k|1\rangle_a|1\rangle_b \to -|+\rangle_k|1\rangle_a|1\rangle_b. \end{aligned} \quad (22)$$

It is worthwhile noticing that the non-standardized phase factors that can be eliminated by an appropriate setting of the phase of subsequent logic operations.[11]

It is necessary to give a brief discussion on the influence of decoherence during the gate operation process to examine how large the efficiency for our method is. We now perform an analysis similar to that in Section 2 to evaluate the fidelity of the swap gate and the photon loss, with resort to the conditional Hamiltonian.[35] This effective non-Hermitian Hamiltonian can be expressed as

$$\begin{aligned}H'_{\rm con} = &-\frac{\lambda}{2}(a_h^{\dagger}a_v + a_h a_v^{\dagger})|+\rangle\langle+| \\ &-\frac{\mathrm{i}}{2}\sum_{m=h,v}\kappa_m a_m^{\dagger}a_m. \end{aligned} \quad (23)$$

For the initial state of the system

$$|\Psi_0\rangle = \prod_{k=1}^{N}|+\rangle_k \otimes (|0\rangle_p + |1\rangle_p)(|0\rangle_b + |1\rangle_b)/2,$$

we plot the fidelity of the photonic qubit swap gate versus κ/g as shown in Fig. 3(a). From that, it can be seen that the gate fidelity monotonically drops when κ/g grows during the gate implementation. However, the fidelity remains exceeding 99.95% when κ/g is smaller than 2×10^{-1}. Due to a quantum jump figire 3(b) shows the numerical calculation $P_{\rm loss} = 1 - \langle\Psi_0|U^{+}(t)U(t)|\Psi_0\rangle$ of the photon loss after a full gate operation as a function of κ/g. The photon loss is proportional to κ/g on the assumption that $g_h = g_v$.

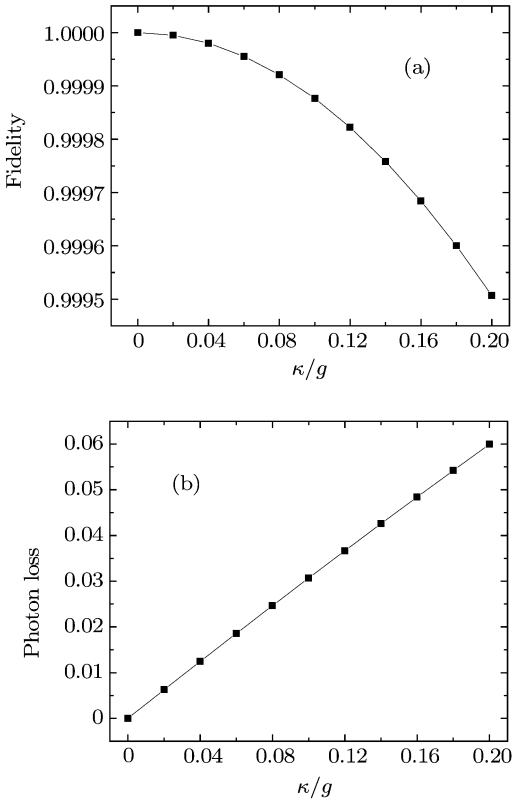


Fig. 3. Fidelity and photon loss of swap gate of the quantum qubits encoded in the Fock photonic states versus κ/g in (a) and (b), respectively. Other common parameters are $\kappa_h = \kappa_v = \kappa$, $g_h = g_v = g$, $N = 10^4$, and $\Delta = 10^3 g$.

4. Discussion and conclusion

We now give a brief discussion of the experimental feasibilities of the proposed schemes within current cavity QED. First, according to recent experiments[36,37] we can assume Cs atoms trapped in an optical cavity to meet the required level structures for our proposals. The ground states $|g\rangle$ and $|s\rangle$ may correspond to Zeeman levels of $|F = 3, m = 1\rangle$ and $|F = 3,\ m = -1\rangle$ of $6S_{1/2}$ respectively, while the excited state $|e\rangle$ is denoted by the hyperfine level $|F' = 3, m' = 0\rangle$ of $6P_{3/2}$. Because the many-atom interference effects result in the collective enhancement of the atom–cavity coupling, the presented schemes

do not require a strong-coupling regime, which relaxes the dependence of the experiment parameters (mainly in strong coupling). It is noticed that in the case of atomic ensemble, the operation speed of the above swap gate is proportional to $1/N$ (N is the number of atoms). So the gate operation process can be greatly speeded up with the large number of atoms, compared with that in the single atom scheme. This makes it possible to complete the swap gates in a reasonable time. For instance, according to the concerned experiment parameters i.e., $g/2\pi = 34$ MHz and $\kappa/2\pi = 4.1$ MHz reported in Refs. [36] and [37] choosing the number of the atoms $N = 10^4$ and the detuning $\Delta = 10^3 g$ the required atom–cavity–field interaction time is about 1.5 ns. Obviously, it is much shorter than the lifetime 0.01 s of the high-Q superconducting cavity and the lifetime of the atom 0.03 s.[10] Furthermore, the atoms have no probability to populate in the excited states and so the atomic spontaneous emission can be strongly suppressed.

In summary, we have proposed two schemes to realize two-qubit quantum swap gates only by one step with the context of cavity QED. Based on the large detuning interaction of a three-level Λ-type atom assemble with two cavity modes and two classical fields, the excited states of the atoms can be adiabatically eliminated. Thus the schemes are insensitive to atomic spontaneous emission. In the presence of a large number of atoms, our scheme may relax the requirements for the strong coupling condition due to the collective enhancement of the coupling parameter. In particular, the required atom–field interaction time decreases as the atomic number increases, which is very important in view of decoherence.

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