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Linear matrix inequality approach for robust stability analysis for stochastic neural networks with time-varying delay*

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This paper studies the problem of linear matrix inequality (LMI) approach to robust stability analysis for stochastic neural networks with a time-varying delay. By developing a delay decomposition approach, the information of the delayed plant states can be taken into full consideration. Based on the new Lyapunov–Krasovskii functional, some inequality techniques and stochastic stability theory, new delay-dependent stability criteria are obtained in terms of LMIs. The proposed results prove the less conservatism, which are realized by choosing new Lyapunov matrices in the decomposed integral intervals. Finally, numerical examples are provided to demonstrate the less conservatism and effectiveness of the proposed LMI method.

Keywords: delay-dependent stability, linear matrix inequality, Lyapunov–Krasovskii functional, stochastic neural networks

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1. Introduction

In the past two decades, neural networks (NNs) have received increasing interest owing to their applications in various areas such as aerospace, defense, robotic, telecommunications, signal processing, pattern recognition, static image processing, associative memory and combinatorial optimization.[1] Time-delay often appears in many physical systems such as aircraft, chemical, and biological systems. Since the integration and communication delays are unavoidably encountered both in biological and artificial neural systems, which may result in oscillation and instability, increasing interest has been focused on the stability analysis of NNs with time-delays. Furthermore, time delay is frequently a source of oscillation, divergence, or even instability and deterioration of NNs. Generally speaking, stability results for delayed NNs can be classified into two types; that is, delay-independent stability[2] and delay-dependent stability;[3−9] the former does not include any information on the size of delay while the latter employs such information. For a delay-dependent type, much attention has been paid to reducing the conservatism of stability conditions. In general, there are two kinds of disturbances to be considered for the addressed NNs. They are parameter uncertainties and stochastic perturbations. First, uncertainties are frequently encountered in various engineering and communication systems. The characteristics of dynamic systems are significantly affected by presence of the uncertainty, the instability in extreme situation.[10] Next, in hardware implementation of NNs, stochastic disturbances are nearly inevitable owing to thermal noise in electronic devices. Due to stochastic disturbances, stability of the NNs may be affected. Recently, some results on stability of stochastic NNs with time-varying delays have been reported in Refs. [11]–[28]. In Ref. [29], the authors have discussed delay decomposition approach and tuning parameter α to derive stability results. A similar concept is followed for stability analysis of stochastic NNs. To the best of our knowledge, the robust stability criteria for stochastic NNs with uncertainties using delay decomposition method has not been investigated so far.

Motivated by the above discussion, in this paper, we investigate the problem of LMI approach to robust stability analysis for stochastic NNs with time-varying delay. By constructing a new Lyapunov–Krasovskii functional and employing some analysis techniques, sufficient conditions are derived for the considered stochastic system in terms of LMIs and are easily solved by MATLAB LMI control Toolbox. Numerical examples are given to illustrate the effectiveness

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and less conservativeness of the proposed method.

Notations Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times n$ real matrices. The superscript T denotes the transposition and the notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive-definite). The I_n is the $n\times n$ identity matrix. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. Moreover, let $(\Omega, \mathscr{F}, \mathscr{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. filtration contains all $\mathscr{P}$-null sets and is right continuous). The notation $*$ always denotes the symmetric block in one symmetric matrix. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. Problem description and preliminaries

Consider the following Hopfield NNs with time-varying delays described by

$$\begin{aligned}\dot{y}_i(t) =& -a_i(y_i(t)) + \sum_{j=1}^{n} b_{ij}^0 g_j(y_j(t)) \\ &+ \sum_{j=1}^{n} c_{ij}^1 g_j(y_j(t-\tau(t))) + I_i, \\ & i = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

or equivalently the vector form

$$\dot{y}(t) = -Ay(t) + Bg(y(t)) + Cg(y(t-\tau(t))) + I, \tag{2}$$

where $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ denotes the state vector associated with n neurons. The matrix $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$ is a diagonal matrix with positive entries $a_i > 0$. The matrices $B = (b_{ij}^0)_{n\times n}$ and $C = (c_{ij}^1)_{n\times n}$ denote connection weights, the discrete delayed connection weights of the j neuron on the i neuron respectively. $g(y) = [g_1(y_1(t)), g_2(y_2(t)), \ldots, g_n(y_n(t))]^{\mathrm{T}} \in \mathbb{R}^n$ is the activation function with $g(0) = 0$. The $I = [I_1, I_2, \ldots, I_n]$ is a constant external input.

In order to obtain our main results, the following assumptions are always made throughout this paper.

Assumption 1 The activation function $g_j(\cdot)$ $j = 1, 2, \ldots, n$, is continuous and bounded which satisfies the following inequality:

$$0 \leq \frac{g_j(s_1) - g_j(s_2)}{s_1 - s_2} \leq l_j, \quad \forall s_1, s_2 \in \mathbb{R},\ s_1 \neq s_2,$$

where $L = \mathrm{diag}(l_1, l_2, \ldots, l_n) > 0$ is a positive diagonal matrix.

Assumption 2 The time-varying delays $\tau(t)$ satisfy

$$0 \leq \tau(t) \leq \eta, \quad \dot{\tau}(t) \leq \mu,$$

where η and μ are constants.

Assuming that $y^* = (y_1^*, y_2^*, \ldots, y_n^*)^{\mathrm{T}}$ is an equilibrium point of system (2), one can derive from system (2) that transformation $x(t) = y(t) - y^*$ transforms system (2) into the following system:

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Cf(x(t-\tau(t))), \tag{3}$$

where $x(t)$ is the state vector of the transformation system, $f_j(x_j(t)) = g_j(x_j(t) + y_j^*) - g_j(y_j^*)$, with $f_j(x_j(0)) = 0$ for $i = 1, 2, \ldots, n$. Note that the function $f_j(\cdot)$, $j = 1, 2, \ldots, n$, satisfies the following condition

$$\begin{aligned} & 0 \leq \frac{f_j(x_j)}{x_j} \leq l_j, \quad f_j(0) = 0, \forall x_j \neq 0, \\ & j = 1, 2, \ldots, n. \end{aligned} \tag{4}$$

Consider the following Hopfield NNs with parameter uncertainties and stochastic perturbations as follows:

$$\begin{aligned} \mathrm{d}x(t) =& [-A(t)x(t) + B(t)f(x(t)) \\ &+ C(t)f(x(t-\tau(t)))]\mathrm{d}t \\ &+ [W_1(t)x(t) + W_2(t)x(t-\tau(t)) \\ &+ W_3(t)f(x(t)) \\ &+ W_4(t)f(x(t-\tau(t)))]\mathrm{d}w(t), \\ x(t) =& \phi(t), \forall t \in [-\eta, 0], \end{aligned} \tag{5}$$

where $w(t)$ denotes a one-dimensional Brownian motion satisfying $\mathbb{E}\{\mathrm{d}w(t)\} = 0$ and $\mathbb{E}\{\mathrm{d}w(t)^2\} = \mathrm{d}t$. The matrices $A(t) = A + \Delta A(t)$, $B(t) = B + \Delta B(t)$, $C(t) = C + \Delta C(t)$, $W_1(t) = W_1 + \Delta W_1(t)$, $W_2(t) = W_2 + \Delta W_2(t)$, $W_3(t) = W_3 + \Delta W_3(t)$ and $W_4(t) = W_4 + \Delta W_4(t)$, where $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$ has positive entries $a_i > 0$, W_1, W_2, W_3 and W_4 are connection weight matrices with appropriate dimensions. In this system, the parametric uncertainties are assumed to be of the form:

$$\begin{aligned} & [\Delta A(t) \quad \Delta B(t) \quad \Delta C(t) \quad \Delta W_1(t) \quad \Delta W_2(t) \quad \Delta W_3(t) \quad \Delta W_4(t)] \\ =& HF(t)[E_1 \quad E_2 \quad E_3 \quad E_4 \quad E_5 \quad E_6 \quad E_7], \end{aligned} \tag{6}$$

where H and E_i $(i = 1, \ldots, 7)$ are known as real constant matrices of appropriate dimensions

$$F^{\mathrm{T}}(t)F(t) \leq I. \tag{7}$$

It is assumed that all elements of $F(t)$ are Lebesque measurable. The matrices $\Delta A(t), \Delta B(t), \Delta C(t)$, $\Delta W_1(t), \Delta W_2(t), \Delta W_3(t)$ and $\Delta W_4(t)$ are said to be admissible if equations (5)–(7) hold. The $\phi(t) \in C([-\eta, 0]; \mathbb{R}^n)$ is the initial function and $f(x) = [f_1(x_1), f_2(x_2), \ldots, f_n(x_n)]^{\mathrm{T}} \in \mathbb{R}^n$ is the activation function with $f(0) = 0$.

Now, we give the following lemmas which are essential for the proof of main results.

Lemma 1 (Schur Complement) Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1^{\mathrm{T}} = \Omega_1$ and $\Omega_2^{\mathrm{T}} = \Omega_2 > 0$, then

$$\Omega_1 + \Omega_3^{\mathrm{T}} \Omega_2^{-1} \Omega_3 < 0,$$

if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^{\mathrm{T}} \\ * & -\Omega_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0.$$

Lemma 2[30] For any vector $x, y \in \mathbb{R}^n$, matrices A, P are real matrices of appropriate dimensions with $P > 0$, $F^{\mathrm{T}}(t)F(t) \leq I$ and scalar $\epsilon > 0$, the following inequalities hold:

(i) $HF(t)N + N^{\mathrm{T}}F^{\mathrm{T}}(t)D^{\mathrm{T}} \leq \epsilon^{-1}DD^{\mathrm{T}} + \epsilon N^{\mathrm{T}}N$;

(ii) if $P^{-1} - \epsilon^{-1}HH^{\mathrm{T}} > 0$, then $(A + HF(t)N)^{\mathrm{T}}P(A + HF(t)N) \leq A^{\mathrm{T}}(P^{-1} - \epsilon^{-1}HH^{\mathrm{T}})^{-1}A + \epsilon N^{\mathrm{T}}N$.

Lemma 3[31] For any $n \times n$ constant matrix $M > 0$, any scalars a and b with $a < b$ and a vector function $x(t) : [a, b] \longrightarrow \mathbb{R}^n$ such that integrations concerned are well defined, then the following inequality holds:

$$\left[\int_a^b x(s)\mathrm{d}s\right]^{\mathrm{T}} M \left[\int_a^b x(s)\mathrm{d}s\right] \leq (b-a)\left[\int_a^b x(s)^{\mathrm{T}} M x(s)\mathrm{d}s\right].$$

3. Main results

Defining two new state variables for the stochastic NNs (5),

$$\begin{aligned} y(t) = {} & [-A(t)x(t) + B(t)f(x(t)) \\ & + C(t)f(x(t-\tau(t)))], \end{aligned} \tag{8}$$

and

$$\begin{aligned} g(t) = {} & [W_1(t)x(t) + W_2(t)x(t-\tau(t)) \\ & + W_3(t)f(x(t)) + W_4(t)f(x(t-\tau(t)))], \end{aligned} \tag{9}$$

we have

$$\mathrm{d}x(t) = y(t)\mathrm{d}t + g(t)\mathrm{d}w(t). \tag{10}$$

Moreover, the following equality holds,

$$\begin{aligned} & x(t) - x(t-\tau(t)) \\ = {} & \int_{t-\tau(t)}^{t} \mathrm{d}x(s) \\ = {} & \int_{t-\tau(t)}^{t} y(s)\mathrm{d}s + \int_{t-\tau(t)}^{t} g(s)\mathrm{d}w(s). \end{aligned} \tag{11}$$

The following theorem gives the mean-square asymptotic stability results for stochastic NNs (5) without uncertainty.

Theorem 1 For given scalars η $(0 < \eta)$, α $(0 < \alpha < 1)$ and μ, the equilibrium solution of stochastic NNs (5) is globally asymptotically stable in the mean square if there exist positive-definite matrices $P = P^{\mathrm{T}} > 0, R_l = R_l^{\mathrm{T}} > 0, l = 1, 2, 3, Q_j = Q_j^{\mathrm{T}} > 0, j = 1, 2, 3, 4$, diagonal matrices $D_1 > 0$ and $D_2 > 0$ such that the following LMIs hold:

$$R_1 + (1-\mu)R_3 > 0, \quad R_2 + (1-\mu)R_3 > 0, \tag{12}$$

$$\Pi_1 = \begin{bmatrix} \Pi_{11} & \Pi_{12} & 0 & 0 & \Pi_{15} & PC & -A^{\mathrm{T}}U_1 & W_1^{\mathrm{T}}P \\ * & \Pi_{22} & \Pi_{23} & 0 & 0 & D_2L & 0 & W_2^{\mathrm{T}}P \\ * & * & \Pi_{33} & \Pi_{34} & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 - \frac{1}{(1-\alpha)\eta}R_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & Q_4 - 2D_1 & 0 & B^{\mathrm{T}}U_1 & W_3^{\mathrm{T}}P \\ * & * & * & * & * & -(1-\mu)Q_4 - 2D_2 & C^{\mathrm{T}}U_1 & W_4^{\mathrm{T}}P \\ * & * & * & * & * & * & -U_1 & 0 \\ * & * & * & * & * & * & * & -P \end{bmatrix} < 0, \tag{13}$$

$$\Pi_2 = \begin{bmatrix} \Pi_{11} & 0 & \Pi_{13}^{(1)} & 0 & \Pi_{15} & PC & -A^{\mathrm{T}}U_2 & W_1^{\mathrm{T}}P \\ * & \Pi_{22}^{(1)} & \Pi_{23}^{(1)} & \Pi_{24}^{(1)} & 0 & D_2L & 0 & W_2^{\mathrm{T}}P \\ * & * & \Pi_{33}^{(1)} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 - \frac{1}{(1-\alpha)\eta}R_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & Q_4 - 2D_1 & 0 & B^{\mathrm{T}}U_2 & W_3^{\mathrm{T}}P \\ * & * & * & * & * & -(1-\mu)Q_4 - 2D_2 & C^{\mathrm{T}}U_2 & W_4^{\mathrm{T}}P \\ * & * & * & * & * & * & -U_2 & 0 \\ * & * & * & * & * & * & * & -P \end{bmatrix} < 0, \quad (14)$$

where

$$\Pi_{11} = Q_1 + Q_3 - PA - A^{\mathrm{T}}P^{\mathrm{T}} - \frac{1}{\alpha\eta}(R_1 + (1-\mu)R_3), \quad \Pi_{12} = \frac{1}{\alpha\eta}(R_1 + (1-\mu)R_3),$$
$$\Pi_{15} = PB + D_1L, \quad \Pi_{22} = -(1-\mu)Q_3 - \frac{1}{\alpha\eta}(R_1 + (1-\mu)R_3) - \frac{1}{\alpha\eta}R_1,$$
$$\Pi_{23} = \frac{1}{\alpha\eta}R_1, \quad \Pi_{33} = -Q_1 + Q_2 - \frac{1}{\alpha\eta}R_1 - \frac{1}{(1-\alpha)\eta}R_2, \quad \Pi_{34} = \frac{1}{(1-\alpha)\eta}R_2,$$
$$\Pi_{13}^{(1)} = \frac{1}{\alpha\eta}(R_1 + (1-\mu)R_3), \quad \Pi_{22}^{(1)} = -(1-\mu)Q_3 - \frac{1}{(1-\alpha)\eta}(R_2 + (1-\mu)R_3) - \frac{1}{(1-\alpha)\eta}R_2,$$
$$\Pi_{23}^{(1)} = \frac{1}{(1-\alpha)\eta}(R_2 + (1-\mu)R_3)^{\mathrm{T}}, \quad \Pi_{24}^{(1)} = \frac{1}{(1-\alpha)\eta}R_2,$$
$$\Pi_{33}^{(1)} = -Q_1 + Q_2 - \frac{1}{\alpha\eta}(R_1 + (1-\mu)R_3) - \frac{1}{(1-\alpha)\eta}(R_2 + (1-\mu)R_3),$$
$$U_1 = (\alpha\eta)R_1 + (1-\alpha)\eta R_2 + \alpha\eta R_3, \quad U_2 = (\alpha\eta)R_1 + (1-\alpha)\eta R_2 + \eta R_3.$$

Proof Consider the Lyapunov–Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad (15)$$

where

$$\begin{aligned} V_1(t) &= x^{\mathrm{T}}(t)Px(t), \\ V_2(t) &= \int_{t-\alpha\eta}^{t} x^{\mathrm{T}}(s)Q_1x(s)\mathrm{d}s \\ &\quad + \int_{t-\eta}^{t-\alpha\eta} x^{\mathrm{T}}(s)Q_2x(s)\mathrm{d}s \\ &\quad + \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s)Q_3x(s)\mathrm{d}s \\ &\quad + \int_{t-\tau(t)}^{t} f^{\mathrm{T}}(x(s))Q_4f(x(s))\mathrm{d}s, \\ V_3(t) &= \int_{-\alpha\eta}^{0}\int_{t+\theta}^{t} y^{\mathrm{T}}(s)R_1y(s)\mathrm{d}s\mathrm{d}\theta \\ &\quad + \int_{-\eta}^{-\alpha\eta}\int_{t+\theta}^{t} y^{\mathrm{T}}(s)R_2y(s)\mathrm{d}s\mathrm{d}\theta \\ &\quad + \int_{-\tau(t)}^{0}\int_{t+\theta}^{t} y^{\mathrm{T}}(s)R_3y(s)\mathrm{d}s\mathrm{d}\theta. \end{aligned}$$

Then, it can be obtained by Ito's formula that

$$\mathrm{d}V(t) = LV(t)\mathrm{d}t + 2x^{\mathrm{T}}(t)Pg(t)\mathrm{d}w(t), \quad (16)$$

where

$$\begin{aligned} LV_1(t) &= 2x^{\mathrm{T}}(t)Py(t) + g^{\mathrm{T}}(t)Pg(t), \quad (17) \\ LV_2(t) &\le x^{\mathrm{T}}(t)Q_1x(t) - x^{\mathrm{T}}(t-\alpha\eta)Q_1x(t-\alpha\eta) \\ &\quad + x^{\mathrm{T}}(t-\alpha\eta)Q_2x(t-\alpha\eta)) \\ &\quad - x^{\mathrm{T}}(t-\eta)Q_2x(t-\eta) + x^{\mathrm{T}}(t)Q_3x(t) \\ &\quad - (1-\mu)x^{\mathrm{T}}(t-\tau(t))Q_3x(t-\tau(t)) \\ &\quad + f^{\mathrm{T}}(x(t))Q_4f(x(t)) \\ &\quad - (1-\mu)f^{\mathrm{T}}x(t-\tau(t))Q_4fx(t-\tau(t)), \quad (18) \\ LV_3(t) &\le y^{\mathrm{T}}(t)((\alpha\eta)R_1 + (1-\alpha)\eta R_2 + \tau(t)R_3)y(t) \\ &\quad - \int_{t-\alpha\eta}^{t} y^{\mathrm{T}}(s)R_1y(s)\mathrm{d}s \\ &\quad - \int_{t-\eta}^{t-\alpha\eta} y^{\mathrm{T}}(s)R_2y(s)\mathrm{d}s \\ &\quad - (1-\mu)\int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)R_3y(s)\mathrm{d}s. \quad (19) \end{aligned}$$

From condition (4), for any

$$D_1 = \mathrm{diag}\{d_{11}, d_{21}, \ldots, d_{n1}\} > 0$$

and

$$D_2 = \mathrm{diag}\{d_{12}, d_{22}, \ldots, d_{n2}\} > 0,$$

it is clear that

$$
\begin{aligned}
0 \leq & -2\sum_{j=1}^{n} d_{j1} f_j(x_j(t))[f_j(x_j(t) - l_j x_j(t)] \\
& - 2\sum_{j=1}^{n} d_{j2} f_j(x_j(t-\tau(t))) \\
& \times [f_j(x_j(t-\tau(t))) - l_j x_j(t-\tau(t))] \\
= & \, 2x^{\mathrm{T}}(t) D_1 L f(x(t)) - 2f^{\mathrm{T}}(x(t)) D_1 f(x(t)) \\
& + 2x^{\mathrm{T}}(t-\tau(t)) D_2 L f(x(t-\tau(t))) \\
& - 2f^{\mathrm{T}}(x(t-\tau(t))) D_2 f(x(t-\tau(t))). \qquad (20)
\end{aligned}
$$

Now, we estimate the upper bound of the last three terms in inequality (19) as follows:

Case 1 If $0 \leq \tau(t) \leq \alpha\eta$, we obtain

$$
\begin{aligned}
& - \int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)(R_1 + (1-\mu)R_3) y(s)\mathrm{d}s \\
& - \int_{t-\alpha\eta}^{t-\tau(t)} y^{\mathrm{T}}(s) R_1 y(s)\mathrm{d}s \\
& - \int_{t-\eta}^{t-\alpha\eta} y^{\mathrm{T}}(s) R_2 y(s)\mathrm{d}s. \qquad (21)
\end{aligned}
$$

From Ref. [28], we can obtain

$$
-1 = -(\alpha\eta)^{-1}\tau(t) - (1 - (\alpha\eta)^{-1}\tau(t))). \qquad (22)
$$

Since $R_1 + (1-\mu)R_3 > 0$, by using Eqs. (11), (22) and Lemma 3, we can obtain an upper bound of the integral term $\int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)(R_1 + (1-\mu)R_3)y(s)\mathrm{d}s$ as follows:

$$
\begin{aligned}
& - \int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)(R_1 + (1-\mu)R_3) y(s)\mathrm{d}s \\
= & -(\alpha\eta)^{-1}\tau(t) \int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)(R_1 + (1-\mu)R_3) y(s)\mathrm{d}s \\
& -(\alpha\eta)^{-1}(1 - (\alpha\eta)^{-1}\tau(t)))(\alpha\eta) \int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)(R_1 + (1-\mu)R_3) y(s)\mathrm{d}s \\
\leq & -(\alpha\eta)^{-1}\tau(t) \int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)(R_1 + (1-\mu)R_3) y(s)\mathrm{d}s \\
& -(\alpha\eta)^{-1}(1 - (\alpha\eta)^{-1}\tau(t)))\tau(t) \int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)(R_1 + (1-\mu)R_3) y(s)\mathrm{d}s \\
\leq & -(\alpha\eta)^{-1}\left[\int_{t-\tau(t)}^{t} y(s)\mathrm{d}s\right]^{\mathrm{T}} (R_1 + (1-\mu)R_3)\left[\int_{t-\tau(t)}^{t} y(s)\mathrm{d}s\right] \\
& -(\alpha\eta)^{-1}(1 - (\alpha\eta)^{-1}\tau(t))\left[\int_{t-\tau(t)}^{t} y(s)\mathrm{d}s\right]^{\mathrm{T}} (R_1 + (1-\mu)R_3)\left[\int_{t-\tau(t)}^{t} y(s)\mathrm{d}s\right] \\
\leq & -(\alpha\eta)^{-1}\left[\int_{t-\tau(t)}^{t} y(s)\mathrm{d}s\right]^{\mathrm{T}} (R_1 + (1-\mu)R_3)\left[\int_{t-\tau(t)}^{t} y(s)\mathrm{d}s\right] \\
= & -(\alpha\eta)^{-1}[x(t) - x(t-\tau(t)) - \int_{(t-\tau(t))}^{t} g(s)\mathrm{d}\omega(s)]^{\mathrm{T}} \\
& \times (R_1 + (1-\mu)R_3)[x(t) - x(t-\tau(t)) - \int_{(t-\tau(t))}^{t} g(s)\mathrm{d}\omega(s)] \\
= & -(\alpha\eta)^{-1}\Bigg([x(t) - x(t-\tau(t))]^{\mathrm{T}}(R_1 + (1-\mu)R_3)[x(t) - x(t-\tau(t))] - 2[x(t) - x(t-\tau(t))]^{\mathrm{T}}(R_1 + (1-\mu)R_3) \\
& \times \int_{(t-\tau(t))}^{t} g(s)\mathrm{d}\omega(s) + \left[\int_{(t-\tau(t))}^{t} g(s)\mathrm{d}\omega(s)\right]^{\mathrm{T}} (R_1 + (1-\mu)R_3)\left[\int_{(t-\tau(t))}^{t} g(s)\mathrm{d}\omega(s)\right]\Bigg). \qquad (23)
\end{aligned}
$$

By making use of a similar method as introduced above, an upper bound of the terms $-\int_{t-\alpha\eta}^{t-\tau(t)} y^{\mathrm{T}}(s)R_1 y(s)\mathrm{d}s$ and $-\int_{t-\alpha\eta}^{t-\eta} y^{\mathrm{T}}(s)R_2 y(s)\mathrm{d}s$ can be estimated as follows:

$$
\begin{aligned}
- \int_{t-\alpha\eta}^{t-\tau(t)} y^{\mathrm{T}}(s) R_1 y(s)\mathrm{d}s \leq & -(\alpha\eta)^{-1}\Bigg([x(t-\tau(t)) - x(t-\alpha\eta)]^{\mathrm{T}} R_1 [x(t-\tau(t)) - x(t-\alpha\eta)] \\
& -2[x(t-\tau(t)) - x(t-\alpha\eta)]^{\mathrm{T}} R_1 \\
& \times \int_{t-\alpha\eta}^{t-\tau(t)} g(s)\mathrm{d}\omega(s) + \left[\int_{t-\alpha\eta}^{t-\tau(t)} g(s)\mathrm{d}\omega(s)\right]^{\mathrm{T}} R_1 \left[\int_{t-\alpha\eta}^{t-\tau(t)} g(s)\mathrm{d}\omega(s)\right]\Bigg), \qquad (24)
\end{aligned}
$$

$$-\int_{t-\eta}^{t-\alpha\eta} y^{\mathrm{T}}(s)R_2y(s)\mathrm{d}s \le -(\eta-\alpha\eta)^{-1}\Bigg([x(t-\alpha\eta)-x(t-\eta)]^{\mathrm{T}}R_2[x(t-\alpha\eta)-x(t-\eta)] -2[x(t-\alpha\eta)-x(t-\eta)]^{\mathrm{T}}R_2\int_{t-\eta}^{t-\alpha\eta} g(s)\mathrm{d}\omega(s) +\Big[\int_{t-\eta}^{t-\alpha\eta} g(s)\mathrm{d}\omega(s)\Big]^{\mathrm{T}}R_2\Big[\int_{t-\eta}^{t-\alpha\eta} g(s)\mathrm{d}\omega(s)\Big]\Bigg). \quad (25)$$

Substituting relations (17)–(25) into Eq. (16), we have

$$\mathrm{d}V(t) \le \xi^{\mathrm{T}}(t)\Pi_1\xi(t)\mathrm{d}t + 2\delta(t)\mathrm{d}w(t), \quad (26)$$

where

$$\begin{aligned}\delta(t) =\ & x^{\mathrm{T}}(t)Pg(t) + (\alpha\eta)^{-1}[x(t)-x(t-\tau(t))]^{\mathrm{T}}(R_1+(1-\mu)R_3)\int_{(t-\tau(t))}^{t} g(s)\mathrm{d}\omega(s)\\ &+(\alpha\eta)^{-1}[x(t-\tau(t))-x(t-\alpha\eta)]^{\mathrm{T}}R_1\int_{t-\alpha\eta}^{t-\tau(t)} g(s)\mathrm{d}\omega(s)\\ &+(\eta-\alpha\eta)^{-1}[x(t-\eta)-x(t-\alpha\eta)]^{\mathrm{T}}R_2\int_{t-\alpha\eta}^{t-\eta} g(s)\mathrm{d}\omega(s).\end{aligned}$$

Taking the mathematical expectation on both sides of relation (26), there exists a positive scalar $\lambda_1 > 0$ such that

$$\mathbb{E}[\mathrm{d}V(t)] \le \mathbb{E}(\xi^{\mathrm{T}}(t)\Pi_1\xi(t)) \le -\lambda_1\mathbb{E}\|x(t)\|^2. \quad (27)$$

Case 2 If $\alpha\eta \le \tau(t) \le \eta$, we obtain

$$-\int_{t-\alpha\eta}^{t} y^{\mathrm{T}}(s)(R_1+(1-\mu)R_3)y(s)\mathrm{d}s - \int_{t-\tau(t)}^{t-\alpha\eta} y^{\mathrm{T}}(s)(R_2+(1-\mu)R_3)y(s)\mathrm{d}s - \int_{t-\eta}^{t-\tau(t)} y^{\mathrm{T}}(s)R_2y(s)\mathrm{d}s. \quad (28)$$

Noticing that $R_1+(1-\mu)R_3 > 0$, $R_2+(1-\mu)R_3 > 0$, we can obtain an upper bound of the integral term $-\int_{t-\alpha\eta}^{t} y^{\mathrm{T}}(s)(R_1+(1-\mu)R_3)y(s)\mathrm{d}s$ as

$$-\int_{t-\alpha\eta}^{t} y^{\mathrm{T}}(s)(R_1+(1-\mu)R_3)y(s)\mathrm{d}s \le -(\alpha\eta)^{-1}\Bigg([x(t)-x(t-\alpha\eta)]^{\mathrm{T}}(R_1+(1-\mu)R_3)[x(t)-x(t-\alpha\eta)] -2[x(t)-x(t-\alpha\eta)]^{\mathrm{T}}(R_1+(1-\mu)R_3)\int_{t-\alpha\eta}^{t} g(s)\mathrm{d}\omega(s) +\Big[\int_{t-\alpha\eta}^{t} g(s)\mathrm{d}\omega(s)\Big]^{\mathrm{T}}(R_1+(1-\mu)R_3)\Big[\int_{t-\alpha\eta}^{t} g(s)\mathrm{d}\omega(s)\Big]\Bigg). \quad (29)$$

By using $-1 = -(\eta-\alpha\eta)^{-1}(\tau(t)-\alpha\eta)-(1-(\eta-\alpha\eta)^{-1}(\tau(t)-\alpha\eta))$, $-1 = -(\eta-\alpha\eta)^{-1}(\eta-\tau(t))-(1-(\eta-\alpha\eta)^{-1}(\eta-\tau(t)))$ and Lemma 3, an upper bound of the terms $-\int_{t-\tau(t)}^{t-\alpha\eta} y^{\mathrm{T}}(s)(R_2+(1-\mu)R_3)y(s)\mathrm{d}s$, $-\int_{t-\eta}^{t-\tau(t)} y^{\mathrm{T}}(s)R_2y(s)\mathrm{d}s$ are obtained

$$-\int_{t-\tau(t)}^{t-\alpha\eta} y^{\mathrm{T}}(s)(R_2+(1-\mu)R_3)y(s)\mathrm{d}s \le -(\eta-\alpha\eta)^{-1}\Bigg([x(t-\alpha\eta)-x(t-\tau(t))]^{\mathrm{T}}(R_2+(1-\mu)R_3) \times[x(t-\alpha\eta)-x(t-\tau(t))] - 2[x(t-\alpha\eta)-x(t-\tau(t))]^{\mathrm{T}} \times(R_2+(1-\mu)R_3)\int_{t-\tau(t)}^{t-\alpha\eta} g(s)\mathrm{d}\omega(s) + \Big[\int_{t-\tau(t)}^{t-\alpha\eta} g(s)\mathrm{d}\omega(s)\Big]^{\mathrm{T}} \times(R_2+(1-\mu)R_3)\Big[\int_{t-\tau(t)}^{t-\alpha\eta} g(s)\mathrm{d}\omega(s)\Big]\Bigg), \quad (30)$$

$$-\int_{t-\eta}^{t-\tau(t)} y^{\mathrm{T}}(s)R_2y(s)\mathrm{d}s \le -(\eta-\alpha\eta)^{-1}\Bigg([x(t-\tau(t))-x(t-\eta)]^{\mathrm{T}}R_2[x(t-\tau(t))-x(t-\eta)]$$

$$- 2[x(t-\tau(t)) - x(t-\eta)]^{\mathrm{T}} R_2 \int_{t-\eta}^{t-\tau(t)} g(s)\mathrm{d}\omega(s) + \left[\int_{t-\eta}^{t-\tau(t)} g(s)\mathrm{d}\omega(s)\right]^{\mathrm{T}} R_2 \left[\int_{t-\eta}^{t-\tau(t)} g(s)\mathrm{d}\omega(s)\right]\Big). \tag{31}$$

Substituting relations (17)–(20) and (29)–(31) into Eq. (16), we have

$$\mathrm{d}V(t) \le \xi^{\mathrm{T}}(t)\Pi_2\xi(t)\mathrm{d}t + \zeta(t)\mathrm{d}w(t), \tag{32}$$

where

$$\begin{aligned}\zeta(t) =\ & 2x^{\mathrm{T}}(t)Pg(t) + 2(\alpha\eta)^{-1}[x(t) - x(t-\alpha\eta)]^{\mathrm{T}}(R_1 + (1-\mu)R_3)\int_{t-\alpha\eta}^{t} g(s)\mathrm{d}\omega(s)\\ & + 2(\eta-\alpha\eta)^{-1}[x(t-\alpha\eta) - x(t-\tau(t))]^{\mathrm{T}}(R_2 + (1-\mu)R_3)\int_{t-\tau(t)}^{t-\alpha\eta} g(s)\mathrm{d}\omega(s)\\ & + 2(\eta-\alpha\eta)^{-1}[x(t-\tau(t)) - x(t-\eta)]^{\mathrm{T}} R_2 \int_{t-\eta}^{t-\tau(t)} g(s)\mathrm{d}\omega(s).\end{aligned}$$

Taking the mathematical expectation on both sides of relation (32), there exists a positive scalar $\lambda_2 > 0$ satisfying

$$\mathbb{E}[\mathrm{d}V(t)] \le \mathbb{E}(\xi^{\mathrm{T}}(t)\Pi_2\xi(t)) \le -\lambda_2\mathbb{E}\|x(t)\|^2. \tag{33}$$

The Π_1 and Π_2 are defined in Theorem 1 with

$$\xi^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-\tau(t)) \quad x^{\mathrm{T}}(t-\alpha\eta) \quad x^{\mathrm{T}}(t-\eta) \quad f^{\mathrm{T}}(x(t)) \quad f^{\mathrm{T}}(x(t-\tau(t)))].$$

Thus if $\Pi_i < 0$, $(i = 1, 2)$ then the stochastic system (5) is globally asymptotically stable in the mean-square. The proof is completed.

Remark 1 In the proof of Theorem 1, the interval $[t-\eta, t]$ is divided into two subintervals $[t-\eta, t-\alpha\eta]$ and $[t-\alpha\eta, t]$, the information of delayed state $x(t-\alpha\eta)$ can be taken into account.

Remark 2 In this paper, the LMIs (12)–(14) obtained for Case 1 and Case 2 by considering integral terms such as

$$-\int_{t-\alpha\eta}^{t} y^{\mathrm{T}}(s)R_1y(s)\mathrm{d}s - \int_{t-\eta}^{t-\alpha\eta} y^{\mathrm{T}}(s)R_2y(s)\mathrm{d}s$$

and

$$-(1-\mu)\int_{t-\tau(t)}^{t} y^{\mathrm{T}}(s)R_3y(s)\mathrm{d}s$$

are different from those of Refs. [11]–[17] and this may lead to less conservatism.

Now, we will discuss the following Theorem 2 to study robust stability analysis for stochastic NNs (5).

Theorem 2 For given scalars η $(0 < \eta)$, α $(0 < \alpha < 1)$ and μ, the equilibrium solution of stochastic NNs (5) is globally robustly asymptotically stable in the mean square if there exist positive-definite matrices $P = P^{\mathrm{T}} > 0, Z_l = Z_l^{\mathrm{T}} > 0$, $l = 1, 2, 3$, $Q_j = Q_j^{\mathrm{T}} > 0$, $j = 1, 2, 3, 4$, diagonal matrices $D_1 > 0$, $D_2 > 0$ and scalars $\epsilon_i > 0$, $(i = 1, 2)$ such that the following LMIs hold:

$$R_1 + (1-\mu)R_3 > 0, \quad R_2 + (1-\mu)R_3 > 0, \tag{34}$$

$$\begin{bmatrix} \Xi_1 & \hat{P}H & \Gamma_1U_1 & 0 & \epsilon_1\Gamma_2 & \Gamma_3P & 0 & \epsilon_2\Gamma_4 \\ * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -U_1 & U_1H & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon_1 I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & -P & PH & 0 \\ * & * & * & * & * & * & -\epsilon_2 I & 0 \\ * & * & * & * & * & * & * & -\epsilon_2 I \end{bmatrix} < 0, \tag{35}$$

$$\begin{bmatrix} \Xi_2 & \hat{P}H & \Gamma_1 U_2 & 0 & \epsilon_1 \Gamma_2 & \Gamma_3 P & 0 & \epsilon_2 \Gamma_4 \\ * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -U_2 & U_2 H & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon_1 I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & -P & PH & 0 \\ * & * & * & * & * & * & -\epsilon_2 I & 0 \\ * & * & * & * & * & * & * & -\epsilon_2 I \end{bmatrix} < 0, \tag{36}$$

where

$$\Xi_1 = \begin{bmatrix} \hat{\Pi}_{11} & \Pi_{12} & 0 & 0 & \Pi_{15} & PC \\ * & \Pi_{22} & \Pi_{23} & 0 & 0 & D_2 L \\ * & * & \Pi_{33} & \Pi_{34} & 0 & 0 \\ * & * & * & -Q_2 - \frac{1}{(1-\alpha)\eta} R_2 & 0 & 0 \\ * & * & * & * & Q_4 - 2D_1 + \epsilon_1 E_2^{\mathrm{T}} E_2 & 0 \\ * & * & * & * & * & -(1-\mu)Q_4 - 2D_2 + \epsilon_1 E_3^{\mathrm{T}} E_3 \end{bmatrix},$$

$$\Xi_2 = \begin{bmatrix} \hat{\Pi}_{11} & 0 & \Pi_{13}^{(1)} & 0 & \Pi_{15} & PC \\ * & \Pi_{22}^{(1)} & \Pi_{23}^{(1)} & \Pi_{24}^{(1)} & 0 & D_2 L \\ * & * & \Pi_{33}^{(1)} & 0 & 0 & 0 \\ * & * & * & -Q_2 - \frac{1}{(1-\alpha)\eta} R_2 & 0 & 0 \\ * & * & * & * & Q_4 - 2D_1 + \epsilon_1 E_2^{\mathrm{T}} E_2 & 0 \\ * & * & * & * & * & -(1-\mu)Q_4 - 2D_2 + \epsilon_1 E_3^{\mathrm{T}} E_3 \end{bmatrix},$$

$$\hat{P} = [P \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^{\mathrm{T}}, \quad \Gamma_1 = [-A^{\mathrm{T}} \quad 0 \quad 0 \quad 0 \quad B^{\mathrm{T}} \quad C^{\mathrm{T}}]^{\mathrm{T}}, \; \Gamma_2 = [-E_1 \quad 0 \quad 0 \quad 0 \quad E_2 \quad E_3]^{\mathrm{T}},$$
$$Ga_3 = [W_1^{\mathrm{T}} \quad W_2^{\mathrm{T}} \quad 0 \quad 0 \quad W_3^{\mathrm{T}} \quad W_4^{\mathrm{T}}]^{\mathrm{T}}, \quad \Gamma_4 = [E_4 \quad E_5 \quad 0 \quad 0 \quad E_6 \quad E_7]^{\mathrm{T}}, \quad \hat{\Pi} = \Pi_{11} + \epsilon_1 E_1^{\mathrm{T}} E_1.$$

Proof Replacing A, B, C, W_1, W_2, W_3 and W_4 in LMIs (12) to (14) with $A + HF(t)E_1$, $B + HF(t)E_2, C + HF(t)E_3, W_1 + HF(t)E_4, W_2 + HF(t)E_5, W_3 + HF(t)E_6, W_4 + HF(t)E_7$ and using Lemmas 1 and 2, we obtain the LMIs (34) to (36). Therefore the proof is completed.

4. Numerical examples

In this section, we will give numerical examples showing the effectiveness of established theories.

Example 1 Consider the system (5) with the following matrices

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4 & -0.7 \\ 0.1 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.2 & 0.6 \\ 0.5 & -0.1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 0 & -0.5 \\ -0.5 & 0 \end{bmatrix}, \quad W_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$W_4 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad H = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix},$$
$$L = 0.5I, \quad E_1 = [0.2 \quad 0.3],$$
$$E_2 = [0.2 \quad -0.3], \quad E_3 = [-0.2 \quad -0.3],$$
$$E_4 = E_5 = E_6 = E_7 = [0.1 \quad 0.1],$$
$$f(x(t)) = 0.5 \tanh(x(t)).$$

For this system, when the differential of $\tau(t)$ is unknown, applying Theorem 2 in Refs. [12] and [18], we find that the equilibrium solution of the stochastic neural network (5) is robustly exponentially stable in the mean square for any delay $\tau(t)$ satisfying

$0 < \tau(t) \leq 0.5730$ and $0 < \tau(t) \leq 0.7056$. However, using Theorem 2 and taking $R_3 = Q_3 = Q_4 = 0$, we can obtain the allowable bound as $\eta = 1.3346$ for $(\alpha = 0.5)$.

Specially, when $\Delta W_1(t) = \Delta W_2(t) = W_3 = \Delta W_3(t) = W_4 = \Delta W_4(t) = 0$, the system is same as the one in Ref. [14]. It is shown in Ref. [14] that the uncertain stochastic neural network is globally stable in mean square for the maximum allowed time delay being 0.4109. In Refs. [12] and [17]–[19], the maximum allowable upper bounds are obtained as 0.6196, 0.6740, 0.7795 and 0.8269 respectively. However, using Theorem 2 and taking $R_3 = Q_3 = Q_4 = 0$, we can obtain the allowable bound as $\eta = 1.5187$ for $(\alpha = 0.5)$. The system (5) is robustly asymptotically stable in mean square. Therefore, for this example, the results given in this paper are less conservative than those in Refs. [12], [14] and [17]–[19]. The response of the state trajectories for the stochastic NNs (5) which converges to zero asymptotically in the mean square are given in Fig. 1.

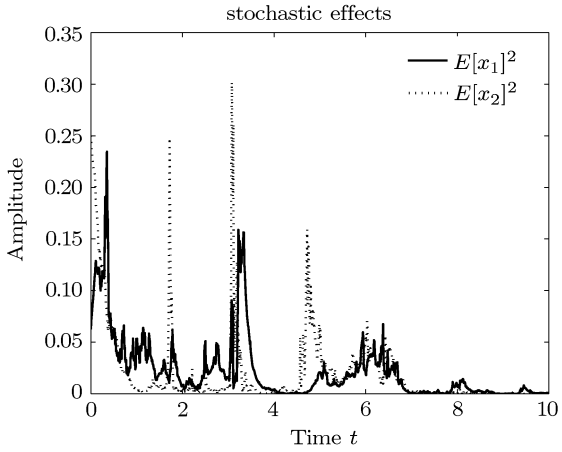


Fig. 1. State trajectories for Example 1 with $\eta = 1.3346$ and initial conditions $(-0.25, 0.5)$.

Example 2 Consider the following uncertain stochastic NNs

$$\begin{aligned} \mathrm{d}x(t) = {} & [-(A + \Delta A(t))x(t) \\ & + (C + \Delta C(t))f(x(t - \tau(t)))]\mathrm{d}t \\ & + [\Delta W_1(t)x(t) \\ & + \Delta W_2(t)x(t - \tau(t))]\mathrm{d}w(t), \end{aligned} \tag{37}$$

where

$$A = \begin{bmatrix} 2.2 & 0 & 0 \\ 0 & 2.4 & 0 \\ 0 & 0 & 2.6 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.3 & -1.8 & 0.5 \\ -1.1 & 1.6 & 1.1 \\ 0.6 & 0.4 & -0.3 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.3 \end{bmatrix},$$

$$E_1 = 0.6I, \quad E_3 = E_4 = E_5 = 0.2I.$$

It was reported in Ref. [15] that the system (37) with the above matrices is robustly asymptotically stable in mean square when $0 < h \leq 0.8$. However, using Theorem 2, taking $R_3 = Q_3 = Q_4 = 0$ and $f(x(t)) = 0.3\tanh(x(t))$, we can obtain the allowable bound as $\eta = 2.1084$ $(\alpha = 0.5)$ when $L = 0.3I$. It is found that the equilibrium solution of uncertain stochastic NNs (37) is globally robustly asymptotically stable in the mean square. In the case of

$$C = \begin{bmatrix} 0.5 & 0.6 & 0.9 \\ 1.7 & 1.9 & 1.8 \\ 1.3 & 1.5 & 1.9 \end{bmatrix},$$

it was reported in Ref. [16] that the delay-dependent stability conditions in Ref. [15] were not feasible and delay-dependent stability conditions in Ref. [16] were satisfied for $h = 0.8$. However, using Theorem 2, taking $R_3 = Q_3 = Q_4 = 0$ and $f(x(t)) = 0.2\tanh(x(t))$, we can obtain the allowable bound as $\eta = 1.9857$ $(\alpha = 0.5)$ when $L = 0.2I$. This implies that the system (37) is globally robustly asymptotically stable in the mean square. The response of the state trajectories for the stochastic NNs (37) which converges to zero asymptotically in the mean square are given in Fig. 2.

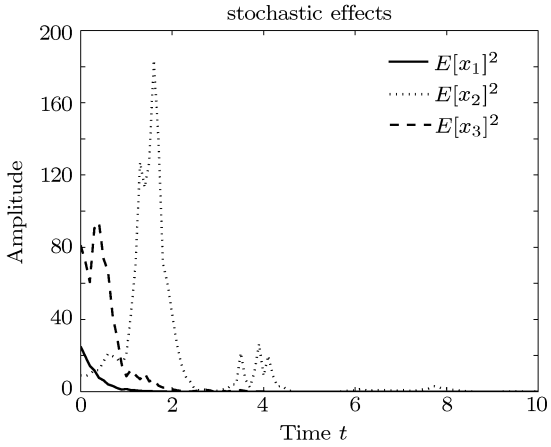


Fig. 2. State trajectories of Example 2 with $\eta = 2.1084$ and initial conditions $(-5, -3, -9)$.

Example 3 Consider the following stochastic system[24]

$$\begin{aligned} \mathrm{d}x(t) = {} & [-(A+\Delta A(t))x(t)+(B+\Delta B(t))f(x(t)) \\ & +(C+\Delta C(t))f(x(t-\tau(t)))]\mathrm{d}t \\ & +\sigma(t,x(t),x(t-\tau(t)))\mathrm{d}w(t), \end{aligned} \tag{38}$$

where

$$A=\begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 2.3 \end{bmatrix},$$

$$B=\begin{bmatrix} 0.3 & -0.19 & 0.3 \\ -0.15 & 0.2 & 0.36 \\ -0.17 & 0.29 & -0.3 \end{bmatrix},$$

$$C=\begin{bmatrix} 0.19 & -0.13 & 0.2 \\ 0.26 & 0.09 & 0.1 \\ 0.02 & -0.15 & 0.07 \end{bmatrix},$$

$W_1=W_2=0.1I$, $H=0.1I$, $E_1=E_2=E_3=I$, $f(x(t))=0.5\tanh(x(t))$.

When $K_p=\mathrm{diag}\{1,1,1\}$, $K_m=\mathrm{diag}\{-0.5,-0.5,-0.5\}$, it is proved in Refs. [24] and [28] that the system is feasible with maximum allowable upper bound $\eta=2.2471$ and for any $\eta>0$ respectively. For the case of $K_p=\mathrm{diag}\{1.2,0.5,1.3\}$, $K_m=\mathrm{diag}\{0,0,0\}$, in Refs. [24] and [28], it is found that the obtained allowable upper bounds are $9.6876, 19.9261$ if $\mu=0.85$ and $2.3879, 4.6364$ if μ is unknown respectively. However, our results, for the case of $L=0.5I$, when $\mu=0.85$ and unknown μ holds for any $\eta>0$, $(0<\alpha<1)$ and 5.0161 $(\alpha=0.2)$, which shows that Theorem 2 improves the feasible region of stability criterion. The response of the state trajectories for the stochastic NNs (38) which converges to zero asymptotically in the mean square is given in Fig. 3.

Example 4 Consider the following uncertain stochastic NNs with time-varying delays:

$$\begin{aligned} \mathrm{d}x(t) = {} & [-Ax(t)+Bf(x(t)) \\ & +Cf(x(t-\tau(t))]\mathrm{d}t \\ & +[\Delta W_1(t)x(t) \\ & +\Delta W_2(t)x(t-\tau(t))]\mathrm{d}w(t), \end{aligned} \tag{39}$$

where

$$A=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad B=\begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix},$$

$$C=\begin{bmatrix} -2 & 4 \\ 2 & -4 \end{bmatrix},\quad H=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$W_1=W_2=W_3=W_4=0$,
$E_1=E_2=E_3=E_5=E_7=0$,
$E_4=0.01I$, $E_5=0.02I$,
$f(x(t))=0.5\tanh(x(t))$.

When $L=0.5I$, the obtained delay upper bounds by Theorem 2 are listed in Table 1. It is clear that these results are significantly improved in comparison with those obtained in Refs. [12], [17], [19] and [28]. This implies that the system (39) is globally robustly asymptotically stable in the mean square. The response of the state trajectories for the stochastic NNs (39) which converges to zero asymptotically in the mean square is given in Fig. 4.

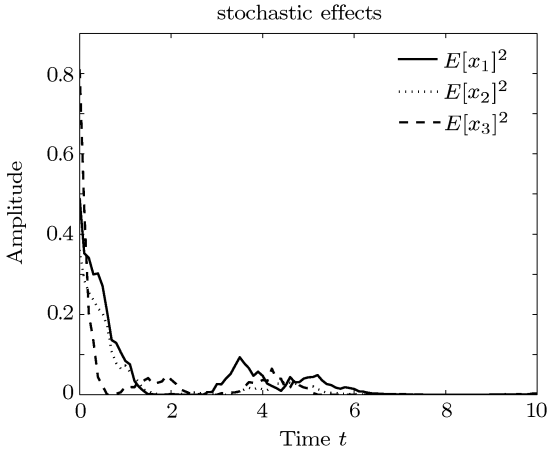


Fig. 3. State trajectories of Example 3 with $\eta=5.0161$ and initial conditions $(-0.7, 0.6, -0.9)$.

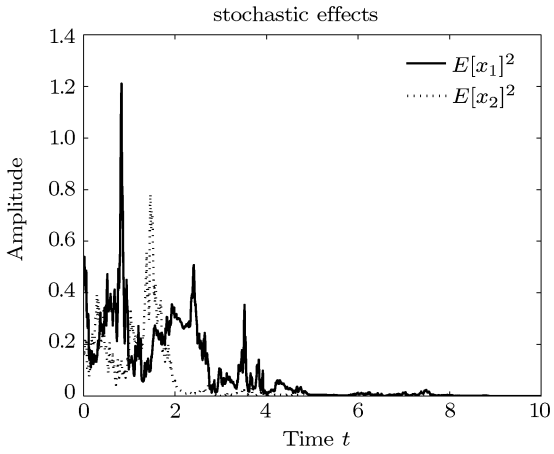


Fig. 4. State trajectories of Example 4 with $\eta=0.3851$ and initial conditions $(-0.7, 0.4)$.

Table 1. Maximum allowable upper bound of η with different μ.

	$\mu = 0.5$	$\mu = 1.1$
	η	η
Ref. [12]	0.264	–
Ref. [17]	0.264	0.195
Ref. [19]	0.273	–
Ref. [28]	0.284	–
Theorem 2	0.3851 ($\alpha = 0.4$)	0.2584 ($\alpha = 0.4$)

5. Conclusion

In this paper, we have dealt with the problem of robust stability analysis for stochastic NNs with time-varying delay. By developing a delay decomposition approach and defining Lyapunov–Krasovskii functional, we find that new LMI-term is concerned more exactly and presents less conservative results. From the numerical comparisons, significant improvements over the recent existing results have been observed.

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