Performance optimization analysis of a thermoelectric refrigerator with two resonances*

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Based on electron transport theory, the performance of k_x and k_r filtered thermoelectric refrigerators with two resonances are studied in this paper. The performance characteristic curves between the cooling rate and the coefficient of performance are plotted by numerical calculation. It is shown that the maximum cooling rate of the thermoelectric refrigerator with two resonances increases but the maximum coefficient of performance decreases compared with those with one resonance. No matter which resonance mechanism is used (k_x or k_r filtered), the cooling rate and the performance coefficient of the k_r filtered refrigerator are much better than those of the k_x filtered one.

Keywords: thermoelectric refrigerator, two resonances, cooling rate, coefficient of performance
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1. Introduction

It is well known that the electrons in two reservoirs may move along a temperature gradient or a electrochemical potential gradient. If the two reservoirs, in which the cold reservoir possesses a high electrochemical potential, are assembled, some electrons will be transported against the temperature gradient or the electrochemical potential gradient. When the high-energy transported electrons move from the cold reservoir to the hot one, the device operates as a refrigerator. Recently, thermoelectric heat engines and refrigerators have aroused considerable interest, particularly nanowire, quantum dot and superlattice thermionic and thermoelectric devices,[1−3] reversible quantum Brownian refrigerators and heat engines for electrons,[4] thermionic refrigerators and power generators,[5−7] quantum tunneling ratchets,[8] thermoelectric refrigerators with nanowire heterostructures,[9] thermoelectric refrigerators with double resonances,[10] etc. If electrons with an energy level $E_{\rm S}$ transmit between the two reservoirs, the efficiency of the thermoelectric device may, in principle, reach the Carnot limit. The special energy $E_{\rm S}$ is[4]

$$E_{\rm S} \equiv \frac{\mu_{\rm C} T_{\rm H} - \mu_{\rm H} T_{\rm C}}{T_{\rm H} - T_{\rm C}}, \tag{1}$$

where $\mu_{\rm H}$ and $\mu_{\rm C}$ are the electrochemical potentials of the hot and cold reservoirs ($\mu_{\rm C} > \mu_{\rm H}$), $T_{\rm H}$ and $T_{\rm C}$ are the temperatures of the hot and cold reservoirs ($T_{\rm H} > T_{\rm C}$). When the energy of the transported electrons is in a range of $\mu_{\rm C} < E < E_{\rm S}$, the thermoelectric device operates as a refrigerator.

2. Transport theory

Generally, in an appropriate range of the energy spectrum for selecting electrons, the wider the energy spectrum width is, the higher the cooling rate is, but the lower the coefficient of performance (COP) is.[4,9] Thus, it is necessary to make an optimal analysis between the cooling rate and the COP of a thermoelectric refrigerator. There are two classes of thermoelectric devices. The first one corresponds to a conventional device, which filters the transmitted electrons according to their momenta in the direction of transport only. This type of device is called a 'k_x-filter' device. The second one is a device that filters transmitted electrons according to their total energies. This type of device is referred to as a 'k_r-filter' device. Humphrey[11] and O'Dwyer *et al.*[12] introduced the 'k_x-filter' and the 'k_r-filter', in which electrons whose momenta meet some requirements in the k_x and the k_r directions of k-space, respectively, may be allowed to be transported, as shown in Fig. 1. Figs. 1(a) and 1(b) show the simple Fermi sphere models of the 'k_x-filter' and the 'k_r-filter', respectively.

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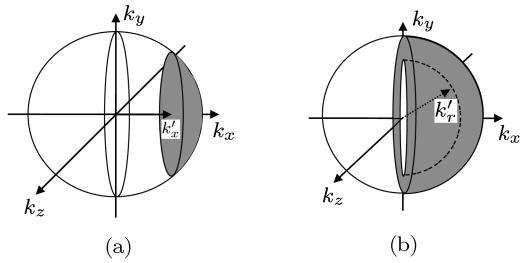


Fig. 1. Fermi sphere indicating the electrons (a) transmitted by a k_x filtered device and (b) transmitted by a k_r filtered device, where the shaded areas are effective.[12]

The density of states (DOS) of the three-dimensional reservoir may be expressed as

$$g(\theta,\phi,k_r) = (2\pi)^{-3} k_r^2 \cos\theta \mathrm{d}\theta\mathrm{d}\phi\delta k_r, \tag{2}$$

and the number of electrons with energy E arriving at the three-dimensional reservoir interface per unit area per unit time is

$$n_r^{\mathrm{H/C}}(k_r)\,\delta k_r = 2\int_0^{\pi}\int_{-\pi/2}^{\pi/2} g(\theta,\phi,k_r) \times v_x(\theta,\phi,k_r) f_{\mathrm{H/C}}(k)\,\mathrm{d}\theta\mathrm{d}\phi, \tag{3}$$

where the velocity of electrons in the x direction (perpendicular to the reservoir interface) is $v_x = \hbar m^{*-1} k_r \sin\phi\cos\theta$ and the factor of 2 accounts for the degeneracy due to electron spin. Assuming that the electrons in the reservoir abide by Fermi–Dirac distribution, and changing the variables by $E = (\hbar k)^2/2m^*$, we have

$$f_{\mathrm{H/C}}(E) = \left[1+\exp\left(\frac{E-\mu_{\mathrm{H/C}}}{k_B T_{\mathrm{H/C}}}\right)\right]^{-1}. \tag{4}$$

Then it can be obtained that

$$n_r^{H/C} = \frac{m^* E}{2\pi^2\hbar^3} f_{\mathrm{H/C}}(E). \tag{5}$$

An electron leaving from or entering into the hot or cold reservoir will let the reservoir remove or add an amount of heat $E-\mu_{\mathrm{H/C}}$. Thus, the expression of the net heat flux out of the hot/cold reservoir is

$$\dot{Q}_r^{\mathrm{H/C}} = \mp\int_0^{\infty}\left(E-\mu_{\mathrm{H/C}}\right)\left(n_r^{\mathrm{C}}-n_r^{\mathrm{H}}\right)\zeta(E)\,\mathrm{d}E, \tag{6}$$

where $\zeta(E)$ is the transmission probability of electrons with energy E. It can be derived that $\left(n_r^{\mathrm{C}}-n_r^{\mathrm{H}}\right)>0$ and $\left(E-\mu_{\mathrm{H/C}}\right)>0$ which leads the net heat flux $\dot{Q}_r^{\mathrm{H}}$ to be negative and cause $\dot{Q}_r^{\mathrm{C}}$ to be positive when $\mu_{\mathrm{H}}<\mu_{\mathrm{C}}<E<E_{\mathrm{S}}$. The thermoelectric device operates as a refrigerator under these circumstances. The coefficient of performance (COP) can be expressed as

$$\varepsilon_r = \frac{\dot{Q}_r^{\mathrm{C}}}{-\dot{Q}_r^{\mathrm{H}}-\dot{Q}_r^{\mathrm{C}}}, \tag{7}$$

where $\dot{Q}_r^{\mathrm{C}}$ is the cooling rate.

The number of electrons with a kinetic energy in the x direction $E_x = \hbar^2 k_x^2/2m^*$ arriving at the reservoir interface per unit area per unit time is[12]

$$n_x^{\mathrm{H/C}} = \frac{m^* k_{\mathrm{B}} T_{\mathrm{H/C}}}{2\pi^2\hbar^3} \times \log\left[1+\exp\left(-\frac{E_x-\mu_{\mathrm{H/C}}}{k_{\mathrm{B}}T_{\mathrm{H/C}}}\right)\right]. \tag{8}$$

It is worthy to note the energy change in an electron with energy E_x arriving at or leaving from the hot/cold reservoir. Under the Maxwell–Boltzmann statistics, we take the energy change $E_x + k_{\mathrm{B}}T_{\mathrm{H/C}} - \mu_{\mathrm{H/C}}$ to denote the energy of its leaving, and take $E_x + k_{\mathrm{B}}T_{\mathrm{C/H}} - \mu_{\mathrm{H/C}}$ to represent the energy of its arriving, where $k_{\mathrm{B}}T_{\mathrm{C/H}}$ is the average energy of the other two dimensions. The heat flux out of the two reservoirs in a 'k_x-filter' device is

$$\dot{Q}_x^{\mathrm{H/C}} = \mp\int_0^{\infty}\left[\left(E_x+k_{\mathrm{B}}T_{\mathrm{H/C}}-\mu_{\mathrm{H/C}}\right)n_r^{\mathrm{C}} - \left(E_x+k_{\mathrm{B}}T_{\mathrm{C/H}}-\mu_{\mathrm{H/C}}\right)n_r^{\mathrm{H}}\right]\zeta(E_x)\,\mathrm{d}E, \tag{9}$$

where $\zeta(E_x)$ is the transmission probability of electrons with energy E_x. When the net heat flux $\dot{Q}_x^{\mathrm{H}}$ is negative and $\dot{Q}_x^{\mathrm{C}}$ is positive, the device will operate as a refrigerator. The coefficient of performance (COP) is

$$\varepsilon_x = \frac{\dot{Q}_x^{\mathrm{C}}}{-\dot{Q}_x^{\mathrm{H}}-\dot{Q}_x^{\mathrm{C}}}. \tag{10}$$

3. Model of a thermoelectric refrigerator with two resonances

The schematic diagram of a conventional thermoelectric refrigerator with two resonances is shown in Fig. 2, where it consists of two electron reservoirs with different temperatures and electrochemical potentials. E_1 or E_2 is the resonance energy centre, and $E_2 = E_1 + \Delta E$. For convenience, we assume that the transmission probability is a Gaussian shaped one,[13]

$$\zeta(E_{\mathrm{i}}) = \exp\left(-\frac{(E-E_{\mathrm{i}})^2}{w}\right), \tag{11}$$

where i is 1 or 2, w is the width parameter of the resonance energy level. According to Eqs. (6) and (7), the net heat flux out of the two reservoirs with a 'k_r-filter' may be expressed as

$$\dot{Q}_{r(1/2)}^{\mathrm{H/C}} = \mp\int_0^{\infty}\left(E-\mu_{\mathrm{H/C}}\right)$$

$$\times \left(n_r^{\rm C} - n_r^{\rm H}\right) \zeta \left(E_{1/2}\right) {\rm d}E, \qquad (12)$$

the cooling rate is written as

$$\dot{Q}_r^{\rm C} = \dot{Q}_{r(1)}^{\rm C} + \dot{Q}_{r(2)}^{\rm C}, \qquad (13)$$

the power input as

$$P_r = -\dot{Q}_{r(1)}^{\rm H} - \dot{Q}_{r(2)}^{\rm H} - \dot{Q}_{r(1)}^{\rm C} - \dot{Q}_{r(2)}^{\rm C}, \qquad (14)$$

and the COP as

$$\varepsilon_r = \frac{\dot{Q}_{r(1)}^{\rm C} + \dot{Q}_{r(2)}^{\rm C}}{-\dot{Q}_{r(1)}^{\rm H} - \dot{Q}_{r(2)}^{\rm H} - \dot{Q}_{r(1)}^{\rm C} - \dot{Q}_{r(2)}^{\rm C}}, \qquad (15)$$

where the superscript H/C in $\dot{Q}_{r(1/2)}^{\rm H/C}$ means hot or cold reservoir, and the subscript $r\,(1/2)$ means the first or the second resonance in a 'k_r-filter' refrigerator with two resonances. Accordingly, the net heat flux out of the two reservoirs with a 'k_x-filter' may be expressed as

$$\begin{aligned}\dot{Q}_{x(1/2)}^{\rm H/C} = \mp \int_0^\infty & \left[\left(E_x + k_{\rm B}T_{\rm H/C} - \mu_{\rm H/C}\right) n_r^{\rm C}\right. \\ & \left. - \left(E_x + k_{\rm B}T_{\rm C/H} - \mu_{\rm H/C}\right) n_r^{\rm H}\right] \\ & \times \zeta \left(E_{x(1/2)}\right) {\rm d}E, \end{aligned} \qquad (16)$$

the cooling rate is written as

$$\dot{Q}_x^{\rm C} = \dot{Q}_{x(1)}^{\rm C} + \dot{Q}_{x(2)}^{\rm C}, \qquad (17)$$

the power input as

$$P_x = -\dot{Q}_{x(1)}^{\rm H} - \dot{Q}_{x(2)}^{\rm H} - \dot{Q}_{x(1)}^{\rm C} - \dot{Q}_{x(2)}^{\rm C}, \qquad (18)$$

and the COP as

$$\varepsilon_x = \frac{\dot{Q}_{x(1)}^{\rm C} + \dot{Q}_{x(2)}^{\rm C}}{-\dot{Q}_{x(1)}^{\rm H} - \dot{Q}_{x(2)}^{\rm H} - \dot{Q}_{x(1)}^{\rm C} - \dot{Q}_{x(2)}^{\rm C}}. \qquad (19)$$

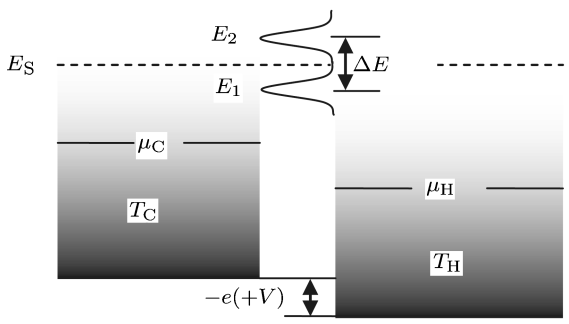


Fig. 2. Schematic diagram of a conventional thermoelectric refrigerator with two resonances.

4. Numerical simulation

The parameters $T_{\rm H} = 1.5$ K, $T_{\rm C} = 1$ K, $\mu_{\rm H}/k_{\rm B} = 14$ K, $\mu_{\rm C}/k_{\rm B} = 16$ K, $w/\left(k_{\rm B}\right)^2 = 0.01$ K^2, $\Delta E/k_{\rm B} = 0.5$ K are given in numerical calculation. According to Eqs. (16) and (19), the curves of cooling rate $\dot{Q}^{\rm C}$ versus the relative performance coefficient (RCOP) $\varepsilon/\varepsilon_{\rm C}$ of a k_x filtered refrigerator with one or two resonances are shown in Fig. 3, where $\varepsilon_{\rm C} = T_{\rm C}/\left(T_{\rm H} - T_{\rm C}\right)$ is the COP of the Carnot refrigerator. It is found that the maximum cooling rate produced by the refrigerator with two resonances is 1.9 times that with one resonance, but the maximum RCOP is reduced by 3.7%. It is also shown that the thermoelectric refrigerator with a 'k_x-filter' cannot reach the Carnot limit.[11]

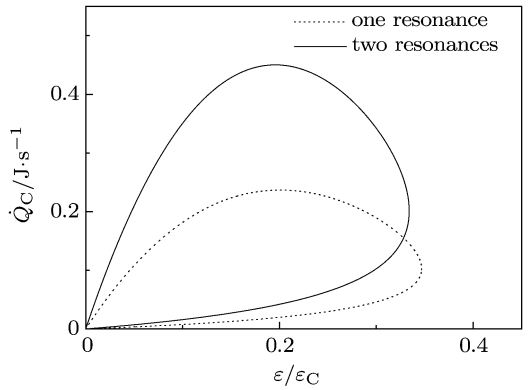


Fig. 3. Curves of cooling rate versus RCOP of a k_x filtered refrigerator.

Similarly, according to Eqs. (12) and (15), the curves of cooling rate $\dot{Q}^{\rm C}$ versus RCOP $\varepsilon/\varepsilon_{\rm C}$ of a k_r filtered refrigerator with one or two resonances are shown in Fig. 4.

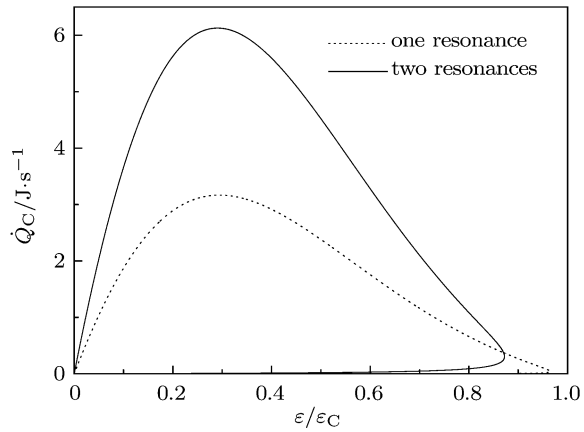


Fig. 4. Curves of cooling rate versus RCOP of a k_r filtered refrigerator.

It is found that the maximum cooling rate produced by the refrigerator with two resonances is 1.935

times that with one resonance, but the maximum RCOP is reduced by 9.6%. The energy selective centre $E_1 = E_S$ and width $w \to 0$ of the 'k_r-filter' thermoelectric refrigerator with one resonance is able to reach the Carnot limit.

In the thermoelectric refrigerator with two resonances, the cooling rate curves and the RCOP versus resonance energy centre E_1/k_B are shown in Figs. 5(a) and 5(b), respectively. For a k_x filtered refrigerator, the energy axis represents the average of both cold and hot reservoir energies, i.e. $E_x + 1/2\,(k_B T_C + k_B T_H)$.

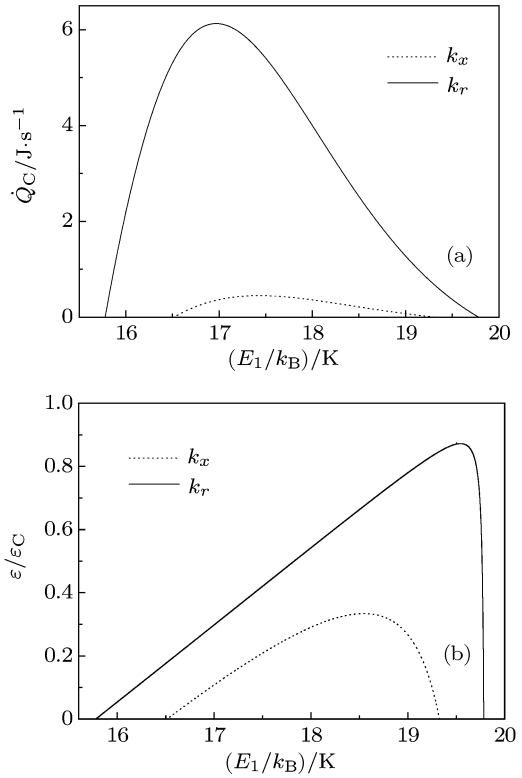


Fig. 5. (a) Curves for cooling rate versus resonance energy centre E_1/k_B, and (b) curves for RCOP versus resonance energy centre E_1/k_B.

Obviously, it is seen from Fig. 5 that the cooling rate $\dot{Q}^C$ and relative coefficient of performance $\varepsilon/\varepsilon_C$ of the 'k_r-filtered' thermoelectric refrigerator are larger than those of the 'k_x-filtered' one. The maximum cooling rate of the 'k_r-filtered' refrigerator is 13.6 times that of the 'k_x-filtered' one, and the maximum RCOP of the 'k_r-filtered' refrigerator is 2.6 times that of 'k_x-filtered' one at the given parameters. The curves for cooling rate versus RCOP of the 'k_r-filtered' and 'k_x-filtered' refrigerators with two resonances are plotted as shown in Fig. 6.

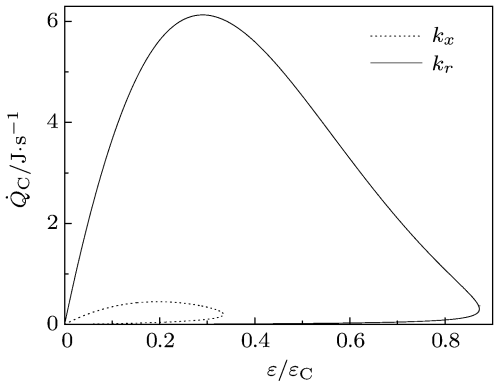


Fig. 6. Curves for cooling rate versus RCOP of 'k_r-filtered' and 'k_x-filtered' refrigerators with two resonances.

It is also shown from Figs. 5 and 6 that the cooling rate of the 'k_r-filtered' refrigerator is much larger than that of the 'k_x-filtered' one as the transmitted electrons in the 'k_r-filtered' one are much more than those in the 'k_x-filtered' one. While the 'k_r-filtered' refrigerator achieves reversibility when the transmission energy is equal to E_S, 'k_x-filtered' one does not, owing to the finite energy spread associated with the two unfiltered degrees of freedom, which causees the RCOP of the 'k_r-filtered' refrigerator to be more than that of 'k_x-filtered' one.

5. Conclusions

The performances of 'k_r-filtered' and 'k_x-filtered' thermoelectric refrigerators with one resonance or two resonances have been studied. For a given filter mechanism, the cooling rate of the thermoelectric refrigerators with two resonances increases but the COP decreases. For a given two resonances, the cooling rate and the COP of the 'k_r-filtered' thermoelectric refrigerators are both larger than those of the 'k_x-filtered' one. Therefore, it can be concluded that in the two-resonance energy selective mechanism, the 'k_r-filtered' thermoelectric refrigerator may operate better than the 'k_x-filtered' one.

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