Plastic analysis of the crack problem in two-dimensional decagonal Al-Ni-Co quasicrystalline materials of point group $10, \overline{10}$*

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The fundamental plastic nature of the quasicrystalline materials remains an open problem due to its essential complicacy. By developing the proposed generalized cohesive force model, the plastic deformation of crack in point group $10, \overline{10}$ decagonal quasicrystals is analysed strictly and systematically. The crack tip opening displacement (CTOD) and the size of the plastic zone around the crack tip are determined exactly. The quantity of the crack tip opening displacement can be used as a parameter of nonlinear fracture mechanics of quasicrystalline material. In addition, the present work may provide a way for the plastic analysis of quasicrystals.

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1. Introduction

Quasicrystals as a kind of new material, which possess unusual physical properties, have been studied extensively and widely.[1] Among various properties the plastic one is particularly interesting. Wollgarten *et al.*[2] were the first to research the plastic deformation of quasicrystals by a dislocation mechanism. The microstructures of deformed Al-Pd-Mn single quasicrystals were studied extensively by Rosenfeld and Feuerbacher in 1995,[3] Schall *et al.* in 1999,[4] and Geyer *et al.* in 2000.[5] A qualitative model for the plastic deformation mechanism of icosahedral quasicrystals was proposed by Feuerbacher *et al.* in 1997[6] and refined by Messerschmidt *et al.* in 1999.[7] And the first plastic deformation experiments on quasicrystals were performed by Takeuchi *et al.*[8] on coarse-grained material containing an unidentified second phase. Of course, there are several experiments on the plastic deformation of quasicrystals as indicated in Refs. [9] and [10]. Fan *et al.*[11] have studied plastic deformation of some one- and two-dimensional quasicrystals by dislocation mechanism theoretically. Fan and Fan[12] put forward the generalized Dugdale model (generalized cohesive force model) for one-dimensional hexagonal and two-dimensional decagonal point group $10mm$ quasicrystals. In this study we develop the generalized cohesive force model for decagonal point group $10, \overline{10}$ quasicrystals and present the systematic and detailed analysis procedure.

Because there are no constitutive equations of plasticity of quasicrystals, the generalized cohesive force model and other phenomenological models for plastic analysis are of fundamental significance. The basic idea of this scheme lies in the linearization of the plastic and nonlinear deformation problem. After linearization, the foundation for further solution is relevant linear elastic solutions. For this purpose we introduce the relevant linear theory solutions first, some of which are cited from Refs. [13]–[18].

2. Linear theory for decagonal quasicrystals of point group $10, \overline{10}$

We assume that the atom arrangement is periodic along the z-direction and quasiperiodic along the plane, and denote $x = x_1$, $y = x_2$, and $z = x_3$ for a two-dimensional decagonal quasicrystal. According to quasicrystal elasticity theory, the equations of deformation geometry are

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad w_{ij} = \frac{\partial w_i}{\partial x_j}. \tag{1}$$

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When the body force is neglected, the equilibrium equations are

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0, \quad \frac{\partial H_{ij}}{\partial x_j} = 0, \tag{2}$$

and the concrete generalized Hooke's law of decagonal quasicrystals of point group $10, \overline{10}$ is

$$\begin{aligned}
\sigma_{xx} &= L(\varepsilon_{xx}+\varepsilon_{yy}) + 2M\varepsilon_{xx} \\
&\quad + R_1(w_{xx}+w_{yy}) + R_2(w_{xy}-w_{yx}), \\
\sigma_{yy} &= L(\varepsilon_{xx}+\varepsilon_{yy}) + 2M\varepsilon_{yy} \\
&\quad - R_1(w_{xx}+w_{yy}) - R_2(w_{xy}-w_{yx}), \\
\sigma_{xy} &= \sigma_{yx} = 2M\varepsilon_{xy} + R_1(w_{yx}-w_{xy}) \\
&\quad + R_2(w_{xx}+w_{yy}), \\
H_{xx} &= K_1 w_{xx} + K_2 w_{yy} + R_1(\varepsilon_{xx}-\varepsilon_{yy}) + 2R_2\varepsilon_{xy}, \\
H_{yy} &= K_1 w_{yy} + K_2 w_{xx} + R_1(\varepsilon_{xx}-\varepsilon_{yy}) + 2R_2\varepsilon_{xy}, \\
H_{xy} &= K_1 w_{xy} - K_2 w_{yx} - 2R_1\varepsilon_{xy} + R_2(\varepsilon_{xx}-\varepsilon_{yy}), \\
H_{yx} &= K_1 w_{yx} - K_2 w_{xy} + 2R_1\varepsilon_{xy} \\
&\quad - R_2(\varepsilon_{xx}-\varepsilon_{yy}),
\end{aligned} \tag{3}$$

where u_i and w_i denote the phonon displacement and the phason displacement, σ_{ij} and ε_{ij} the phonon stresse and the phonon strain, H_{ij} and w_{ij} phason stress and strain and C_{ijkl}, K_{ijkl} and R_{klij} the phonon, phason and phonon–phason coupling elastic constants respectively. We assume that a plane notch penetrates through the solid along the period direction. In this case, it is evident that all the field variables are independent of z.

According to the deformation geometry equation (1), deformation compatibility equations are as follows:

$$\begin{aligned}
&\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y}, \\
&\frac{\partial w_{xx}}{\partial y} = \frac{\partial w_{xy}}{\partial x}, \quad \frac{\partial w_{yy}}{\partial x} = \frac{\partial w_{yx}}{\partial y}.
\end{aligned} \tag{4}$$

By introducing the stress potential functions $\phi(x,y)$, $\psi_1(x,y)$ and $\psi_2(x,y)$ such that

$$\begin{aligned}
&\sigma_{xx} = \frac{\partial^2 \varphi}{\partial y^2}, \quad \sigma_{yy} = \frac{\partial^2 \varphi}{\partial x^2}, \quad \sigma_{xy} = \sigma_{yx} = -\frac{\partial^2 \varphi}{\partial x \partial y}, \\
&H_{xx} = \frac{\partial \psi_1}{\partial y}, \quad H_{xy} = -\frac{\partial \psi_1}{\partial x}, \\
&H_{yx} = -\frac{\partial \psi_2}{\partial y}, \quad H_{yy} = \frac{\partial \psi_2}{\partial x}
\end{aligned} \tag{5}$$

equilibrium equation (2) will be automatically satisfied.

Based on the generalized Hooke's law (3), all strain components can be written as all stress components. Equation (5) will be satisfied when we define a new function G called the stress potential function, such that

$$\begin{aligned}
\phi &= c_1 \nabla^2 \nabla^2 G, \\
\psi_1 &= -\left(R_1 \frac{\partial}{\partial y} \Pi_1 + R_2 \frac{\partial}{\partial x} \Pi_2\right) \nabla^2 G, \\
\psi_2 &= \left(R_1 \frac{\partial}{\partial x} \Pi_2 - R_2 \frac{\partial}{\partial y} \Pi_1\right) \nabla^2 G,
\end{aligned} \tag{6}$$

in which

$$\begin{aligned}
&L = C_{12}, \quad M = C_{66}, \\
&c = M(K_1+K_2) - 2(R_1^2+R_2^2), \\
&c_1 = \frac{c}{K_1-K_2} + M.
\end{aligned}$$

And we also have

$$\nabla^2 \nabla^2 \nabla^2 \nabla^2 G = 0. \tag{7}$$

It is clear that the general solution of Eq. (7) is

$$\begin{aligned}
G &= 2\,\mathrm{Re}\Big(g_1(z) + \overline{z} g_2(z) + \frac{1}{2}\overline{z}^2 g_3(z) \\
&\quad + \frac{1}{6}\overline{z}^3 g_4(z)\Big),
\end{aligned} \tag{8}$$

where $g_i(z)$, $(i = 1, 2, 3, 4)$, are analytic functions of a single complex variable z, and $z = x + yi = r\mathrm{e}^{\mathrm{i}\,\theta}$. Substituting expression (8) into Eq. (6) then substituting the resulting equations into Eq. (5) lead to

$$\begin{aligned}
\sigma_{xx} &= -32c_1\,\mathrm{Re}(\Omega(z) - 2g_4'''(z)), \\
\sigma_{yy} &= 32c_1\,\mathrm{Re}(\Omega(z) + 2g_4'''(z)), \\
\sigma_{xy} &= \sigma_{yx} = 32c_1\,\mathrm{Im}\,\Omega(z), \\
H_{xx} &= 32R_1\,\mathrm{Re}(\Theta'(z) - \Omega(z)) \\
&\quad - 32R_2\,\mathrm{Im}(\Theta'(z) - \Omega(z)), \\
H_{xy} &= -32R_1\,\mathrm{Im}(\Theta'(z) + \Omega(z)) \\
&\quad - 32R_2\,\mathrm{Re}(\Theta'(z) + \Omega(z)), \\
H_{yx} &= -32R_1\,\mathrm{Im}(\Theta'(z) - \Omega(z)) \\
&\quad - 32R_2\,\mathrm{Re}(\Theta'(z) - \Omega(z)), \\
H_{yy} &= -32R_1\,\mathrm{Re}(\Theta'(z) + \Omega(z)) \\
&\quad + 32R_2\,\mathrm{Im}(\Theta'(z) + \Omega(z)),
\end{aligned} \tag{9}$$

in which

$$\begin{aligned}
&\Theta(z) = g_2^{(\mathrm{IV})}(z) + \overline{z} g_3^{(\mathrm{IV})}(z) + \frac{1}{2}\overline{z}^2 g_4^{(\mathrm{IV})}(z), \\
&\Omega(z) = g_3^{(\mathrm{IV})}(z) + \overline{z} g_4^{(\mathrm{IV})}(z).
\end{aligned}$$

So we will also have

$$\begin{aligned}
u_x + \mathrm{i}\,u_y &= 32(4c_1c_2 - c_3 - c_1c_4)g_4''(z) \\
&\quad - 32(c_1c_4 - c_3)(\overline{g_3'''(z)} + z\overline{g_4'''(z)}),
\end{aligned}$$

$$w_x + \mathrm{i}\, w_y = \frac{32(R_1 - \mathrm{i}\, R_2)}{K_1 - K_2}\overline{\Theta(z)}, \tag{10}$$

in which

$$c_2 = \frac{(L+M)(K_1+K_2)+c}{4c(L+M)}, \quad c_3 = \frac{(R_1^2+R_2^2)}{c},$$
$$c_4 = \frac{(K_1+K_2)}{c}.$$

For convenience, we introduce new symbols

$$g_2^{(\mathrm{IV})}(z) = h_2(z), \quad g_3'''(z) = h_3(z),$$
$$g_4''(z) = h_4(z). \tag{11}$$

3. Plastic analysis of generalized cohesive force model of quasicrystals

Now we probe into the generalized cohesive force model of quasicrystals, which belongs to nonlinear problem mathematically and physically due to the irreversible deformation. There is a Griffith crack with a length of $2(l+b)$ along the z axis in the generalized cohesive force model of quasicrystals(see Fig. 1).

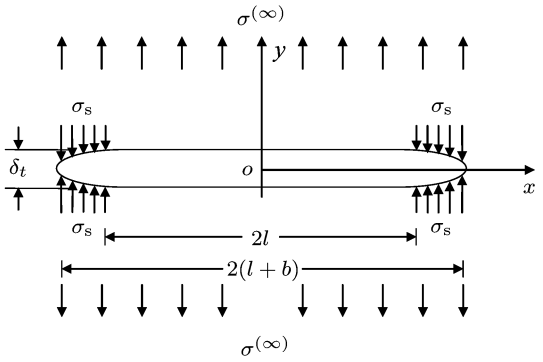


Fig. 1. Generalized cohesive force model of quasicrystal.

We have assumed already that the stress distribution within the plastic zone is known. So the generalized cohesive force model of quasicrystals can be expressed as follows:

$$\begin{cases} \sigma_{yy} = \sigma^{(\infty)},\ H_{yy} = 0,\ \sigma_{xx} = \sigma_{xy} = 0,\ H_{xx} = H_{xy} = 0, & \sqrt{x^2+y^2} \to \infty, \\ \sigma_{yy} = \sigma_{xy} = 0,\ H_{yy} = H_{xy} = 0, & y = 0,\ \ |x| < l, \\ \sigma_{yy} = \sigma_{\mathrm{s}},\ \sigma_{xy} = 0,\ H_{yy} = 0,\ H_{xy} = 0, & y = 0,\ \ l < |x| < l+b. \end{cases} \tag{12}$$

This problem can be reduced into a superposition of the following two problems:

$$\begin{cases} \sigma_{xx} = \sigma_{xy} = \sigma_{yy} = 0,\ H_{xx} = H_{xy} = H_{yy} = 0, & \sqrt{x^2+y^2} \to \infty, \\ \sigma_{yy} = \sigma_{xy} = 0, \quad H_{xy} = H_{yy} = 0, & y = 0,\ \ |x| < l, \\ \sigma_{yy} = \sigma_{\mathrm{s}},\ \sigma_{xy} = 0, \quad H_{xy} = H_{yy} = 0, & l < |x| < l+b, \end{cases} \tag{12a}$$

and

$$\begin{cases} \sigma_{yy} = \sigma^{(\infty)}, \quad \sigma_{xx} = \sigma_{xy} = 0, H_{xx} = H_{xy} = 0, & \sqrt{x^2+y^2} \to \infty, \\ \sigma_{yy} = \sigma_{xy} = 0,\ H_{yy} = H_{xy} = 0, & y = 0,\ \ |x| < l+b. \end{cases} \tag{12b}$$

So the nonlinear problem is reduced into an 'equivalent' elasticity problem. However, the calculation cannot be completed in the z-plane due to the complicacy of the evaluation, and we use the conformal mapping

$$z = \omega(\zeta) = \frac{(l+b)}{2}\left(\zeta + \frac{1}{\zeta}\right) \tag{13}$$

to transform the exterior of the crack in the z-plane into the exterior or interior of the unit circle in the ζ-plane. It is worth noting that $\widetilde{h}_2(\zeta)$ and $h_2(\zeta)$ will no longer be referred in the following text. It is clear that the boundary condition can be written as follows:

$$f_4(\zeta) + \overline{f_3(\zeta)} + \frac{\omega(\zeta)}{\overline{\omega'(\zeta)}}\overline{f_4'(\zeta)} = \frac{\mathrm{i}}{32c_1}\int (T_x + \mathrm{i}\, T_y)\mathrm{d}s\ , \tag{14}$$

in which $f_4(\zeta)$ denotes $h_4(\zeta)$ or $\widetilde{h}_4(\zeta)$, and $f_3(\zeta)$ represents $h_3(\zeta)$ or $\widetilde{h}_3(\zeta)$, T_x and T_y are the generalized surface tractions in the x-direction and the y-direction respectively. In the above analysis process, the phason filed can be discussed similarly, so it is omitted here.

For problem (12a), we have the solution

$$\begin{cases} h_4(\zeta) = \dfrac{1}{32c_1}\cdot\dfrac{\sigma_{\rm s}(l+b)\varphi_2}{\pi}\cdot\dfrac{1}{\zeta} - \dfrac{1}{32c_1}\cdot\dfrac{\sigma_{\rm s}}{2\pi i}\left[z\left(\ln\dfrac{\sigma_2-\zeta}{\overline{\sigma_2}-\zeta}+\ln\dfrac{\sigma_2+\zeta}{\overline{\sigma_2}+\zeta}\right) - l\ln\dfrac{(\zeta-\sigma_2)(\zeta+\overline{\sigma_2})}{(\zeta+\sigma_2)(\zeta-\overline{\sigma_2})}\right], \\ h_3(\zeta) = \dfrac{1}{32c_1}\cdot\dfrac{\sigma_{\rm s}(l+b)\varphi_2}{\pi}\cdot\dfrac{2\zeta}{\zeta^2-1} - \dfrac{1}{32c_1}\cdot\dfrac{\sigma_{\rm s}l}{2\pi\,{\rm i}}\ln\dfrac{(\zeta-\sigma_2)(\zeta+\overline{\sigma_2})}{(\zeta+\sigma_2)(\zeta-\overline{\sigma_2})}, \end{cases} \tag{15}$$

and for problem (12b), the solution is

$$\begin{cases} \widetilde{h}_4(\zeta) = -\dfrac{1}{32c_1}\dfrac{\sigma^{(\infty)}}{2}(l+b)\dfrac{1}{\zeta}, \\ \widetilde{h}_3(\zeta) = -\dfrac{\sigma^{(\infty)}}{32c_1}(l+b)\left[\dfrac{\zeta}{(\zeta^2-1)}\right], \end{cases} \tag{16}$$

in which $\sigma_2 = {\rm e}^{{\rm i}\,\varphi_2}$, $l = (l+b)\cos\varphi_2$, and we must point out that $\sigma_2 = {\rm e}^{{\rm i}\,\varphi_2}$ corresponds to $z = l$ in the z-plane.

So we have the complex potentials of the generalized cohesive force model of quasicrystals as follows:

$$\begin{aligned} \widehat{h}_4(\zeta) = h_4(\zeta) + \widetilde{h}_4(\zeta) =\ & \frac{(l+b)}{32c_1}\cdot\left(\frac{\sigma_{\rm s}\varphi_2}{\pi}-\frac{\sigma^{(\infty)}}{2}\right)\frac{1}{\zeta} \\ & -\frac{1}{32c_1}\cdot\frac{\sigma_{\rm s}}{2\pi\,{\rm i}}\left[z\left(\ln\frac{\sigma_2-\zeta}{\overline{\sigma_2}-\zeta}+\ln\frac{\zeta+\sigma_2}{\zeta+\overline{\sigma_2}}\right) - l\ln\frac{(\zeta-\sigma_2)(\zeta+\overline{\sigma_2})}{(\zeta+\sigma_2)(\zeta-\overline{\sigma_2})}\right], \\ \widehat{h}_3(\zeta) = h_3(\zeta) + \widetilde{h}_3(\zeta) =\ & \frac{(l+b)}{32c_1}\cdot\left(\frac{2\sigma_{\rm s}\varphi_2}{\pi}-\sigma^{(\infty)}\right)\frac{2\zeta}{\zeta^2-1} - \frac{1}{32c_1}\cdot\frac{\sigma_{\rm s}l}{2\pi\,{\rm i}}\ln\frac{(\zeta-\sigma_2)(\zeta+\overline{\sigma_2})}{(\zeta+\sigma_2)(\zeta-\overline{\sigma_2})}. \end{aligned} \tag{17}$$

Because the stresses are finite values in the tip of the plastic zone, we have

$$\frac{(l+b)}{32c_1}\cdot\left(\frac{\sigma_{\rm s}\varphi_2}{\pi}-\frac{\sigma^{(\infty)}}{2}\right)\frac{1}{\zeta} = 0, \tag{18}$$

and we obtain

$$\varphi_2 = \frac{\pi}{2}\frac{\sigma^{(\infty)}}{\sigma_{\rm s}}. \tag{19}$$

At the same time, noting that $l = (l+b)\cos\varphi_2$, one can have the size of plastic zone

$$b = l\left[\sec\left(\frac{\pi}{2}\frac{\sigma^{(\infty)}}{\sigma_{\rm s}}\right)-1\right]. \tag{20}$$

Now we calculate the crack tip opening displacement δ_t (CTOD). According to solutions (15) and (16) and displacement formula (10), after a lengthy calculation, we have

$$u_y(x,0) = (128c_1c_2 - 64c_3)\,{\rm Im}(\widehat{h}_4(\zeta)). \tag{21}$$

Substituting Eq. (17) into Eq. (21) and noting Eq. (18), one can obtain

$$\begin{aligned} u_y(x,0) =\ & \frac{(4c_1c_2-2c_3)}{c_1}\cdot\frac{\sigma_{\rm s}(l+b)}{2\pi} \\ & \cdot\left[\cos\varphi\ln\frac{\sin(\varphi_2-\varphi)}{\sin(\varphi_2+\varphi)} - \cos\varphi_2\ln\frac{(\sin\varphi_2-\sin\varphi)}{(\sin\varphi_2+\sin\varphi)}\right]. \end{aligned} \tag{22}$$

So we have

$$\begin{aligned} \delta_t =\ & {\rm CTOD} = \lim_{x\to l}2u_y(x,0) = \lim_{\varphi\to\varphi_2}2u_y(x,0) \\ =\ & \frac{(8c_1c_2-4c_3)\sigma_{\rm s}l}{c_1\pi}\ln\sec\left(\frac{\pi}{2}\frac{\sigma^{(\infty)}}{\sigma_{\rm s}}\right). \end{aligned} \tag{23}$$

Assume that $R_1 = R$ and $R_2 = 0$ in Eq. (23), then δ_t will be the corresponding solution of point group $10mm$ quasicrystals

$$\begin{aligned} \delta_t = {\rm CTOD} =\ & \frac{2\sigma_{\rm s}l}{\pi}\left[\frac{1}{L+M}+\frac{K_1}{MK_1-R^2}\right] \\ & \times\ln\sec\left(\frac{\pi}{2}\frac{\sigma^{(\infty)}}{\sigma_{\rm s}}\right). \end{aligned} \tag{24}$$

Let the phonon–phason coupling constant $R = o(K_1)$, $K_1 \to 0$, then one will have the crack tip opening displacement for conventional crystal, such as

$$\delta_t = {\rm CTOD} = \frac{2\sigma_{\rm s}l}{\pi}\frac{(L+2M)}{(L+M)M}\ln\sec\left(\frac{\pi}{2}\frac{\sigma^{(\infty)}}{\sigma_{\rm s}}\right), \tag{25}$$

in which $L = C_{12}$ and $M = C_{66}$. If the crystals are of isotropic solid, there are $L = \lambda$, and $M = \mu$, which are Lame constants, i.e.

$$\delta_t = \text{CTOD} = \begin{cases} \dfrac{(1+\kappa)\sigma_s l}{\pi\mu} \ln\sec\left(\dfrac{\pi}{2}\dfrac{\sigma^{(\infty)}}{\sigma_s}\right) = \dfrac{4(1-\nu)\sigma_s l}{\pi\mu} \ln\sec\left(\dfrac{\pi}{2}\dfrac{\sigma^{(\infty)}}{\sigma_s}\right), & \text{plane strain state} \\ \dfrac{(1+\kappa')\sigma_s l}{\pi\mu} \ln\sec\left(\dfrac{\pi}{2}\dfrac{\sigma^{(\infty)}}{\sigma_s}\right) = \dfrac{4\sigma_s l}{(1+\nu)\pi\mu} \ln\sec\left(\dfrac{\pi}{2}\dfrac{\sigma^{(\infty)}}{\sigma_s}\right), & \text{plane stress state} \end{cases} \tag{26}$$

in which $\kappa = 3 - 4\nu$ is for plane strain, and $\kappa' = 3 - \nu/1 + \nu$ is for plane stress, with ν being the poisson ratio of the isotropic solid, which is exactly reduced into the well-known classic dugdale solution in nonlinear fracture mechanics of conventional structural (engineering) materials.

4. Numerical example

For the relevant parameters of an Al–Ni–Co decagonal quasicrystal, elastic moduli are taken as $C_{11} = 2.3430$, $C_{12} = 0.5741 \times 10^7$ N/cm^2 which are obtained by resonant ultrasound spectroscopy,[19] we have also chosen $K_1 = 1.22$ and $K_2 = 0.24 \times 10^7$ N/cm^2 estimated by Monte–Carlo simulation.[20] Because the coupling constants R_1, R_2 have been measured, we take the latest results $R_1 = -1.1$, $R_2 = 0.2$ GPa as coupling case corresponding to quasicrystals.[21] For a crystal, we take $\lambda = 52.1$ and $\mu = 26.8$ GPa as the corresponding constants, which are the Lame constants of metal aluminum. And we adopt $\sigma_s = \sigma_c = 400$ MPa for quasicrystal and $\sigma_s = \sigma_c = 585$ MPa for crystal and also adopt dimensionless factors δ_t/l and $\sigma^{(\infty)}/\sigma_s$. The normalized crack tip opening displacements of point group $10, \overline{10}$ decagonal Al–Ni–Co quasicrystal versus normalized stress and its comparison with those of point group $10mm$ decagonal quasicrystal and conventional crystal are shown in Fig. 2.

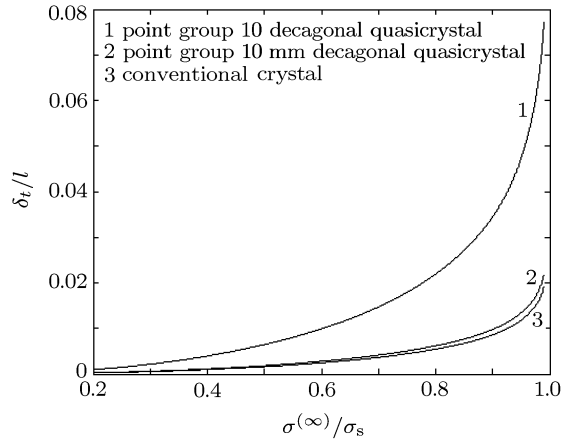


Fig. 2. Normalized crack tip opening displacements versus normalized applied stress and their comparisons.

Because the present solution can be exactly reduced into that of point group $10mm$ decagonal quasicrystal and conventional crystal given above, the figure reveals this character excellently. The results show that the effects of phason variables on the plastic deformation are evident, and indicate that the phason modes constrain the plastic deformation of quasicrystals, in addition, the phonon–phason coupling has also the constrain effect on the plastic deformation.

5. Conclusions and discussion

Quasicrystal as a brittle material in low and normal temperature ranges can present typical plastic behaviour in high temperature, and this phenomenon attracts a great deal of attention of researchers. In addition, a high stress concentration may appear near a crack tip in the material, and the stress level can go beyond the plastic limit σ_c (or τ_c) of the material in the high stress zone, thereby also leading to plastic flow. The assumption of infinite sharp crack leads to the infinity of stress concentration, and the stress state does not in fact exist, because the plastic deformation happens near the crack tip. The physical essence can remove the paradox. The phenomenological model-cohesive force model presents double sense, which removes the paradox as well as realizing the linearization of the nonlinear problem due to the plasticity, and provides a feasible solution scheme. Application of complex function and conformal mapping makes the scheme an excellent success. We obtain the crack tip opening displacement (CTOD) according to this method. A more intriguing point is that the plastic zone of the generalized cohesive force model of quasicrystals is obtained explicitly and perfectly. The above results show that they are useful for the next work which is related to the plasticity of quasicrystals. So the phenomenological methodology can provide a new insight into the study of the nonlinear theory of quasicrystals. From the results, we can see that the role of the phason modes in plastic deformation and fracture is the key to analyzing material, even though the constitutive law for nonlinear behaviour has not been established. The results of Eqs. (20) and (23) may be used as elasto-plastic fracture parameters for the material in the case of medium and large scale plastic deformations.

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