Numerical investigation of slow solitons in Bragg gratings with a hyperbolic tangent apodization*

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This paper numerically and analytically investigates the formation and propagation motion of optical soliton in the Bragg grating. We choose the fibre Bragg grating with hyperbolic tangent apodization in the middle section in order to obtain slower solitons. Optical fibre soliton but not Bragg grating soliton is used as input pulse in the discussion, which is much more approximate to the light source for the practical purpose. We discuss in detail the effects of the soliton's velocity with some parameters in the process of transmission. The results show that by choosing special parameters, one can make the soliton slow-down with a little distortion and energy decay and obtain tunable time-delay on a small scale.

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1. Introduction

Recently, there has been a lot of interest in slow light which relies on their potential applications for high speed optical information processing, such as random access memory and jitter correlation.[1,2] Many reports have demonstrated that optical pulses have delayed in some kinds of physical systems, such as the Mach–Zehnder intensity modulator,[3] photonic crystal waveguide[4] and optical fibres.[5] Because it is in the large bandwidth communication system, it has a requirement to responding the short pulse quickly, so we must make the pulse have some adjustable time-delay and little distortion. In this paper, we investigate a slow light system, in which the input pulse transmitted through the fibre Bragg grating (FBG) as solitons, which can avoid the distortion.

In 1985, Winful pointed out that the Bragg grating (BG) solitons can exist in the FBG.[6] In 1996, Eggleton *et al.* first observed the BG solitons in the experiment.[7] The BG solitons propagation in FBGs at 50%–75% of vacuum light speed c in the experiments was achieved. Theoretically the BG solitons can be slowed down and even be captured as a standing Bragg soliton.[8] In 2004, William *et al.* reported that the BG solitons can be captured by an apodized fibre grating[9] which is analysed by the nonlinear coupled mode theory. When the frequency of the input pulse is near the band-gap edge of the FBG, a 0.68 ns pulse transmitted through a 10 cm FBG can cause a time delay of about 2.4 pulse widths.[10]

In previous reports, for the input pulse propagated through the gratings, the influence of the δ (the detuning of the pulse from the BG's frequency) and Γ (the nonlinear coefficient of the fibre) is overlooked. Moreover, because the solutions non-linear coupled mode equations (NLCMEs) have many strict limits, they cannot be conveniently obtained in the experiment. The optical fibre soliton is the solution of the non-linear Schrödinger equation (NLSE), which is in the form of the Secant function and much more similar to the output pulse shape of mode-locked laser than the solution from the NLCMEs. If the NLSE is established in some approximations,[11−14] it can be conveniently used to describe the input pulse (the optical soliton) which will propagate through the gratings. In this paper, the optical fibre soliton based on NLSE instead of the BG soliton is used to describe the input pulse. The fibre grating is apodized with hyperbolic tangent function in the middle section. When the soliton is transmitted in this apodized FBG, the time-displacements of solitons are obtained, and the stabilities of slow solitons on the system's parameter are analysed. Furthermore, it is shown that the velocity of the soliton can be controlled by changing the

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parameters, such as, the apodized function κ, the detuning δ and the nonlinear coefficient Γ. The results have some important potential applications on the optical buffer device.

2. Theoretical model

For a uniform grating, the nonlinear refractive index $\widetilde{n}$ is given by

$$\widetilde{n}(\omega, x) = \overline{n}(\omega) + \overline{\delta n_{\mathrm{g}}(x)} + n_2 |E|^2, \tag{1}$$

where $\overline{n}$ is the effective refractive index, considering the sum of the index of core n_{core} and index DC modulation n_{dc}, $\overline{\delta n_{\mathrm{g}}}$ is the AC amplitude of the refractive index modulation of the grating, n_2 is the nonlinear refractive index, E is the electric field in a waveguide, which is the sum of the electric fields of the forward wave and the backward wave,

$$\begin{aligned} E(x,t) = {} & [U(x,t)\exp(\mathrm{i}\,kx) + V(x,t)\exp(-\mathrm{i}\,kx)] \\ & \times \exp(-\mathrm{i}\,\omega t) + \text{c.c.}, \end{aligned} \tag{2}$$

where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength in the FBG, ω is the frequency in the grating, and c.c. means the complex conjugate, U and V mean the envelopes of the forward and backward propagating modes, respectively, x means the axial direction along the BG and t is the time.

The most accurate description of pulse propagation through a grating is the NLCMEs. Based on a system of coupled equations for the local amplitudes of the forward and backward travelling waves, we can write the NLCMEs as[9]

$$\begin{aligned} & \mathrm{i}\frac{\partial U}{\partial x} + \mathrm{i}\frac{\overline{n}}{c}\frac{\partial U}{\partial t} + \delta U + \kappa V \\ & + \Gamma\left(|U|^2 + 2|V|^2\right)U = 0, \\ & -\mathrm{i}\frac{\partial V}{\partial x} + \mathrm{i}\frac{\overline{n}}{c}\frac{\partial V}{\partial t} + \delta V + \kappa U \\ & + \Gamma\left(|V|^2 + 2|U|^2\right)V = 0, \end{aligned} \tag{3}$$

where $c = 3.0 \times 10^8$ m/s means the vacuum light velocity,

$$\delta = \frac{\overline{n}}{c}(\omega - \omega_{\mathrm{B}}) = 2\pi\overline{n}\left(\frac{1}{\lambda} - \frac{1}{\lambda_{\mathrm{B}}}\right)$$

is the frequency detuning, λ_{B} is the Bragg wavelength, ω_{B} is the associated Bragg frequency of the grating, $\kappa = \pi\eta\overline{\delta n_{\mathrm{g}}}/\lambda$ is the coupling coefficient of the fibre grating, $\eta \approx 0.8$ means the fraction of the power in the core, $\Gamma = 4\pi\overline{n}n_2/\lambda Z_0$ is the nonlinear coefficient of the fibre, and Z_0 is the vacuum impedance. The fifth and the sixth terms in Eq. (3) correspond to the self-phase and cross-phase modulation.

Consider that a pulse tuned close to the grating's Bragg frequency is propagated through the apodized fibre grating. At the input edge of fibre grating, V can be ignored because U is much larger than V. The $\varepsilon = U$ means the electric field, $v_{\mathrm{G}} = c/\overline{n} \approx 2.07 \times 10^8$ m/s means the group velocity in the absence of the grating, and the NLCMEs can be written as the expression of the NLSE[10−12]

$$\mathrm{i}\frac{\partial \varepsilon}{\partial x} - \frac{\beta_2}{2}\frac{\partial^2 \varepsilon}{\partial t^2} + \overline{\Gamma}|\varepsilon|^2\varepsilon = 0; \tag{4}$$

$$\overline{\Gamma} = \frac{3-\nu^2}{2\nu}\Gamma, \quad \beta_2 = -\frac{1}{V_{\mathrm{G}}^2}\frac{1}{\kappa\gamma^3\nu^3}, \quad \gamma = 1/\sqrt{1-\nu^2}, \tag{5}$$

where ν can take all values $-1 < \nu < 1$.

The fundamental soliton of Eq. (4) can be written as

$$\varepsilon = \sqrt{\left|\frac{\beta_2}{\overline{\Gamma}}\right|}\frac{1}{t_0}\operatorname{sech}(\tau)\exp(\mathrm{i}\,\xi/2), \tag{6}$$

$$\tau = t/t_0, \quad \xi = x/L_{\mathrm{D}}, \quad L_{\mathrm{D}} = t_0^2/|\beta_2|. \tag{7}$$

The relation between t_0 and w_{e} (the fullwidth at half maximum of incoming pulse) is

$$w_{\mathrm{e}} = 2\ln(1+\sqrt{2})t_0 = 1.763t_0. \tag{8}$$

It is well known that the solitons can propagate through the grating at a velocity of $0.5v_{\mathrm{G}}$,[7,9] and we can use the apodized grating if we want to obtain much slower solitons. In this paper, the grating is apodized by a hyperbolic tangent function in the middle of the grating. The hyperbolic tangent function is given as

$$\kappa(x) = \kappa_0 + \Delta\kappa \cdot \tanh\frac{x - x_0}{w}, \tag{9}$$

where $x_0 = 0$ means that the point is the central location of the apodization function, and w is the step width.

Combining Eq. (9) with Eq. (3), we can obtain the NLCME to describe the apodized gratings.

$$\begin{aligned} & \mathrm{i}\frac{\partial U}{\partial x} + \mathrm{i}\frac{\overline{n}}{c}\frac{\partial U}{\partial t} + \delta U + \left(\kappa_0 + \Delta\kappa \cdot \tanh\frac{x-x_0}{w}\right)V \\ & + \Gamma\left(|U|^2 + 2|V|^2\right)U = 0, \\ & -\mathrm{i}\frac{\partial V}{\partial x} + \mathrm{i}\frac{\overline{n}}{c}\frac{\partial V}{\partial t} + \delta V + \left(\kappa_0 + \Delta\kappa \cdot \tanh\frac{x-x_0}{w}\right)U \\ & + \Gamma\left(|V|^2 + 2|U|^2\right)V = 0. \end{aligned} \tag{10}$$

3. Numerical results and discussion

In our simulation by using the split-step fast-Fourier-transform method, the gap soliton being the exact solution of Eq. (6) is launched into the apodized grating with initial values of $\nu = 0.8$. The stabilities of slow solitons on the fibre grating's coupling coefficient κ, the frequency detuning δ and the nonlinear coefficient Γ are analysed, and the curves of solitons' time-displacement (which means the relationship between the displacement with time when the solitons are transmitting through the FBG) are shown. The input solitons are propagating from the initial displacement $s = -0.05$ which is at the left edge of the grating, and the parameters are chosen as follows: $\overline{n} = 1.447$, $n_2 = 2.6 \times 10^{-20}$ m^2/W, $\lambda_{\rm B} = 1550$ nm, $w_{\rm e} = 30$ ps.

3.1. The apodized function

Launching the solitons with parameters $\delta = 0$ and $\Gamma = 22$ (km $\cdot$ W)$^{-1}$, for the apodization functions, $\kappa_0 = 420$ m^{-1} and $w = 0.01$ m, we find that the step's height $\Delta\kappa$ has different values. The parameters are chosen as follows: (a) $\Delta\kappa = 0$, (b) $\Delta\kappa = 40$ m^{-1}, (c) $\Delta\kappa = 65$ m^{-1}, and (d) $\Delta\kappa = 80$ m^{-1}. Figure 1 shows the typical results of the stability and instability of collision between the propagating solitons and apodization step. The lower panels show the evolution of the soliton in terms of $|U|$ level contours, and the waveforms $|U|$ (solid curve) and $|V|$ (dashed curve) of the end of the simulation are shown on the upper panels together with the apodization profile $\kappa(x)$ (which is described by the thick solid line in the figures).

As shown in Fig. 1(a) with $\Delta\kappa = 0$, the soliton propagates through the BG with a constant velocity of $0.58v_{\rm G}$, which agrees well with Eggleton's experiment.[7] In Fig. 1(b), when the step's height is enough small, the soliton can pass through the apodization profile with decreasing velocity. However, the most interesting case is that the soliton is captured by a larger $\Delta\kappa$ as 'standing light', as shown in Fig. 1(c). Actually, there are some theories to analyse the standing light which can also be called standing wave.[15] In Fig. 1(d) where the same case is considered except for a much stronger $\Delta\kappa$, the soliton is bounced back, showing a bigger repulsion in comparison with the transmission.

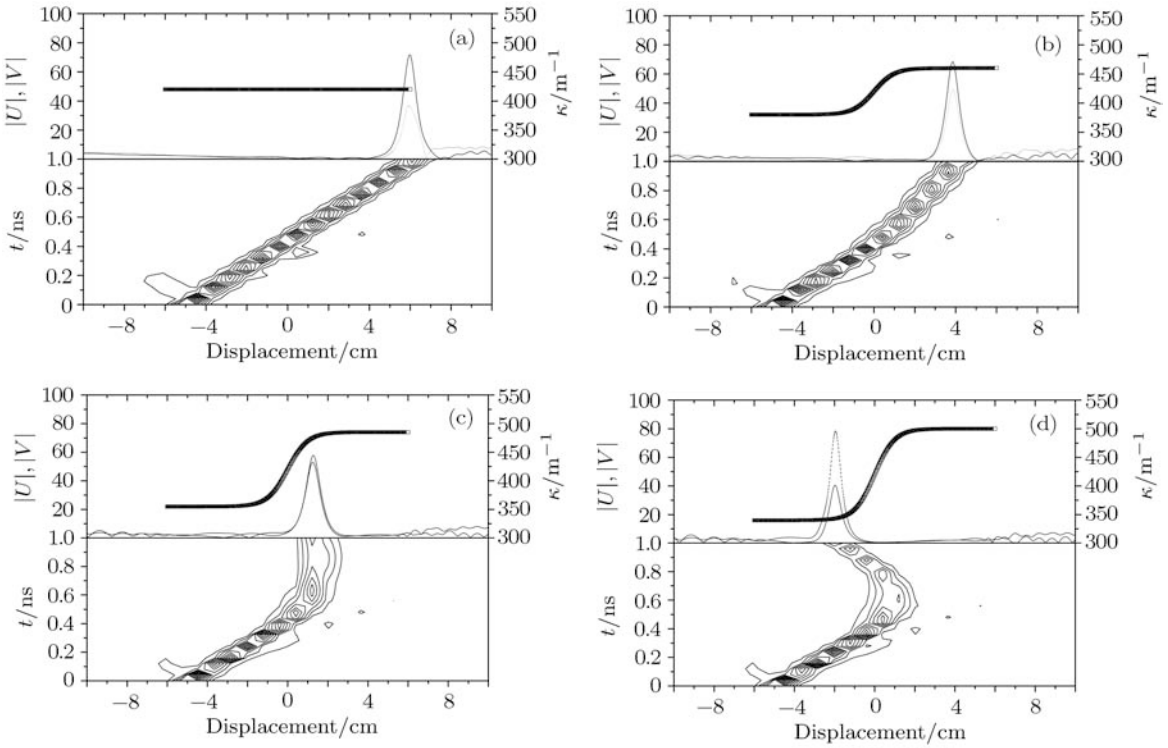


Fig. 1. The impact of the soliton in the FBG with the apodization step which has different apodization profiles. (a) $\Delta\kappa = 0$. (b) $\Delta\kappa = 40$ m^{-1}, (c) $\Delta\kappa = 65$ m^{-1}, (d) $\Delta\kappa = 80$ m^{-1}.

It is also obvious that $\Delta\kappa$ has a significant effect on the soliton's velocity, so we can obtain a soliton with any velocity by changing the apodization profile's step $\Delta\kappa$. Because the apodized function κ has a relation with the modulation amplitude of refractive index $\delta n_{\rm g}$, in the grating manufacture, $\delta\kappa$ can be obtained by controlling the exposure value. Because optical soliton is not the exact solution of the NLCME, in order to have a stable transmission there will be some pulse reshaping. Therefore, there is some energy dispersion in the simulation.

3.2. The fibre's nonlinear coefficient

We also discussed the effect of fibre's nonlinear coefficient Γ on the solitons' velocity. The launching pulse is the same as that in the above discussion. The BG's apodization profile is $\kappa_0 = 420\ {\rm m}^{-1}$, $w = 0.01$ m, $\Delta\kappa = 60$. Figures 2(a), 2(b) and 2(c) show typical results of the collision between moving solitons and the apodization step with $\Gamma = 10\ ({\rm km\cdot W})^{-1}$, $\Gamma = 20\ ({\rm km\cdot W})^{-1}$ and $\Gamma = 30\ ({\rm km\cdot W})^{-1}$, respectively.

As shown in Fig. 2(a), it is known that the output pulse has an irreversible deformation. For the reason that soliton is based on the balance between the fibre dispersion and Γ, consequently the yielding pulse will have some dispersion when Γ is small. We obtained the result that the soliton can move with a velocity about $0.3v_{\rm G}$ on the output end of the grating by calculating the data of the simulation. When Γ becomes larger, the soliton can pass through the apodization profile with a final velocity about $0.2v_{\rm G}$, and a little energy is radiated, as presented in Fig. 2(b). When Γ became much larger, the velocity of the soliton nears the last one. It also shows the output pulse without seriously irreversible deformation, although the soliton's energy has a remarkable radiation which is shown in Fig. 2(c). Therefore, Γ has a significant effect on the soliton's shape and energy. The fibre with appropriate Γ can be chosen in order that the soliton can propagate without irreversible deformation and energy radiation.

3.3. Frequency detuning

When a BG is used as an optical buffer device, the fibre's Γ and the grating's apodization profile are constants. The soliton's velocity can be controlled only by changing the input pulse's parameter or $\lambda_{\rm B}$ of the BG. A discussion on changing δ is in the simulations. The BG's apodization profile and Γ are chosen as $\kappa_0 = 420\ {\rm m}^{-1}$, $\Delta\kappa = 60$ and $\Gamma = 22\ ({\rm km\cdot W})^{-1}$. Figures 3(a), 3(b), and 3(c) show the typical results of the collision between moving solitons and the apodization step with $\delta \approx 1892\ {\rm m}^{-1}$, $2650\ {\rm m}^{-1}$ and $3029\ {\rm m}^{-1}$ by changing BG's wavelength $\lambda_{\rm B}$.

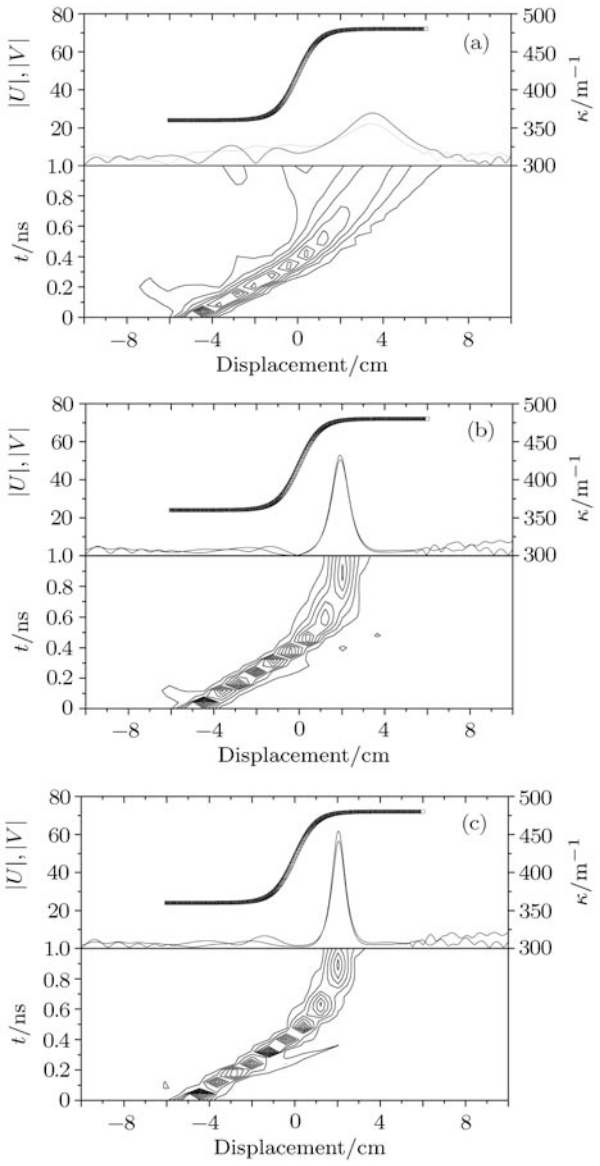


Fig. 2. The impact of the soliton in the FBG with the apodization step which has different nonlinear coefficient Γ. (a) $\Gamma = 10\ ({\rm km\cdot W})^{-1}$, (b) $\Gamma = 20\ ({\rm km\cdot W})^{-1}$, (c) $\Gamma = 30\ ({\rm km\cdot W})^{-1}$.

In Fig. 3, the solitons can be slowed down by the apodization step, but their velocities are not constant. First, the parameter $\delta \approx 1892\ {\rm m}^{-1}$, the case shows that the solitons can move without irreversible deformation in Fig. 3(a).

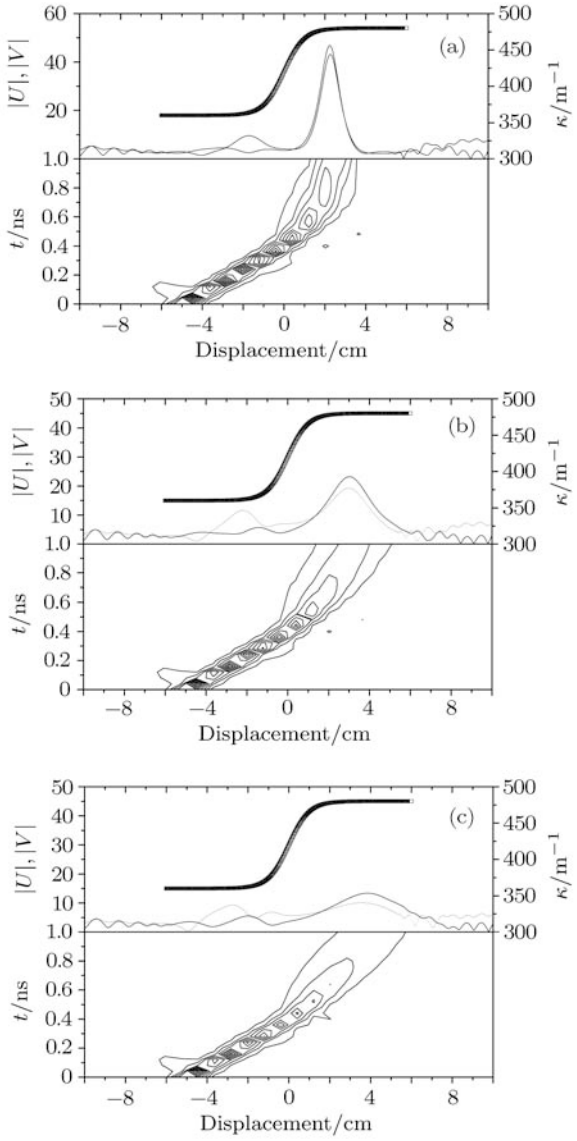


Fig. 3. The impact of the soliton transmitted in the apodized FBG with varying the frequency detuning δ. (a) $\delta \approx 1892$ m^{-1}, (b) $\delta \approx 2650$ m^{-1}, (c) $\delta \approx 3029$ m^{-1}.

The soliton's velocity is about $0.18v_{\rm G}$ at the grating's right end. In Fig. 3(b), $\delta \approx 2650$ m^{-1}, the pulse can be broadened severely and the final velocity is about $0.27v_{\rm G}$. The causation of the pulse broadening is that the impact of nonlinear effect is not as large as the dispersion in this condition. When $\delta \approx 3029$ m^{-1}, the pulse cannot propagate through the grating as a soliton, and the velocity of pulse peak is about $0.32v_{\rm G}$. With δ increasing, not only the soliton's velocity will increase, but also the irreversible deformation of soliton emerges in some degree, which owing to the fact that the dispersion cannot have a balance with the fibre's nonlinear coefficient Γ. Though changing δ may cause some distortion of the soliton, but we can keep the pulse's shape by choosing the fibre with higher Γ, such as photonic crystal fibre. In value of $\delta = 2\pi\overline{n}\left(\frac{1}{\lambda} - \frac{1}{\lambda_{\rm B}}\right)$, the wavelength of input pulse or $\lambda_{\rm B}$ can be changed for the sake of obtaining a tunable velocity on a small scale.

4. Conclusions

In summary, based on the nonlinear coupled theory, we investigated the propagation in apodized gratings with the optical fibre solitons but not the BG solitons. The motion of the soliton in apodized FBG is shown, the influences of different parameters including the step's height $\Delta\kappa$ of apodization functions, fibre's nonlinear coefficient Γ and frequency detuning of input pulse δ on soliton's velocity and deformation are analysed. The simulation results show that the solitons can be slowed down, bounced back or captured when moving in the apodized grating through choosing appropriate parameters.

As the pulse generated by mode-locked laser can be approximated as optical fibre soliton, the theory analyses the optical fibre soliton but the BG soliton is much more suitable for the practical research. We have a conclusion that the apodization step has the greatest impact on the soliton's velocity in the three parameters which we simulated. Thus, in the progress of a grating manufacture, we can select the kind of fibre with larger nonlinear coefficient and appropriate apodization profile. While in the optical buffer system, the soliton's velocity can be controlled by changing δ (changing the wavelength of the input pulse or the Bragg wavelength of FBG). Therefore, the velocity of the solitons can be tunable by changing the frequency detuning on a small scale.

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