Effects of σ^* and Φ mesons on the surface redshift of a neutron star*

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The effects of σ^* and Φ mesons on the surface redshift of a neutron star have been investigated within the framework of relativistic mean field theory for the baryon octet $\{n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0\}$ system. It is found that compared with those without considering the contributions of σ^* and Φ mesons, the surface redshift decreases and that corresponding to the maximum value of the mass also decreases from 0.2540 to 0.2236, about by 12%. Meanwhile, it is also found that including σ^* and Φ mesons, the M/R and that corresponding to the maximum mass decrease.

Keywords: surface redshift, relativistic mean field theory, neutron star

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1. Introduction

The surface redshift of a neutron star is related to the value of M/R and if either M or R is known from other measurements then the other quantity is obtained.[1] It is thus clear that the research on the surface redshift of a neutron star is very important. By now, much work about it has been done. In 1986, Liang[2] fixed the scope of surface redshift of a neutron star by measuring 39 gamma-ray burst pair annihilation lines in a range of 300–511 KeV.[2] Li *et al.*[3] used temperature, density and isospin dependent nuclear equation of state to calculate the surface redshift of a neutron star through resolving Tolman–Openheimer–Volkoff equation in 2001. They found that the surface redshift strongly depend on nuclear equation state.

In 1985, Glendenning[4] applied relativistic mean field theory (RMF), in which the baryon octet is considered, to the research on a neutron star. In 1994, Schaffner *et al.*[5] further suggested that the interaction between hyperons, which interacts by exchanging σ^* and ϕ mesons, should be considered.

In the present paper, the effects of σ^* and ϕ mesons on the surface redshift of a neutron star are investigated in the framework of relativistic mean field theory for the baryon octet system.

2. Relativistic mean field theory and surface redshift of a neutron star

The Lagrangian density of hadron matter is as follows:[6]

$$\begin{aligned}\mathscr{L} &= \sum_B \bar{\Psi}_B(\mathrm{i}\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho B}\gamma_\mu\tau\cdot\rho^\mu)\Psi_B + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - U_\sigma \\ &+ \sum_{\lambda=e,\mu}\bar{\Psi}_\lambda\left(\mathrm{i}\gamma_\mu\partial^\mu - m_\lambda\right)\Psi_\lambda - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu + \mathscr{L}^{\mathrm{YY}}, \qquad (1)\end{aligned}$$

where the last term on the right-hand side represents the contribution of σ^* and ϕ mesons, and for the case of coupling to hyperons it is

$$\mathscr{L}^{\mathrm{YY}} = \sum_B g_{\sigma^*B}\bar{\Psi}_B\Psi_B\sigma^* - \sum_B g_{\phi B}\bar{\Psi}_B\gamma_\mu\Psi_B\phi^\mu + \frac{1}{2}\left(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}\right) - \frac{1}{4}S_{\mu\nu}S^{\mu\nu} + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu, \qquad (2)$$

where $S_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$.

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We solve the field equations in the usual RMF approximation. The equation of baryon field is obtained as follows:

$$(\gamma_\mu k^\mu - m_B + g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* - g_{\omega B}\gamma_0\omega_0 - g_{\phi B}\gamma_0\phi_0 - g_{\rho B}\gamma_0\tau_3\rho_{03})\psi_B(k,\lambda) = 0, \tag{3}$$

and its eigenvalue is

$$e_B(k) = g_{\omega B}\omega_0 + g_{\phi B}\phi_0 + g_{\rho B}\rho_{03}I_{3B} + \sqrt{k^2 + m_B^{*2}}, \tag{4}$$

where m_B^* is the effective mass of baryons and expressed as

$$m_B^* = m_B - g_{\sigma B}\sigma - g_{\sigma^* B}\sigma^*. \tag{5}$$

The mesons field equations are given as

$$m_\sigma^2\sigma = -g_2\sigma^2 - g_3\sigma^3 + \sum_B g_{\sigma B}\rho_{sB}, \tag{6}$$

$$m_{\sigma^*}^2.\sigma^* = \sum_B g_{\sigma^* B}\rho_{sB}, \tag{7}$$

$$\omega_0 = \sum_B \frac{g_{\omega B}}{m_\omega^2}\rho_B, \tag{8}$$

$$m_\phi^2\phi_0 = 2\sum_B g_{\phi B}\rho_B, \tag{9}$$

$$m_\rho^2\rho_{03} = \sum_B g_{\rho B}I_{3B}\rho_B. \tag{10}$$

The chemical potentials for the baryons and the leptons respectively are

$$\begin{aligned}
&\mu_B = e_B(k),\\
&\mu_e = \sqrt{k_e^2 + m_e^2},\\
&\mu_\mu = \mu_e = \sqrt{k_\mu^2 + m_\mu^2}, &(11)\\
&\mu_B = \mu_n - q_B\mu_e. &(12)
\end{aligned}$$

The scalar density for baryon B and the condition of charge neutrality are given as

$$\begin{aligned}
\rho &= \sum_B \rho_B\\
&= \sum_B (2J_B+1)\, b_B k_B^3/\left(6\pi^2\right)\\
&= \text{const}, &(13)\\
Q &= \sum_B Q_B + \sum_\lambda Q_\lambda\\
&= \sum_B (2J_B+1)\, q_B k_B^3/\left(6\pi^2\right) + \sum_\lambda 2q_\lambda k_\lambda^3/(6\pi^2)\\
&= 0. &(14)
\end{aligned}$$

The energy density and the pressure of a neutron star are given as

$$\begin{aligned}
\varepsilon = {} & \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_{\sigma^*}^2\sigma^{*2}\\
&+ \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2\\
&+ \sum_{\boldsymbol{B}} \frac{2J_B+1}{2\pi^2}\int_0^{k_B} k^2\,\mathrm{d}k\sqrt{k^2 + m_B^{*2}}\\
&+ \frac{1}{3}\sum_{\lambda=e,\mu}\frac{1}{\pi^2}\int_0^{k_\lambda} k^2\sqrt{k^2 + m_\lambda^2}\;\mathrm{d}k, &(15)
\end{aligned}$$

$$\begin{aligned}
p = {} & -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - \frac{1}{2}m_{\sigma^*}^2\sigma^{*2}\\
&+ \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2\\
&+ \frac{1}{3}\sum_{\boldsymbol{B}} \frac{2J_B+1}{2\pi^2}\int_0^{k_B} \frac{k^4}{\sqrt{k^2 + m^{*2}}}\,\mathrm{d}k\\
&+ \frac{1}{3}\sum_{\lambda=e,\mu}\frac{1}{\pi^2}\int_0^{k_\lambda} \frac{k^4}{\sqrt{k^2 + m_\lambda^2}}\,\mathrm{d}k. &(16)
\end{aligned}$$

We use the Oppenheimer–Volkoff equation (O–V equation)to obtain the mass and the radius of a neutron star respectively as

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{(p+\varepsilon)\left(M + 4\pi r^3 p\right)}{r\,(r-2M)}, \tag{17}$$

$$M = 4\pi\int_0^r \varepsilon r^2\,\mathrm{d}r. \tag{18}$$

The surface redshift of a neutron star is given as[1]

$$z = \left(1 - \frac{2M}{R}\right)^{-1/2} - 1. \tag{19}$$

3. Parameters

The coupling constants of the nucleons are the GL97[1] constants listed in Table 1.

We define the following ratios: $x_\sigma = g_{\sigma h}/g_\sigma, x_\omega = g_{\omega h}/g_\omega$, and $x_\rho = g_{\rho h}/g_\rho$. For the coupling constants of meson ω, we use ratios

$$g_{\omega N}/3 = g_{\omega\Sigma}/2 = g_{\omega\Lambda}/2 = g_{\omega\Xi}, \tag{20}$$

which are given by the constituent quark model [SU(6) symmetry].[1] The coupling constants of meson σ are

then determined by fitting Λ, Σ and Ξ well depth in nuclear matter to the following expression:[7]

$$U_h^{(N)} = m_B\left(\frac{m_n^*}{m_n}-1\right)\left(\frac{g_{\sigma h}}{g_{\sigma n}}\right) + \left(\frac{g_\omega}{m_\omega}\right)^2\rho_0\left(\frac{g_{\omega h}}{g_{\omega n}}\right), \quad (21)$$

where we choose $U_\Lambda^{(N)} = U_\Sigma^{(N)} = -30$ MeV and $U_\Xi^{(N)} = -28$ MeV.[8,9]

Table 1. Coupling constants of nucleons.

	m	m_σ	m_ω	m_ρ	g_σ	g_ω	g_ρ	g_2
GL97[1]	939	500	782	770	7.9835	8.7	8.5411	20.966
	g_3	C_3	ρ_0	B/A	K	$a_{\rm sym}$	m^*/m	
GL97[1]	–9.835	0	0.153	16.3	240	32.5	0.78	

To obtain the coupling constants of the strange meson ϕ, we use the quark model relationships

$$g_{\phi\Xi} = 2g_{\phi\Lambda} = -2\sqrt{2}g_{\omega N}/3. \quad (22)$$

For the σ^* mesons, we use the mass of the obtained $f_0\,(975)$ meson, but treat its couplings purely phenomenologically so as to satisfy the equation of potential depth $U_\Lambda^{(\Xi)} \approx U_\Xi^{(\Xi)} \approx 2U_\Lambda^{(\Lambda)} \approx 40$ MeV, thereby yielding $g_{\sigma^*\Lambda}/g_{\sigma N} = g_{\sigma^*\Sigma}/g_{\sigma N} = 0.69, g_{\sigma^*\Xi}/g_{\sigma N} = 1.25$.[5]

4. Theoretical results and analyses

The surface redshift as a function of central energy density is given in Fig. 1, from which we can see that compared with those without considering the contributions of σ^* and ϕ mesons, the surface redshift decreases.

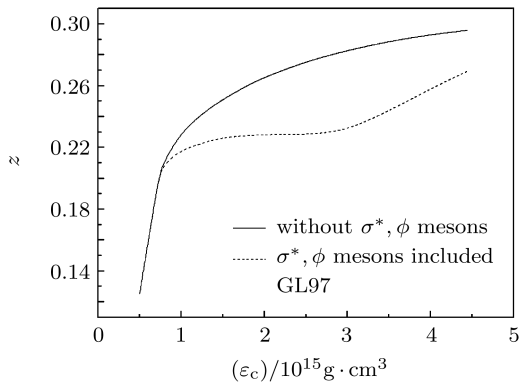


Fig. 1. Surface redshifts each as a function of central energy density.

Figure 2 shows the surface redshifts each as a function of the mass of a neutron star. The maximum values of mass $M_{\rm max}$, corresponding radius R, and gravitational redshift z are given in Table 2. From Fig. 2 and Table 2 we can see that compared with those without considering the contributions of σ^* and ϕ mesons, the maximum value of the mass decreases from 1.5211 $M_\odot$ to 1.4424 $M_\odot$, about by 5 percent and the surface redshift corresponding to the maximum value of the mass decreases from 0.2540 to 0.2236, about by 12 percent. The reason is that the appearing of σ^* and ϕ mesons makes the maximum value of neutron star smaller and the neutron star with smaller maximum mass will provide weaker gravity, leading the surface redshift to decrease. The observational data of the gravitational redshift is in a range of 0.25–0.35.[2] It is thus evident that our results are in agreement with the observational data.

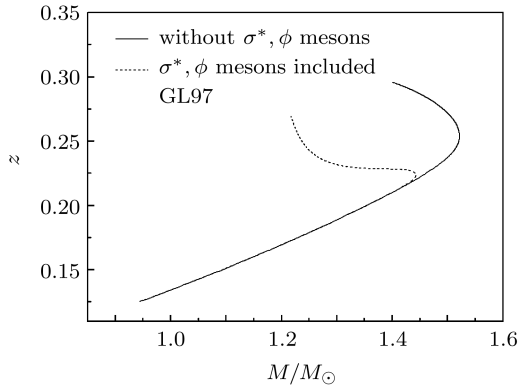


Fig. 2. Gravitational redshifts each as a function of mass.

Table 2. Maximum values of mass $M_{\rm max}$ and corresponding radius R and gravitational redshift z.

	$\varepsilon_c/(10^{15}$ g/cm$^3)$	R/km	$M_{\rm max}/M_\odot$	z
without σ^*, ϕ mesons	1.5942	12.339	1.5211	0.2540
with σ^*, ϕ mesons	1.3040	12.828	1.4424	0.2236

The surface redshifts each as a function of R is given in Fig. 3, from which we can see that the surface redshift increases with radius reducing. Compared with those without considering the contributions of σ^* and ϕ mesons, the surface redshift decreases. This is because, with the mesons σ^* and ϕ included, the mass corresponding to an identical radius R becomes smaller (seen Fig. 4) and thus the weaker gravity will provide the smaller surface redshift.

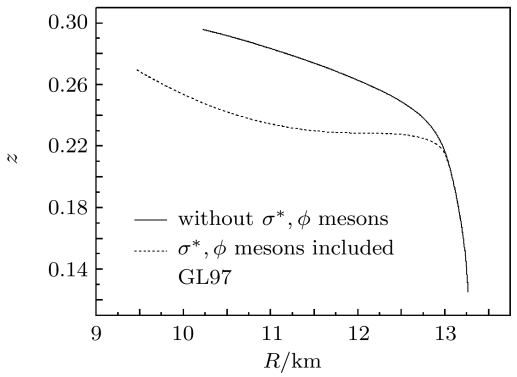


Fig. 3. Surface redshifts each as a function of radius R.

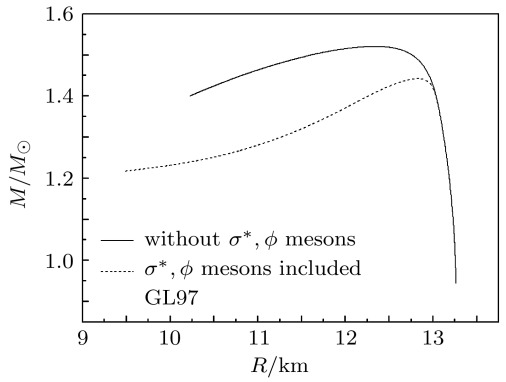


Fig. 4. Values of mass M each as a function of radius R.

Finally, the surface redshifts each as a function of M/R are shown in Fig. 5. It is found that the surface redshift increases with M/R growing. Considering the contributions of σ^* and ϕ mesons, the M/R and the corresponding z decrease. For the maximum value of the mass, the homologous surface redshift z and M/R both decrease too. From Fig. 5 we can see that if the surface redshift is observed then the quantity M/R would be known. Furthermore, if either M or R is known from other measurements then the other quantity is obtained.

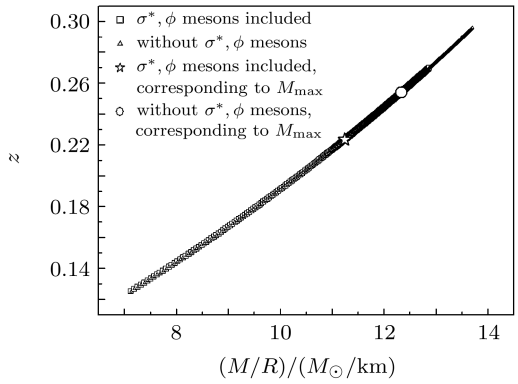


Fig. 5. Surface redshifts each as a function of M/R.

5. Summary

In this paper the effects of σ^* and Φ mesons on the surface redshift of a neutron star has been investigated within the framework of relativistic mean field theory for the baryon octet $\{n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0\}$ system. It is found that compared with those without considering the contributions of σ^* and Φ mesons, the surface redshift decreases and that corresponding to the maximum value of the mass also decreases from 0.2540 to 0.2236, about by 12%. Meanwhile, it is also found that, with σ^* and Φ mesons included, the M/R and that corresponding to the maximum mass decrease.

The strange nuclear matter is very important for both the astrophysics application and the comparison of strange quark matters. Firstly, the effects of σ^* and Φ mesons on the surface redshift of a neutron star conduce to a better understanding of the equation of state of the nuclear matter, through which the possible model of a neutron star can be determined. Secondly, the surface redshift of a neutron star has a significantly observable effect on surface electromagnetic radiation, which arouses the great interest and has been extensively studied.[10,11] Thirdly, the observational data of mass and surface redshift still may constrain the properties of the equation of state for quark matter in compact stars.[12–15]

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