Spatial solitons in photonic lattices with large-scale defects*

Yang Xiao-Yu(杨晓雨)[a], Zheng Jiang-Bo(郑江波)[b], and Dong Liang-Wei(董亮伟)[b]†

[a] College of Mathematics Physics and Information Engineering, Zhejiang Normal University, Jinhua 321004, China
[b] Institute of Information Optics, Zhejiang Normal University, Jinhua 321004, China

(Received 18 September 2010; revised manuscript received 16 October 2010)

We address the existence, stability and propagation dynamics of solitons supported by large-scale defects surrounded by the harmonic photonic lattices imprinted in the defocusing saturable nonlinear medium. Several families of soliton solutions, including flat-topped, dipole-like, and multipole-like solitons, can be supported by the defected lattices with different heights of defects. The width of existence domain of solitons is determined solely by the saturable parameter. The existence domains of various types of solitons can be shifted by the variations of defect size, lattice depth and soliton order. Solitons in the model are stable in a wide parameter window, provided that the propagation constant exceeds a critical value, which is in sharp contrast to the case where the soliton trains is supported by periodic lattices imprinted in defocusing saturable nonlinear medium. We also find stable solitons in the semi-infinite gap which rarely occur in the defocusing media.

Keywords: defect solitons, photonic lattices, saturable nonlinearity
PACS: 42.65.Tg, 42.65.Jx, 42.65.Wi **DOI:** 10.1088/1674-1056/20/3/034208

1. Introduction

Optical solitons in periodic media, such as fabricated waveguide arrays and optically-induced photonic lattices, have aroused the great interest these days, owing to their physics and light-routing applications.[1−4] Linear light propagation exhibits Bloch bands and forbidden bandgaps in periodic lattice modulated media compared with that in homogeneous media. The periodic lattice potentials imprinted in focusing and defocusing nonlinear medium can support soliton trains, vortex solitons, necklace solitons, vector solitons and so on. The properties of such solitons are very different from those in homogeneous media.[5−11]

On the other hand, defects and defect states exist in a variety of lattice modulated linear and nonlinear systems, including solid state physics, photonic crystals, and Bose–Einstein condensates. Linear defect modes in one-dimensional (1D)[12,13] and two-dimensional (2D)[14,15] photonic lattices were investigated. Nonlinear localized modes in both attractive and repulsive single site defects proved to be stable in certain parameter regimes.[16] Stable defect vector gap solitons[17] and surface defect gap solitons[18] in 1D photonic lattices were also predicted. Defect solitons and their stabilities in the Kronig–Penney periodic medium with delta-function linear and nonlinear defects were analysed in Ref. [19]. Very recently, domainlike nonlinear modes supported by a defect covering several lattice sites were investigated by Ye *et al.*[20] By varying the core width, these modes transform into defect solitons in the small core limit and into a pair (for fundamental mode) or an array (for higher-order modes) of kinks[21] in the large core limit.

Although the above progress, defect solitons and their stabilities in nonuniform optical media, especially in large-scale defected lattices imprinted in media with saturable nonlinearity, are little understood. Almost all defect solitons reported so far concentrate on the single site defect with the only few exceptions.[20,22] Moreover, the dynamics of solitons supported by the large-scale defects with nonzero index core in lattices is still an open problem. In this paper, we address the properties of nonlinear modes supported by a planar low-index core with different depths in harmonic lattices imprinted in defocusing media with saturable nonlinearity. In such a scheme, the nonlinear modes in the core region can exhibit the following shapes: (i) flat-top profile, (ii) dipole-like profile, (iii) higher-order multipole-like profile. If the above modes reside in the semi-infinite gaps of lat-

*Project supported by the National Natural Science Foundation of China (Grant No. 10704067) and the Natural Science Foundation of Zhejiang Province, China (Grant No. Y6100381).
†Corresponding author. E-mail: donglw@zjnu.cn
© 2011 Chinese Physical Society and IOP Publishing Ltd

tices, their shapes look like traditional solitons in bulk media. However, the nonlinear modes will exhibit oscillatory decaying wings inside the lattices when they reside in the first finite band gaps. In contrast with the cases in cubic Kerr media, we reveal that the saturable nonlinearity can dramatically enhance the power of defect solitons, which is very important for the practical application of solitons. We also investigate the stability of defect solitons and find various types of solitons to be stable in wide parameter windows.

2. Model

We consider light trapping in a defect covering several lattice sites imprinted in a defocusing medium with saturable nonlinearity. The dynamics of beam propagation can be described mathematically by the nonlinear Schrodinger equation for the dimensionless amplitude q

$$\mathrm{i}\frac{\partial q}{\partial z}+\frac{1}{2}\frac{\partial^2 q}{\partial q^2}-\frac{|q|^2 q}{1+s|q|^2}+V(x)q=0, \qquad (1)$$

where the transverse x and the longitudinal z coordinates are scaled in the terms of beam width and diffraction length respectively, s is a saturation parameter and the function $V(x)$ describes the refractive index profile: $V(x)=p\cos^2(\Omega x)$ for $|x|\geq n_x\pi/2\Omega$ and $V(x)=\gamma$ for otherwise, where γ is a constant and Ω is the lattice frequency. In the following discussions, we set $\gamma=0$, $0.5p$ and $1p$ to illustrate the properties of solitons in the flat-bottomed, repulsive and flat-topped defected lattices. The parameter p characterizes the lattice depth, and n_x is the number of lattice periods removed to form the guiding core. Note that $\gamma=0.5p$ corresponds to the repulsive defect because light tends to escape from the defect to nearby lattice sites.[16] Equation (1) admits several conserved quantities, including the power or energy flow given by $U=\int_{-\infty}^{\infty}qq^*\mathrm{d}x$.

We search for stationary solutions of Eq. (1) in the form $q(x,z)=w(x)\exp(\mathrm{i}\,bz)$, where $w(x)$ is a real function describing the profile of stationary solution and b is a nonlinear propagation constant. Without loss of any generality, we vary the depth parameter p, the saturation parameter s, the number of lattice periods removed to form the guiding core n_x, and fix the frequency of lattice $\Omega=2$ in numerical analysis. Stationary solutions of solitons can be obtained by solving Eq. (1) with a standard relaxation method. To examine the stability properties, we consider the perturbed stationary solution of Eq. (1) in the form $q(x,z)=[w(x)+u(x,z)+\mathrm{i}\,v(x,z)]\exp(\mathrm{i}\,bz)$, where $u(x,z)$ and $v(x,z)$ are respectively real and imaginary parts of perturbation which can grow with a complex rate λ upon propagation. Substituting the perturbed solution into Eq. (1) and linearizing the resulting equation around $w(x)$ yield an eigenvalue problem

$$\begin{aligned}\lambda u&=-\frac{1}{2}\frac{\partial^2 v}{\partial x^2}+bv-Vv+\frac{w^2+sw^4}{(1+sw^2)^2}v,\\ \lambda v&=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}-bu+Vu-\frac{3w^2+sw^4}{(1+sw^2)^2}u. \qquad (2)\end{aligned}$$

To confirm the results of the linear stability analysis, we simulate Eq. (1) by the standard beam propagation method with the input condition as $q(x,z=0)=w(x)[1+\rho(x)]$, where ρ is a random function with Gaussian distribution whose variance $\sigma^2_{\mathrm{noise}}=0.01$.

3. Properties of defect solitons

Before we discuss the properties of defect solitons, it is better to understand the band-gap structure of the undefected periodic lattice since the defect modes must reside in the forbidden gaps of the lattices. The bandgap structure of a periodic optical medium has been extensively studied previously. In the present setting, the band structure of the linear version of Eq. (1) with $R(x)=p\cos^2(\Omega x)$ for $x\in(-\infty,\infty)$ at $\Omega=2$ is shown in Fig. 1. For each lattice depth, all possible propagation constants of the Bloch modes form bands. The gray regions represent Bloch bands, while regions between Bloch bands are bandgaps. They are called sequentially the semi-infinite gap, the first finite gap, the second gap, and so on.

First, we address the properties of solitons residing in the first finite gap of lattices with flat-bottomed defects covering several lattice sites. Typical profiles of solitons residing in the first finite gap are shown in Figs. 1(c) and 1(d). When $n_x=0$, the defected lattice degenerates in to an undefected photonic lattice which supports stable odd, even and (twisted) soliton trains when the saturable nonlinearity is used.[23] However, the soliton profiles exhibit decaying oscillating wings in the lattices when the lattice is defected by nonzero n_x. For $n_x=2$, the solitons become more localized with the increase of propagation constant (Fig. 1(d)). In other words, the solitons expand quickly if the propagation constant approaches to the lower edge of the first finite gap which accompanies the dramatic

growth of the power of solitons (Fig. 1(b)). The delocalization of solitons can be attributed to the saturable nonlinearity which restricts the further growth of nonlinearity when the peak value of soliton approaches to a critical value. When the size of the defected core region is larger, the soliton profiles in the defect exhibit flat-topped shapes (Fig. 1(c)). The solitons penetrate into the lattices surrounding the defect more evidently with the decrease of propagation constant. Comparing the two power curves shown in Fig. 1(b), one can find that the difference in n_x can induce the shift of the existence domain of solitons. We also find that the existence of such solitons satisfies the relation $b_{\rm upp} = b_{\rm low} + 1/s$ (Fig. 1(b)) which holds for the odd and even solitons in the undefected lattices imprinted in the saturable nonlinear media.[23,24] Linear stability analysis based on Eqs. (2) reveals that solitons in defected lattices are stable in a very wide parameter window (Fig. 1(e)). Figure 1(f) displays an example of stable propagation of solitons supported by defected lattice with $n_x = 6$.

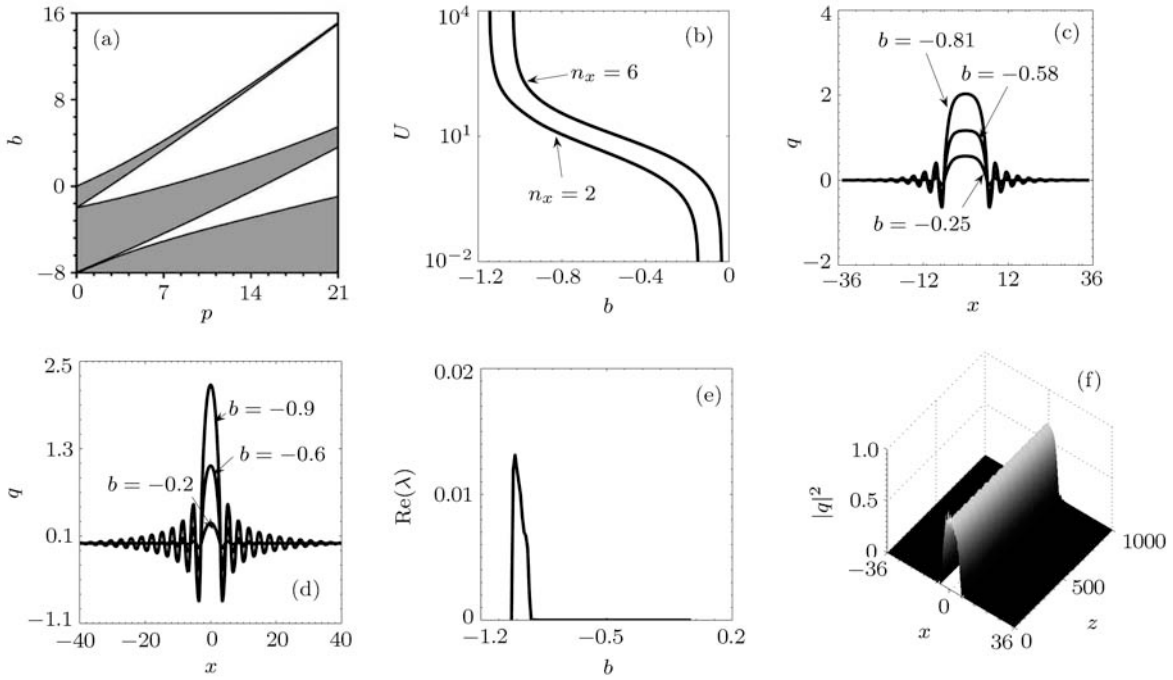


Fig. 1. (a) Bandgap lattice spectrum at $\Omega = 2$. (b) Energy flow versus propagation constant. Profiles of flat-topped solitons for (c) $n_x = 6$ and (d) $n_x = 2$. (e) Instability growth rate versus propagation constant for $n_x = 6$. (f) Stable propagation of flat-topped soliton at $b = -0.4$ and $n_x = 6$. White noise with variance $\sigma^2_{\rm noise} = 0.01$ is added into the initial input. $\gamma = 0$, $p = 4$, $s = 1$ in (b)–(f).

Next, we focus on the solitons supported by the lattice with a repulsive flat defect ($\gamma = 0.5p$). Figure 2(a) gives two soliton profiles each with a flat-topped shape in the core region of the defected lattice, similar to those displayed in Fig. 1. Although the propagation constants of solitons we discussed here are all positive, the solitons still reside in the first finite gap since the lattice depth $p = 7$. Comparing the soliton profiles at propagation constant $b = 1.9$ in Fig. 2(a) and $b = -0.81$ in Fig. 1(c) and their relative positions in Fig. 1(b) and Fig. 2(c), one can find that the solitons become more localized when the lattice is deeper. It can be interpreted in physics as the fact that the deeper lattice can build a deeper potential which prevents the delocalization of solitons. Figure 2(b) shows another kind of soliton, exhibiting dipole-like profile in the core region of defect. The existence domains of flat-topped and dipole-like solitons expand compared with those shown Fig. 1(b) because the saturable parameter becomes smaller. They still satisfy the afore mentioned relation $b_{\rm upp} = b_{\rm low} + 1/s$ which is in sharp contrast to the cases in the undefected lattices imprinted in defocusing saturable nonlinear medium.[23,24] The power approaches to infinity at $b \to b_{\rm low}$ and zero at $b \to b_{\rm upp}$. Comparing the power curves with those in Ref. [20] where the de-

focusing cubic Kerr medium was used, one can find that the power of solitons can be enhanced dramatically by the saturable nonlinearity. The descending power curve is due to the nature of defocusing nonlinearity we selected. Linear stability analysis reveals that flat-topped solitons are almost completely stable in their entire existence domains while dipole-like solitons are unstable in two very narrow bands near the lower edge of the gap. The existence domain of solitons versus saturable parameter is illustrated in Fig. 2(e). It shrinks with the increase of saturable parameter s. The width of existence domain remains the same with the increment of lattice depth for fixed saturable parameter s (Fig. 2(f)). Typical examples of stable propagation of flat-topped and dipole-like solitons are displayed in Figs. 2(g) and 2(h), respectively.

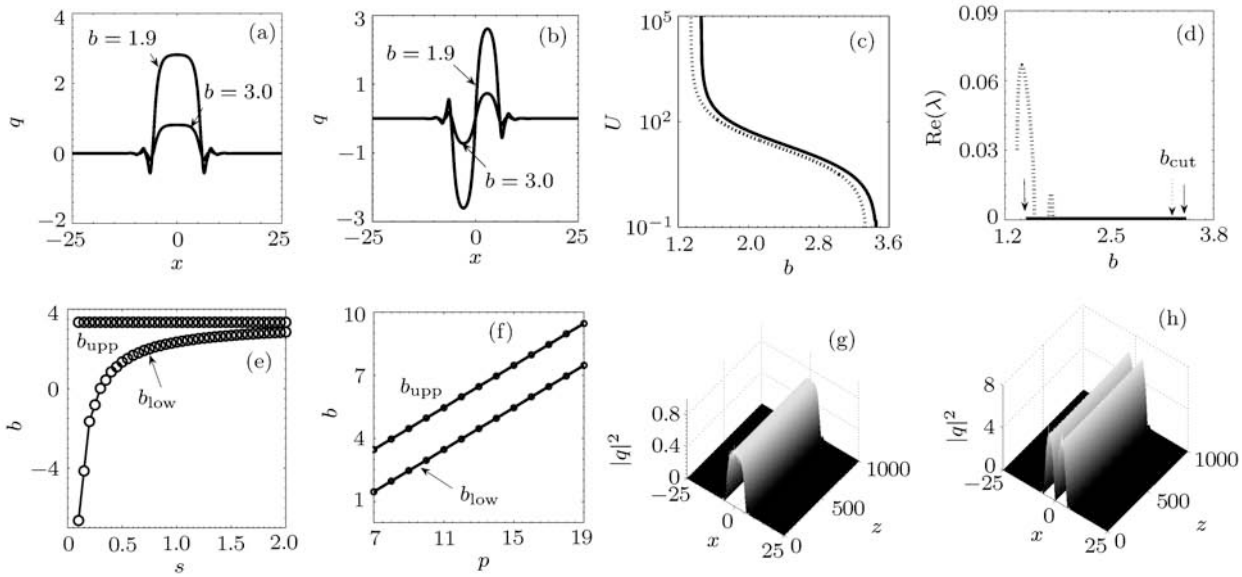


Fig. 2. Profiles of flat-topped (a) and dipole-like solitons (b). (c) Energy flow of flat-topped soliton (solid curve) and dipole-like soliton (dashed) versus propagation constant. (d) Instability growth rate versus propagation for the flat-topped soliton (solid curve) and dipole-like (dash curve) soliton, dash arrow denotes the cutoff of the dipole-like solitons. (e) Lower and upper cutoffs of the dipole-like solitons versus saturable parameter. (f) Existence domains of the dipole-like solitons versus lattice depth p. Stable propagations of the flat-topped soliton at $b = 3.0$ (g) and dipole-like soliton at $b = 1.96$ (h), white noises with variance $\sigma^2_{\rm noise} = 0.01$ are added into the initial inputs. $\gamma = 0.5p$, $n_x = 6$, $s = 0.5$, and $p = 7$ in (a)–(h).

Now, we explore the properties of multipole-like(twisted) solitons supported by the repulsive defects ($\gamma = 0.5p$). We can see clearly the shift of existence domain, induced by the variation of lattice depths (Fig. 3(a)). We stress again that the lower and the upper cutoffs of propagation constant of multipole-like solitons satisfy the relation $b_{\rm upp} = b_{\rm low} + 1/s$ which is very different from that the the higher order twisted solitons in undefected lattices imprinted in defocusing medium where the existence domain shrinks quickly with the increase of soliton order. Thus, the defect plays a role in broadening the existence domain of twisted solitons. Solitons with odd and even symmetries are stable when the propagation constant exceeds a critical value (Fig. 3(d)). Figures 3(e) and 3(f) show two instances of stable multipole-like solitons with white noises added into the initial inputs.

Besides the aforementioned solitons residing in the first finite gaps, we also find the solutions of solitons residing in the semi-infinite gaps which rarely occur in the undefected lattices in defocusing media. We take a flat-topped defect covering five lattice sites as an example. Both flat-topped and dipole solitons in the semi-infinite gaps are truly the fundamental and the dipole solitons since there is no decaying oscillating wing in the lattices. Such modes bifurcate from the first and second linear eigen-functions of the linearized version of Eq. (1). Numerical analysis shows that flat-topped solitons residing in the first finite gap are completely stable while the dipole solitons are stable provided that the propagation constant is larger than a critical value (Fig. 4(d)). For illustration, we plot two examples of stable propagations of flat-topped and dipole solitons in Figs. 4(e) and 4(f), respectively.

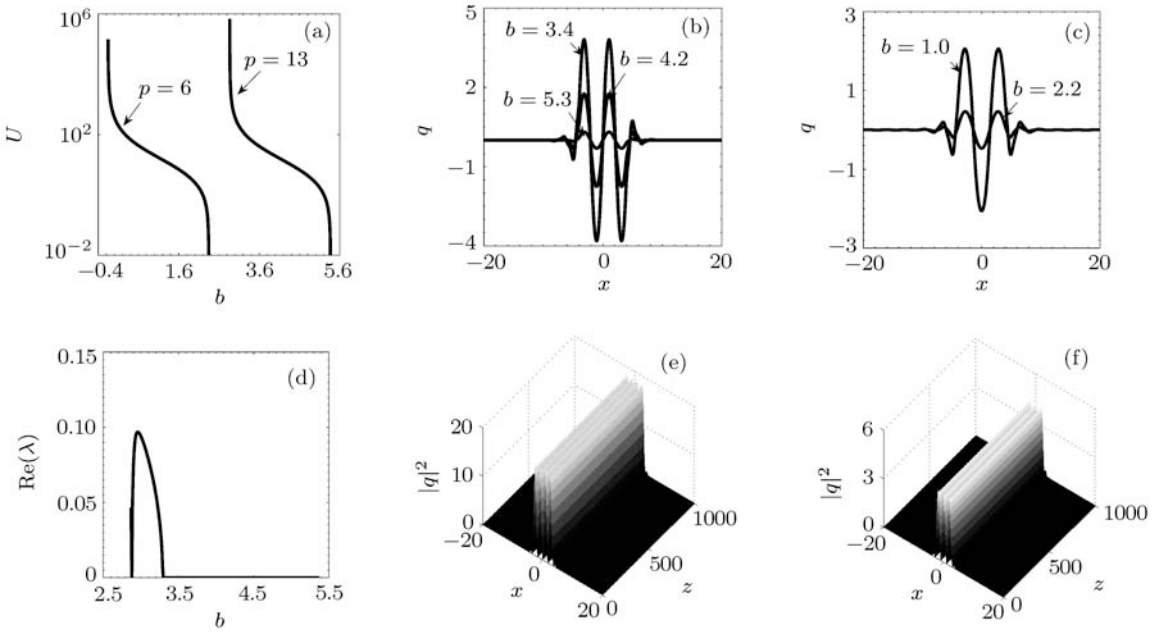


Fig. 3. (a) Energy flows of multipole-like solitons versus propagation constant. Profiles of the odd symmetrical solitons at $p = 13$ (b) and even symmetrical solitons at $p = 6$ (c). (d) Instability growth rate versus propagation for the odd symmetrical solitons shown in (c). (e) Stable evolution of solitons shown in (b) at $b = 3.4$. (f) Stable evolution of solitons shown in and (c) at $b = 1.0$, respectively. White noises with variance $\sigma^2_{\rm noise} = 0.01$ are added into the initial inputs. $\gamma = 0.5p$, $n_x = 4$, $s = 0.4$ in (a)–(f).

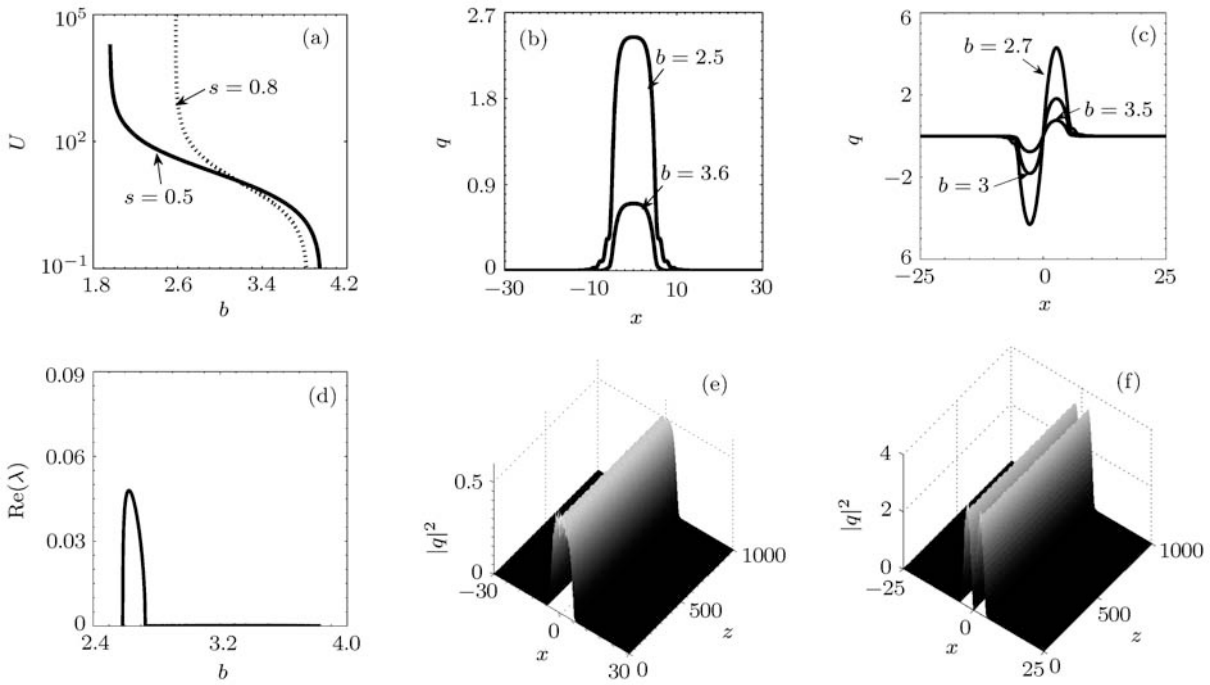


Fig. 4. (a) Energy flows of fundamental soliton (solid curve) and dipole soliton (dash curve) versus propagation constant. (b) Profiles of fundamental solitons. (c) Profiles of dipole solitons. (d) Instability growth rate of the dipole solitons at $s = 0.8$. (e) Stable propagation of the fundamental soliton shown in (b) at $b = 3.6$. (f) Stable propagation of dipole soliton shown in (c) at $b = 3.0$. White noises with variance $\sigma^2_{\rm noise} = 0.01$ are added into the initial inputs. $\gamma = 1p$, $p = 4$, $n_x = 5$ in (a)–(f).

4. Conclusions

We have investigated the properties of the large-scale defect solitons trapped in defected photonic lattices imprinted in defocusing saturable nonlinear medium. Solitons residing in the first finite and semi-infinite gaps

of lattices with different sizes of defects are found to be stable almost in their entire existence domains. The flat-bottomed, repulsive and flat-topped defects can enlarge the existence domain of solitons, especially for the dipole-like and multipole-like solitons, which is contrary to the fact that the existence domain of twisted solitons in undefected lattices shrinks quickly with the increase of peak number. We also discuss the influences of lattice depth, defect size and saturable parameter on the stability properties of solitons. Our research may offer new opportunities for the soliton steering and soliton control.

References

[1] Kivshar Yu S and Agrawal G P *Optical Solitons: from Fibers to Photonic Crystals* 2003 (San Diego: Academic)

[2] Fleischer J W, Segev M, Efremidis N K and Christodoulides D N 2003 *Nature* (London) **422** 147

[3] Christodoulides D N, Lederer F and Silberberg Y 2003 *Nature* (London) **424** 817

[4] Lederer F, Stegeman G I, Christodoulides D N, Assanto G, Segev M and Silberberg Y 2008 *Phys. Rep.* **463** 1

[5] Yang J and Musslimani Z H 2003 *Opt. Lett.* **28** 2094

[6] Yang J, Makasyuk I, Bezryadina A and Chen Z 2004 *Opt. Lett.* **29** 1662

[7] Neshev D N, Alexander T J, Ostrovskaya E A, Kivshar Yu S, Martin H, Makasyuk I and Chen Z 2004 *Phys. Rev. Lett.* **92** 123903

[8] Fleischer J W, Bartal G, Cohen O, Manela O, Segev M, Hudock J and Christodoulides D N 2004 *Phys. Rev. Lett.* **92** 123904

[9] Rodas-Verde M I, Michinel H and Kivshar Yu S 2006 *Opt. Lett.* **31** 607

[10] Desyatnikov A S, Kivshar Yu S and Torner L 2005 *Prog. Opt.* **47** 291

[11] Lou C, Wang X, Xu J, Chen Z and Yang J 2007 *Phys. Rev. Lett.* **98** 213903

[12] Fedele F, Yang J and Chen Z 2005 *Opt. Lett.* **30** 1506

[13] Wang X, Young J, Chen Z, Weinstein D and Yang J 2006 *Opt. Express* **14** 7362

[14] Wang J, Yang J and Chen Z 2007 *Phys. Rev.* A **76** 013828

[15] Makasyuk I, Chen Z and Yang J 2006 *Phys. Rev. Lett.* **96** 223903

[16] Yang J and Chen Z 2006 *Phys. Rev.* E **73** 026609

[17] Wang R, Zhang C, Kong L and Zhang Y 2008 *Opt. Commun.* **281** 4509

[18] Chen W H, He Y J and Wang H Z 2006 *Opt. Express* **14** 11271

[19] Sukhorukov A A and Kivshar Yu S 2001 *Phys. Rev. Lett.* **87** 083901

[20] Ye F, Kartashov Y V, Vysloukh V A and Torner L 2008 *Opt. Lett.* **33** 1288

[21] Kartashov Y V, Vysloukh V A and Torner L 2006 *Opt. Express* **14** 12365

[22] Ye F, Kartashov Y V, Vysloukh V A and Torner L 2008 *Phys. Rev.* A **78** 013847

[23] Kartashov Y V, Vysloukh V A and Torner L 2004 *Opt. Express* **12** 2831

[24] Yin G, Zheng J, Yang X and Dong L 2010 *Chin. Phys.* B **19** 044206