Exact propagator for an electron in a quadratic saddle-point potential and a magnetic field*

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We study the propagator for an electron moving in a two-dimensional (2D) quadratic saddle-point potential, in the presence of a perpendicular uniform magnetic field. A closed-form expression for the propagator is derived using the Feynmann path integrals.

Keywords: Feynmann path integrals, propagator, quadratic saddle-point potential

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1. Introduction

Owing to rapid progress in the fabrication of semiconductor devices,[1] in the past decades there has been substantial interest in the electron systems in perpendicular magnetic fields, especially the quantized Hall effect[2−6] and the quantum dots.[7−10] So it becomes very important to understand the motion of confined electrons theoretically because now many physical phenomena can be investigated experimentally.

Usually semiclassical techniques[11−16] are used to describe the motion of an electron in two dimensions, which is accurate and unambiguous when the perpendicular magnetic field is strong. In the semiclassical approximation, the electron moves along the lines of a constant potential,[11,12] but when two such lines come close, the quantum tunneling begins to play an important role. This phenomenon is very important for describing the transition between plateaus in the quantum Hall effect.[17−21] The barrier between two equipotentials is well described by a saddle-point potential.[19,20] We use the potential

$$V(x,y)=\frac{m}{2}(\omega_y^2y^2-\omega_x^2x^2)$$

with m being the electron mass and a magnetic field $\boldsymbol{B}$ being along the z axis. To our knowledge, this model was first employed by Fertig and Halperin[22] to study the tunneling process for an electron in a two-dimensional (2D) electron gas, under the influence of strong transverse magnetic field, whereby they derived an analytical expression for the transmission coefficient through the saddle point and other various approximate results.

Later, the exact quantum-mechanical propagator for the special case $\omega_y=\omega_x$ was obtained.[23,24] However, for most cases, ω_y must be not equal to ω_x. Therefore in the present paper, we extend the work of Refs. [23] and [24] to a more general case, obtaining the analytical expression for the propagator where the values of ω_y and ω_x are not necessarily the same, this is then the subject of the next section.

2. Derivation of propagator

Choosing a symmetric gauge

$$\boldsymbol{A}=\frac{1}{2}\boldsymbol{B}\times\boldsymbol{r}=\left(\frac{-By}{2},\frac{Bx}{2},0\right),$$

we have the Hamiltonian

$$H=\frac{(p_x-\frac{e}{c}A_x)^2}{2m}+\frac{(p_y-\frac{e}{c}A_y)^2}{2m}+\frac{m}{2}(\omega_y^2y^2-\omega_x^2x^2),$$

where c is the speed of light, and e is the electron charge. The Lagrangian of the corresponding classical system is

$$L=\frac{m}{2}(\dot{\boldsymbol{r}}^2-\tilde{\boldsymbol{r}}\Omega^2\boldsymbol{r}+\omega_{\rm c}\dot{\tilde{\boldsymbol{r}}}\boldsymbol{J}\boldsymbol{r}),\qquad(1)$$

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where $\omega_{\rm c} = eB/mc$ is the cyclotron frequency, $\boldsymbol{r} = \binom{x}{y}$ denotes the component of the electron position vector perpendicular to the magnetic field $\boldsymbol{B}$, $\tilde{\boldsymbol{r}}$ represents the transpose of vector $\boldsymbol{r}$, $J = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$, and $\Omega^2 = \left(\begin{smallmatrix} -\omega_x^2 & 0 \\ 0 & \omega_y^2 \end{smallmatrix}\right)$.

It is well known that a propagator for a quadratic action can be calculated exactly.[25–27] Thus we have

$$K(\boldsymbol{r}_T T|\boldsymbol{r}_0 0) = \frac{m}{2\pi\,\mathrm{i}\,\hbar(\det D(T))^{1/2}} \times \exp\left(\frac{\mathrm{i}}{\hbar} S(\boldsymbol{r}_T T|\boldsymbol{r}_0 0)\right), \quad (2)$$

where $S(\boldsymbol{r}_T T|\boldsymbol{r}_0 0)$ is the classical action and the matrix $D(T)$ is defined as $mD^{-1}(T) = -\partial^2 S/\partial \boldsymbol{r}_T \partial \boldsymbol{r}_0$. From the Lagrangian we obtain the equation of motion as

$$\ddot{\boldsymbol{r}} + \omega_{\rm c} J\dot{\boldsymbol{r}} + \Omega^2 \boldsymbol{r} = 0. \quad (3)$$

Following Ref. [28], we will seek the solution of Eq. (3) in the form $\boldsymbol{r} \sim \mathrm{e}^{Rt}$, where the matrix has the structure $\left(\begin{smallmatrix} 0 & \alpha \\ \beta & 0 \end{smallmatrix}\right)$. Substituting this into Eq. (3), we obtain

$$R^2 + \omega_{\rm c} JR + \Omega^2 = 0, \quad (4)$$

which has two solutions $R_1 = \left(\begin{smallmatrix} 0 & \lambda_1 \\ \lambda_2 & 0 \end{smallmatrix}\right)$ and $R_2 = \left(\begin{smallmatrix} 0 & \lambda_3 \\ \lambda_4 & 0 \end{smallmatrix}\right)$, where $\lambda_{1,3} = (\omega_{\rm c}/2 - \Omega_0 \mp \Delta)$, $\lambda_{2,4} = (-\omega_{\rm c}/2 - \Omega_0 \pm \Delta)$, with $\Omega_0 = (\omega_x^2 + \omega_y^2)/2\omega_{\rm c}$ and $\Delta = \sqrt{(\omega_{\rm c}^2 + \omega_y^2 - \omega_x^2)^2 + 4\omega_x^2\omega_y^2}/2\omega_{\rm c}$.

Thus, the general solution of Eq. (3) satisfying the boundary conditions $\boldsymbol{r}(0) = \boldsymbol{r}_0 = \binom{x_0}{y_0}$ and $\boldsymbol{r}(T) = \boldsymbol{r}_T = \binom{x_T}{y_T}$ is

$$\boldsymbol{r}(t) = \mathscr{A}_1 \boldsymbol{r}(0) + \mathscr{A}_2 \boldsymbol{r}(T), \quad (5)$$

where the matrix $\mathscr{A}_1 = [\mathrm{e}^{R_1 t} - A(t)A^{-1}(T)\mathrm{e}^{R_1 T}]$, and $\mathscr{A}_2 = A(t)A^{-1}(T)$, with $A(t) = \exp(R_2 t) - \exp(R_1 t)$. To find the explicit expressions of the matrix elements in Eq. (5), using the following identities:

$$\mathrm{e}^{R_1 t} = \begin{pmatrix} \cosh s_1 t & \dfrac{\lambda_1}{s_1}\sinh s_1 t \\ \dfrac{\lambda_2}{s_1}\sinh s_1 t & \cosh s_1 t \end{pmatrix},$$
$$\mathrm{e}^{R_2 t} = \begin{pmatrix} \cos s_2 t & \dfrac{\lambda_3}{s_2}\sin s_2 t \\ \dfrac{\lambda_4}{s_2}\sin s_2 t & \cos s_2 t \end{pmatrix}, \quad (6)$$

where $s_1 = \sqrt{\lambda_1\lambda_2} = \sqrt{\Omega_0^2 - (\omega_{\rm c}/2 - \Delta)^2}$ and $s_2 = \sqrt{-\lambda_3\lambda_4} = \sqrt{\Omega_0^2 - (\omega_{\rm c}/2 + \Delta)^2}$, we can rewrite the matrix $A(t)$ as

$$A(t) = \begin{pmatrix} \cos s_2 t - \cosh s_1 t & \dfrac{\lambda_3}{s_2}\sin s_2 t - \frac{\lambda_1}{s_1}\sinh s_1 t \\ \dfrac{\lambda_4}{s_2}\sin s_2 t - \frac{\lambda_2}{s_1}\sinh s_1 t & \cos s_2 t - \cosh s_1 t \end{pmatrix}, \quad (7)$$

and the inverse

$$A^{-1}(t) = \frac{1}{\Pi(t)} \begin{pmatrix} \cos s_2 t - \cosh s_1 t & \dfrac{\lambda_1}{s_1}\sinh s_1 t - \dfrac{\lambda_3}{s_2}\sin s_2 t \\ \dfrac{\lambda_2}{s_1}\sinh s_1 t - \dfrac{\lambda_4}{s_2}\sin s_2 t & \cos s_2 t - \cosh s_1 t \end{pmatrix}, \quad (8)$$

where $\Pi(t)$ is the determinant

$$\Pi(t) = \det(A(t)) = 2 - 2\cosh s_1 t \cos s_2 t + \frac{1}{s_1 s_2}(\lambda_1\lambda_4 + \lambda_2\lambda_3)\sinh s_1 t \sin s_2 t. \quad (9)$$

From Eqs. (6)–(9), we then obtain

$$\mathscr{A}_1 = \frac{1}{\Pi(T)} \begin{pmatrix} a_{11} & a_{12} \\ a_{13} & a_{14} \end{pmatrix}, \quad (10)$$

where

$$a_{11} = \left(\sinh s_1 T \cos s_2 T - \frac{\lambda_1\lambda_4}{s_1 s_2}\sin s_2 T \cosh s_1 T\right)\sinh s_1 t - \left(\sin s_2 T \cosh s_1 T + \frac{\lambda_2\lambda_3}{s_1 s_2}\sinh s_1 T \cos s_2 T\right)\sin s_2 t + \left(1 - \cosh s_1 T \cos s_2 T + \frac{\lambda_1\lambda_4}{s_1 s_2}\sinh s_1 T \sin s_2 T\right)\cosh s_1 t$$

$$+\left(1+\frac{\lambda_2\lambda_3}{s_1s_2}\sinh s_1T\sin s_2T-\cosh s_1T\cos s_2T\right)\cos s_2t, \tag{11}$$

$$\begin{aligned}a_{12} =& \left[\frac{\lambda_1}{s_1}(1-\cosh s_1T\cos s_2T)+\frac{\lambda_3}{s_2}\sinh s_1T\sin s_2T\right]\sinh s_1t\\ &+\left[\frac{\lambda_3}{s_2}(1-\cosh s_1T\cos s_2T)-\frac{\lambda_1}{s_1}\sinh s_1T\sin s_2T\right]\sin s_2t\\ &+\left(\frac{\lambda_1}{s_1}\sinh s_1T\cos s_2T-\frac{\lambda_3}{s_2}\sin s_2T\cosh s_1T\right)\cosh s_1t\\ &+\left(\frac{\lambda_3}{s_2}\sin s_2T\cosh s_1T-\frac{\lambda_1}{s_1}\sinh s_1T\cos s_2T\right)\cos s_2t,\end{aligned} \tag{12}$$

$$\begin{aligned}a_{13} =& \left[\frac{\lambda_2}{s_1}(1-\cosh s_1T\cos s_2T)+\frac{\lambda_4}{s_2}\sinh s_1T\sin s_2T\right]\sinh s_1t\\ &+\left[\frac{\lambda_4}{s_2}(1-\cosh s_1T\cos s_2T)-\frac{\lambda_2}{s_1}\sinh s_1T\sin s_2T\right]\sin s_2t\\ &+\left(\frac{\lambda_2}{s_1}\sinh s_1T\cos s_2T-\frac{\lambda_4}{s_2}\sin s_2T\cosh s_1T\right)\cosh s_1t\\ &+\left(\frac{\lambda_4}{s_2}\sin s_2T\cosh s_1T-\frac{\lambda_2}{s_1}\sinh s_1T\cos s_2T\right)\cos s_2t,\end{aligned} \tag{13}$$

$$\begin{aligned}a_{14} =& (\sinh s_1T\cos s_2T-\frac{\lambda_2\lambda_3}{s_1s_2}\sin s_2T\cosh s_1T)\sinh s_1t\\ &-\left(\frac{\lambda_1\lambda_4}{s_1s_2}\sin s_1T\cos s_2T+\sin s_2T\cos s_1T\right)\sin s_2t\\ &+\left(1-\cos s_1T\cos s_2T+\frac{\lambda_2\lambda_3}{s_1s_2}\sinh s_1T\sin s_2T\right)\cosh s_1t\\ &+\left(1+\frac{\lambda_1\lambda_4}{s_1s_2}\sinh s_1T\sin s_2T-\cosh s_1T\cos s_2T\right)\cos s_2t,\end{aligned} \tag{14}$$

and

$$\mathscr{A}_2=\frac{1}{\Pi(T)}\begin{pmatrix}a_{21} & a_{22}\\ a_{23} & a_{24}\end{pmatrix}, \tag{15}$$

where

$$\begin{aligned}a_{21} =& \left(\frac{\lambda_1\lambda_4}{s_1s_2}\sin s_2T-\sinh s_1T\right)\sinh s_1t+(\cosh s_1T-\cos s_2T)\cosh s_1t\\ &+\left(\frac{\lambda_2\lambda_3}{s_1s_2}\sinh s_1T+\sin s_2T\right)\sin s_2t+(\cos s_2T-\cosh s_1T)\cos s_2t,\end{aligned} \tag{16}$$

$$\begin{aligned}a_{22} =& \frac{\lambda_1}{s_1}(\cosh s_1T-\cos s_2T)\sinh s_1t+\left(\frac{\lambda_3}{s_2}\sin s_2T-\frac{\lambda_1}{s_1}\sinh s_1T\right)\cosh s_1t\\ &+\frac{\lambda_3}{s_2}(\cos s_2T-\cosh s_1T)\sin s_2t+\left(\frac{\lambda_1}{s_1}\sinh s_1T-\frac{\lambda_3}{s_2}\sin s_2T\right)\cos s_2t,\end{aligned} \tag{17}$$

$$\begin{aligned}a_{23} =& \frac{\lambda_2}{s_1}(\cosh s_1T-\cos s_2T)\sinh s_1t+\left(\frac{\lambda_4}{s_2}\sin s_2T-\frac{\lambda_2}{s_1}\sinh s_1T\right)\cosh s_1t\\ &+\frac{\lambda_4}{s_2}(\cos s_2T-\cosh s_1T)\sin s_2t+\left(\frac{\lambda_2}{s_1}\sinh s_1T-\frac{\lambda_4}{s_2}\sin s_2T\right)\cos s_2t,\end{aligned} \tag{18}$$

$$\begin{aligned}a_{24} =& \left(\frac{\lambda_2\lambda_3}{s_1s_2}\sin s_2T-\sinh s_1T\right)\sinh s_1t+(\cosh s_1T-\cos s_2T)\cosh s_1t\\ &+\left(\frac{\lambda_1\lambda_4}{s_1s_2}\sinh s_1T+\sin s_2T\right)\sin s_2t+(\cos s_2T-\cosh s_1T)\cos s_2t.\end{aligned} \tag{19}$$

Having obtained the matrix elements, we can also write solution Eq. (5) as

$$x(t)=\frac{\lambda_1}{s_1}a\sin s_1t+b\cos s_1t+\frac{\lambda_3}{s_2}d\sin s_2t+c\cos s_2t, \tag{20}$$

and

$$y(t) = \frac{\lambda_2}{s_1} b \sin s_1 t + a \cos s_1 t + \frac{\lambda_4}{s_2} c \sin s_2 t + d \cos s_2 t, \tag{21}$$

where the coefficients

$$\begin{aligned} a \;=\; & \frac{1}{\Pi(T)} \Bigg\{ x_0 \left(\frac{\lambda_2}{s_1} \sinh s_1 T \cos s_2 T - \frac{\lambda_4}{s_2} \sin s_2 T \cosh s_1 T \right) \\ & + y_0 \left(1 - \cos s_1 T \cos s_2 T + \frac{\lambda_2 \lambda_3}{s_1 s_2} \sinh s_1 T \sin s_2 T \right) \\ & + x_T \left(\frac{\lambda_4}{s_2} \sin s_2 T - \frac{\lambda_2}{s_1} \sinh s_1 T \right) + y_T (\cosh s_1 T - \cos s_2 T) \Bigg\}, \end{aligned} \tag{22}$$

$$\begin{aligned} b \;=\; & \frac{1}{\Pi(T)} \Bigg\{ x_0 \left(1 - \cosh s_1 T \cos s_2 T + \frac{\lambda_1 \lambda_4}{s_1 s_2} \sinh s_1 T \sin s_2 T \right) \\ & + y_0 \left(\frac{\lambda_1}{s_1} \sinh s_1 T \cos s_2 T - \frac{\lambda_3}{s_2} \sin s_2 T \cosh s_1 T \right) \\ & + x_T (\cosh s_1 T - \cos s_2 T) + y_T \left(\frac{\lambda_3}{s_2} \sin s_2 T - \frac{\lambda_1}{s_1} \sinh s_1 T \right) \Bigg\}, \end{aligned} \tag{23}$$

$$\begin{aligned} c \;=\; & \frac{1}{\Pi(T)} \Bigg\{ x_0 \left(1 + \frac{\lambda_2 \lambda_3}{s_1 s_2} \sinh s_1 T \sin s_2 T - \cosh s_1 T \cos s_2 T \right) \\ & + y_0 \left(\frac{\lambda_3}{s_2} \sin s_2 T \cosh s_1 T - \frac{\lambda_1}{s_1} \sinh s_1 T \cos s_2 T \right) \\ & + x_T (\cos s_2 T - \cosh s_1 T) + y_T \left(\frac{\lambda_1}{s_1} \sinh s_1 T - \frac{\lambda_3}{s_2} \sin s_2 T \right) \Bigg\}, \end{aligned} \tag{24}$$

and

$$\begin{aligned} d \;=\; & \frac{1}{\Pi(T)} \Bigg\{ x_0 \left(\frac{\lambda_4}{s_2} \sin s_2 T \cosh s_1 T - \frac{\lambda_2}{s_1} \sinh s_1 T \cos s_2 T \right) \\ & + y_0 \left(1 + \frac{\lambda_1 \lambda_4}{s_1 s_2} \sinh s_1 T \sin s_2 T - \cosh s_1 T \cos s_2 T \right) \\ & + x_T \left(\frac{\lambda_2}{s_1} \sinh s_1 T - \frac{\lambda_4}{s_2} \sin s_2 T \right) + y_T (\cos s_2 T - \cosh s_1 T) \Bigg\}. \end{aligned} \tag{25}$$

Now, the classical action between points $(0, \boldsymbol{r}_0)$ and $(T, \boldsymbol{r}_T)$ becomes

$$S(\boldsymbol{r}_T T | \boldsymbol{r}_0 0) \;=\; \frac{m}{2} \{ \tilde{\boldsymbol{r}}(T) \cdot \dot{\boldsymbol{r}}(T) - \tilde{\boldsymbol{r}}(0) \cdot \dot{\boldsymbol{r}}(0) \}. \tag{26}$$

Inserting expression (5) into Eq. (26), after lengthy algebraic manipulations one finally obtains

$$\begin{aligned} S(\boldsymbol{r}_T T | \boldsymbol{r}_0 0) \;=\; & \frac{m}{2\Pi(T)} \Bigg\{ (x_0^2 + x_T^2) \left[\left(\frac{\lambda_2 \lambda_3}{s_1} - s_1 \right) \sinh s_1 T \cos s_2 T + \left(s_2 + \frac{\lambda_1 \lambda_4}{s_2} \right) \sin s_2 T \cosh s_1 T \right] \\ & + (y_0^2 + y_T^2) \left[\left(\frac{\lambda_1 \lambda_4}{s_1} - s_1 \right) \sinh s_1 T \cos s_2 T + \left(\frac{\lambda_2 \lambda_3}{s_2} + s_2 \right) \sin s_2 T \cosh s_1 T \right] \\ & + (x_T y_T - x_0 y_0) \left[- 2\Omega_0 \left(2 - 2 \cosh s_1 T \cos s_2 T + \left(\frac{s_1}{s_2} - \frac{s_2}{s_1} \right) \sinh s_1 T \sin s_2 T \right) \right] \\ & + (x_0 y_T - x_T y_0) [4\Delta (\cosh s_1 T - \cos s_2 T)] \\ & + 2 x_0 x_T \left[\left(s_1 - \frac{\lambda_2 \lambda_3}{s_1} \right) \sinh s_1 T - \left(s_2 + \frac{\lambda_1 \lambda_4}{s_2} \right) \sin s_2 T \right] \\ & + 2 y_0 y_T \left[\left(s_1 - \frac{\lambda_1 \lambda_4}{s_1} \right) \sinh s_1 T - \left(s_2 + \frac{\lambda_2 \lambda_3}{s_2} \right) \sin s_2 T \right] \Bigg\}. \end{aligned} \tag{27}$$

For the determinant of the matrix $D^{-1}(T)$ we obtain

$$\det(D^{-1}(T)) = \frac{4\Delta^2}{\Pi(T)}. \tag{28}$$

Hence, combining Eqs. (3), (28), and (29) one can obtain the full expression of the propagator. Various special cases can be obtained by setting the values of the parameters. For example, one can easily check that our result will reduce into those in Refs. [23] and [24] if $\omega_y = \omega_x$. In the case $\omega_c = 0$ we obtain the propagator for an electron in the saddle point potential, which is the product of two independent one-dimensional propagators. If we replace ω_x with $\mathrm{i}\omega_x$, we obtain the propagator of an electron moving in an anisotropic quantum dot subjected to a perpendicular uniform magnetic field.

3. Discussion and conclusions

In this paper, we have presented the details of the exact evaluation of the propagator for a 2D electron in a saddle-point potential and a uniform perpendicular magnetic field by making use of path integrals. The presented result can be employed to study the behaviour of a tunneling electron and is related to the transmission coefficient through the saddle point calculated exactly in Ref. [22]. Various other applications such as obtaining the energy eigenvalues, the magnetization and the magnetic susceptibility of the system will be the topic in our future research.

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