Global synchronization of general delayed complex networks with stochastic disturbances*

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In this paper, global synchronization of general delayed complex networks with stochastic disturbances, which is a zero-mean real scalar Wiener process, is investigated. The networks under consideration are continuous-time networks with time-varying delay. Based on the stochastic Lyapunov stability theory, Itô's differential rule and the linear matrix inequality (LMI) optimization technique, several delay-dependent synchronous criteria are established, which guarantee the asymptotical mean-square synchronization of drive networks and response networks with stochastic disturbances. The criteria are expressed in terms of LMI, which can be easily solved using the Matlab LMI Control Toolbox. Finally, two examples show the effectiveness and feasibility of the proposed synchronous conditions.

Keywords: global synchronization, general delayed complex networks with stochastic disturbances, linear matrix inequality, mean-square stability

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1. Introduction

It is well known that complex networks exist in many natural and man-made systems, e.g., food webs, neural networks, cellular and metabolic networks, electrical power grids, computer networks, technological networks, the World Wide Web, coauthorship and citation networks, social networks, etc.[1,2] Over the past few years, complex networks have received increasing research attention from all fields of basic science and technological practice. Analysis and control of complex behaviours in complex networks in particular have attracted the attention of researchers from different fields.[3−8]

Synchronization is an interesting phenomenon that occurs universally in nature and society.[1] In essence, synchronization is a form of self-organization. It has been demonstrated that many real-world problems are closely related to network synchronization.[1,2] For example, theoretical and experimental results show that a mammalian brain not only displays storage of associative memories, but also modulates oscillatory neuronal synchronization by selective perception attention.[8] The study of synchronization between dynamical systems was initiated in the early 1980s for deterministic chaos. Since then, the synchronization problem has attracted increasing research interest from many scientific and technical disciplines for more complex systems.[9−18] Furthermore, time-delay inevitably exists in natural and man-made networks. Due to the fact that time delays may cause undesirable dynamic network behaviours such as oscillation and instability, the synchronization problem of complex networks with time delays has gained increasing research attention.[11−13,15,18] Chen *et al.*[11] investigated the global synchronization of coupled delayed neural networks and its application to the chaotic cellular neural network (CNN) model. Lü and Chen[12] explored the synchronization criteria for a time-varying complex dynamical network. Zhou and Chen[13] studied synchronization in general complex delayed dynamical networks. In Ref. [15], pinning synchronization techniques were investigated for complex delayed dynamical networks.

However, stochastic disturbances have seldom been considered in the research on the synchronization of complex networks in past years. In real complex networks, signal transmission could be a noisy process brought on by random fluctuations from the release of probabilistic causes such as neurotransmitters. Analysis of the dynamical behaviours of complex networks shows that the obtained results are often largely affected by stochastic disturbances.[19−21] Recently, the synchronization problem of noise-perturbed net-

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works has received some initial research interest.[22−28] In Refs. [22]–[24], the exponential synchronizations of stochastic delayed discrete-time complex networks have been intensively investigated. Sun *et al.*[25] proposed some techniques to obtain the synchronization of stochastic perturbed chaotic delayed neural networks. Li and Yue [27] investigated the synchronization stability problem for a class of complex dynamical networks with Markovian jumping parameters and mixed time delays. In Ref. [28], an H_∞ pinning control scheme for a linear complex dynamical network with disturbance was proposed.

Motivated by the above observations, in this paper, we aim to deal with the global synchronization of general delayed complex networks with stochastic disturbances, which is a zero-mean real scalar Wiener process. Compared with Refs. [22]–[28], the present paper has two most important contributions: dealing with two general continuous-time networks with time-varying delay and stochastic disturbances; and proposing some novel delay-dependent criteria of globally asymptotic mean-square synchronization for a drive network and a response network. It should be pointed out that our hypotheses and the proposed controllers for network synchronization are very simple and easy to use.

The rest of the present paper is organized as follows. In Section 2 general complex dynamical networks with stochastic disturbances are introduced and several useful hypotheses are proposed. Some novel globally asymptotic mean-square synchronization criteria are given in Section 3, which guarantee the global asymptotical mean-square synchronization of drive networks and response networks with stochastic disturbances. In Section 4 two representative examples are given to show the effectiveness of the proposed synchronous schemes. Finally, in Section 5, some conclusions are drawn from the present study.

Notations. Throughout this paper, $\boldsymbol{X} > 0$ means that matrix $\boldsymbol{X}$ is real symmetric and positive definite. The superscript 'T' represents the matrix transposition. $\|\cdot\|$ refers to the Euclidean vector norm, and $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue. $\Re^n$ and $\Re^{n\times m}$ are the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. Let $(\Omega,\mathcal{F},\mathcal{P})$ be a complete probability space which is relevant to an increasing family $(\mathcal{F}_t)_{t>}$ of σ algebras $(\mathcal{F}_t)_{t>}\subset\mathcal{F}$, where Ω is the sample space, $\mathcal{F}$ is σ algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\|\cdot\|_{E_2}=\|E(\cdot)\|_2$ is the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. $\boldsymbol{E}$ is the identity matrix with an appropriate dimension.

2. Problem formulation and preliminaries

Consider a general complex network consisting of N linearly coupled identical nodes. Each node of the network is an n-dimensional dynamical system. The state equations of the entire network are

$$\dot{x}_i = f(x_i) + \sum_{j=1}^{N} a_{ij} h(x_j(t-\tau(t))),$$
$$i = 1,2,\ldots,N, \tag{1}$$

where $f:\Re^n\to\Re^n$ is an unknown but bounded nonlinear function; $x_i=(x_{i1},x_{i2},\ldots,x_{in})^T\in\Re^n$ and $x_j(t-\tau)=(x_{j1}(t-\tau(t)),\ldots,x_{jn}(t-\tau(t)))^{\mathrm{T}}\in\Re^n$ are the state variables of node i and j respectively; $\tau(t)$ is the time-varying delay and continuously differentiable satisfying

$$\dot{\tau}(t)\le\alpha<\infty,$$

in which α is a known nonzero constant; $A=(a_{ij})_{N\times N}$ is the coupling configuration matrix of the network; $h:\Re^n\to\Re^n$ is the inner-coupling function and also unknown but bounded nonlinear function. Especially, when it is linear, model (1) can be described by

$$\dot{x}_i = f(x_i) + \sum_{j=1}^{N} a_{ij}\boldsymbol{B}x_j(t-\tau(t)),$$
$$i = 1,2,\ldots,N, \tag{2}$$

where $\boldsymbol{B}=(b_{ij})_{n\times n}\in R^{n\times n}$ is an inner connecting matrix in each node.

Defining $\boldsymbol{x}(t)=(x_1(t),\ldots,x_N(t))^{\mathrm{T}}$, $F(x(t))=(f(x_1(t)),\ldots,f(x_N(t)))^{\mathrm{T}}$, and

$$H(x(t-\tau(t))) = (h(x_1(t-\tau(t))),\ldots,h(x_N(t-\tau(t))))^{\mathrm{T}},$$

network (1) can be rewritten as

$$\mathrm{d}x(t) = (F(x(t)) + AH(x(t-\tau(t))))\,\mathrm{d}t. \tag{3}$$

In this paper, we will take network (3) as a drive system. The noise-perturbed response network is given by

$$\mathrm{d}y(t) = (F(y(t)) + AH(y(t-\tau(t))) + u(t))\,\mathrm{d}t + G(t,y(t)-x(t),y(t-\tau(t))$$

$$-x\left(t-\tau\left(t\right)\right))\mathrm{d}w\left(t\right), \tag{4}$$

where $u\left(t\right)=\left(u_1\left(t\right),u_2\left(t\right),\ldots,u_N\left(t\right)\right)\in\Re^N$ is the state feedback controller given to achieve the global delay-dependent synchronization of drive network (3) with response network (4); $w\left(t\right)$ is a zero-mean real scalar Wiener process (Brownian motion) defined in the complete probability space $(\Omega,\mathcal{F},\mathcal{P})$, satisfying

$$E\left(\mathrm{d}w\left(t\right)\right)=0,\quad \text{and}\quad E\left(\mathrm{d}w\left(t\right)^2\right)=\mathrm{d}t, \tag{5}$$

and $G:\Re\times\Re^N\times\Re^N\to\Re^N$ is the diffusion coefficient vector and bounded.

Remark 1 Compared with the earlier discussion in Refs. [22]–[26], the discussion in the present paper is devoted mainly to the global delay-dependent synchronization of two continuous-time networks with time-varying delay.

Generally speaking, network (3) and noise-perturbed network (4) are said to archive asymptotic mean-square synchronization if

$$\lim_{t\to\infty}E\left\{\left\|x_i\left(t\right)-y_i\left(t\right)\right\|^2\right\}=0,\quad 1\leq i\leq N. \tag{6}$$

Throughout this paper, the following assumptions are made.

Assumption 1 There exists a matrix $\boldsymbol{Q}\in\Re^{N\times N}$ such that

$$\left\|F\left(x\left(t\right)\right)-F\left(y\left(t\right)\right)\right\|\leq\left\|Q\left(x\left(t\right)-y\left(t\right)\right)\right\|, \tag{7}$$

for all $x,y\in\Re^N$.

Assumption 2 There exists a matrix $R\in\Re^{N\times N}$ such that

$$\left\|H\left(x\left(t\right)\right)-H\left(y\left(t\right)\right)\right\|\leq\left\|R\left(x\left(t\right)-y\left(t\right)\right)\right\|, \tag{8}$$

for all $x,y\in\Re^N$.

Assumption 3 The function $G\left(\cdot,\cdot,\cdot\right)$ satisfies the Lipschitz condition and there exist matrices G_1 and G_2 such that

$$\left\|G\left(t,x,y\right)\right\|^2\leq\left\|G_1x\right\|^2+\left\|G_2y\right\|^2, \tag{9}$$

for all $x,y\in\Re^N$ and $t\in\Re$.

Defining the synchronization error $e_i\left(t\right)=y_i\left(t\right)-x_i\left(t\right)$ $(i=1,2,\ldots,N)$, and $e\left(t\right)=\left(e_1\left(t\right)\ e_2\left(t\right)\ \ldots\ e_N\left(t\right)\right)^{\mathrm{T}}$, the error stochastic system can be obtained as

$$\begin{aligned}\mathrm{d}e\left(t\right)=&\left(F_{\mathrm{c}}\left(e\left(t\right)\right)+AH_{\mathrm{c}}\left(e\left(t-\tau\left(t\right)\right)\right)+u\left(t\right)\right)\mathrm{d}t\\&+G\left(t,e\left(t\right),e\left(t-\tau\left(t\right)\right)\right)\mathrm{d}w\left(t\right),\end{aligned} \tag{10}$$

where

$$\begin{aligned}F_{\mathrm{c}}\left(e\left(t\right)\right)=&\left(f\left(y_1\left(t\right)\right)-f\left(x_1\left(t\right)\right),\ldots,f\left(y_N\left(t\right)\right)\right.\\&\left.-f\left(x_N\left(t\right)\right)\right)^{\mathrm{T}}\end{aligned}$$

and

$$\begin{aligned}&H_{\mathrm{c}}\left(e\left(t-\tau\left(t\right)\right)\right)\\=&\left(h\left(y_1\left(t-\tau\left(t\right)\right)\right)-h\left(x_1\left(t-\tau\left(t\right)\right)\right),\ldots,\right.\\&\left.h\left(y_N\left(t-\tau\left(t\right)\right)\right)-h\left(x_N\left(t-\tau\left(t\right)\right)\right)\right)^{\mathrm{T}}.\end{aligned}$$

According to assumptions 1, 2 and 3, we have the following inequalities:

$$F_{\mathrm{c}}^{\mathrm{T}}\left(e\left(t\right)\right)F_{\mathrm{c}}\left(e\left(t\right)\right)\leq e^{\mathrm{T}}\left(t\right)Q^{\mathrm{T}}Qe\left(t\right), \tag{11}$$

$$H_{\mathrm{c}}^{\mathrm{T}}\left(e\left(t\right)\right)H_{\mathrm{c}}\left(e\left(t\right)\right)\leq e^{\mathrm{T}}\left(t\right)R^{\mathrm{T}}Re\left(t\right), \tag{12}$$

and

$$\begin{aligned}&G^{\mathrm{T}}\left(t,e\left(t\right),e\left(t-\tau\left(t\right)\right)\right)G\left(t,e\left(t\right),e\left(t-\tau\left(t\right)\right)\right)\\\leq&\ e^{\mathrm{T}}\left(t\right)G_1^{\mathrm{T}}G_1e\left(t\right)\\&+e^{\mathrm{T}}\left(t-\tau\left(t\right)\right)G_2^{\mathrm{T}}G_2e\left(t-\tau\left(t\right)\right).\end{aligned} \tag{13}$$

Remark 2 Drive network (3) and noise-perturbed response network (4) are said to be globally synchronized if, for an appropriately designed feedback controller, the trivial solution of error system (10) is globally stable in the mean square. That is to say, system (3) and system (4) are synchronized with each other if

$$\lim_{t\to\infty}E\left\{\left\|e(t)\right\|^2\right\}=0. \tag{14}$$

In the investigation in this paper, we mainly use linear matrix inequality (LMI), so we first give the LMI,[29] that is, the matrix inequality

$$\left[\begin{array}{cc}\bar{Q}\left(x\right)&\bar{S}\left(x\right)\\\bar{S}\left(x\right)^{\mathrm{T}}&\bar{R}\left(x\right)\end{array}\right]<0, \tag{15}$$

where $\bar{Q}\left(x\right)=\bar{Q}\left(x\right)^{\mathrm{T}}$, $\bar{R}\left(x\right)=\bar{R}\left(x\right)^{\mathrm{T}}$ and $\bar{S}\left(x\right)$ depend affinely on x, and they are equivalent to

$$\begin{aligned}&\bar{R}\left(x\right)<0,\quad \text{and}\\&\bar{Q}\left(x\right)-\bar{S}\left(x\right)\bar{R}\left(x\right)^{-1}\bar{S}\left(x\right)^{\mathrm{T}}<0.\end{aligned} \tag{16}$$

3. Main results

In this section, we consider the global delay-dependent mean square synchronization of system (3) with system (4). Some elegant results on synchronization criteria are derived. The delay-dependent results are obtained based on Lyapunov functional method combined with the stochastic analysis, as well as the

feedback control technique, Itô's differential rule and the LMI technique.

In order to realize the synchronization of drive system (3) with response system (4), we design a suitable feedback controller, which is

$$u(t) = K_1 e(t) + K_2 e(t-\tau(t)), \tag{17}$$

where K_1 and K_2 are the feedback gains.

With assumptions 1, 2, and 3 and let feedback controller (17) be given, error system (10) can be rewrite as

$$\begin{aligned} \mathrm{d}e(t) = {} & (F_{\mathrm{c}}(e(t)) + AH_{\mathrm{c}}(e(t-\tau(t))) \\ & + K_1 e(t) + K_2 e(t-\tau(t)))\mathrm{d}t \\ & + G(t, e(t), e(t-\tau(t)))\,\mathrm{d}w(t). \end{aligned} \tag{18}$$

In order to obtain delay-dependent synchronization criteria for drive system (3) and noise-perturbed response system (4), the following Lyapunov–Krasovskii functional is constructed:

$$V(t) = V_1(t) + V_2(t) \tag{19}$$

in which

$$V_1(t) = e^{\mathrm{T}}(t) P e(t), \tag{20}$$

$$V_2(t) = \int_{t-\tau(t)}^{t} H_{\mathrm{c}}^{\mathrm{T}}(e(s)) A^{\mathrm{T}} \boldsymbol{P} A H_{\mathrm{c}}(e(s))\,\mathrm{d}s, \tag{21}$$

with $\boldsymbol{P}$ being a positive matrix. Therefore, the derivations of Eqs. (20) and (21) along with Eq. (18) are

$$\begin{aligned} \mathrm{d}V_1(t) = {} & \mathrm{d}\left(e^{\mathrm{T}}(t)\boldsymbol{P}e(t)\right) \\ = {} & (F_{\mathrm{c}}^{\mathrm{T}}(e(t))\boldsymbol{P}e(t) + H_{\mathrm{c}}^{\mathrm{T}}(e(t-\tau(t)))A^{\mathrm{T}}\boldsymbol{P}e(t) + e^{\mathrm{T}}(t)K_1^{\mathrm{T}}\boldsymbol{P}e(t) \\ & + e^{\mathrm{T}}(t-\tau(t))K_2^{\mathrm{T}}\boldsymbol{P}e(t))\mathrm{d}t + G^{\mathrm{T}}(t, e(t), e(t-\tau(t)))\,\mathrm{d}w(t)\boldsymbol{P}e(t) \\ & + (e^{\mathrm{T}}(t)\boldsymbol{P}F_{\mathrm{c}}(e(t)) + e^{\mathrm{T}}(t)\boldsymbol{P}AH_{\mathrm{c}}(e(t-\tau(t))) + e^{\mathrm{T}}(t)\boldsymbol{P}K_1 e(t) \\ & + e^{\mathrm{T}}(t)\boldsymbol{P}K_2 e(t-\tau(t)))\mathrm{d}t + e^{\mathrm{T}}(t)\boldsymbol{P}G(t, e(t), e(t-\tau(t)))\,\mathrm{d}w(t), \end{aligned} \tag{22}$$

and

$$\mathrm{d}V_2(t) = H_{\mathrm{c}}^{\mathrm{T}}(e(t))A^{\mathrm{T}}\boldsymbol{P}AH_{\mathrm{c}}(e(t))\,\mathrm{d}t - H_{\mathrm{c}}^{\mathrm{T}}(e(t-\tau(t)))A^{\mathrm{T}}\boldsymbol{P}AH_{\mathrm{c}}(e(t-\tau(t)))(1-\dot{\tau})\,\mathrm{d}t. \tag{23}$$

Applying the Itô formula, the derivation of Lyapunov–Krasovskii functional (19) given above can described by

$$\mathrm{d}V(t) = \Phi(t)\,\mathrm{d}t + 2e^{\mathrm{T}}(t)\boldsymbol{P}G(t, e(t), e(t-\tau(t)))\,\mathrm{d}w(t), \tag{24}$$

in which

$$\begin{aligned} \Phi(t) = {} & e^{\mathrm{T}}(t)K_1^{\mathrm{T}}\boldsymbol{P}e(t) + e^{\mathrm{T}}(t)\boldsymbol{P}K_1 e(t) + e^{\mathrm{T}}(t-\tau(t))K_2^{\mathrm{T}}\boldsymbol{P}e(t) + e^{\mathrm{T}}(t)\boldsymbol{P}K_2 e(t-\tau(t)) \\ & + H_{\mathrm{c}}^{\mathrm{T}}(e(t))A^{\mathrm{T}}\boldsymbol{P}AH_{\mathrm{c}}(e(t)) + H_{\mathrm{c}}^{\mathrm{T}}(e(t-\tau(t)))A^{\mathrm{T}}\boldsymbol{P}e(t) + e^{\mathrm{T}}(t)\boldsymbol{P}AH_{\mathrm{c}}(e(t-\tau(t))) \\ & - H_{\mathrm{c}}^{\mathrm{T}}(e(t-\tau(t)))A^{\mathrm{T}}\boldsymbol{P}AH_{\mathrm{c}}(e(t-\tau(t)))(1-\dot{\tau}) + F_{\mathrm{c}}^{\mathrm{T}}(e(t))\boldsymbol{P}e(t) \\ & + e^{\mathrm{T}}(t)\boldsymbol{P}F_{\mathrm{c}}(e(t)) + G^{\mathrm{T}}(t, e(t), e(t-\tau(t)))\boldsymbol{P}G(t, e(t), e(t-\tau(t))). \end{aligned} \tag{25}$$

Moreover,

$$\begin{aligned} & F_{\mathrm{c}}^{\mathrm{T}}(e(t))\boldsymbol{P}e(t) + e^{\mathrm{T}}(t)\boldsymbol{P}F_{\mathrm{c}}(e(t)) \\ = {} & -\left[P^{1/2}F_{\mathrm{c}}(e(t)) - P^{1/2}e(t)\right]^{\mathrm{T}}\left[P^{1/2}F_{\mathrm{c}}(e(t)) - P^{1/2}e(t)\right] + F_{\mathrm{c}}^{\mathrm{T}}(e(t))\boldsymbol{P}F_{\mathrm{c}}(e(t)) + e^{\mathrm{T}}(t)\boldsymbol{P}e(t) \\ \le {} & F_{\mathrm{c}}^{\mathrm{T}}(e(t))\boldsymbol{P}F_{\mathrm{c}}(e(t)) + e^{\mathrm{T}}(t)\boldsymbol{P}e(t), \end{aligned} \tag{26}$$

and

$$\begin{aligned} & H_{\mathrm{c}}^{\mathrm{T}}(e(t-\tau(t)))A^{\mathrm{T}}\boldsymbol{P}e(t) + e^{\mathrm{T}}(t)\boldsymbol{P}AH_{\mathrm{c}}(e(t-\tau(t))) - (1-\dot{\tau})H_{\mathrm{c}}^{\mathrm{T}}(e(t-\tau(t)))A^{\mathrm{T}}\boldsymbol{P}AH_{\mathrm{c}}(e(t-\tau(t))) \\ = {} & -\left[(1-\dot{\tau})^{1/2}P^{1/2}AH_{\mathrm{c}}(e(t-\tau(t))) - (1-\dot{\tau})^{-\frac{1}{2}}P^{1/2}e(t)\right]^{\mathrm{T}} \\ & \times\left[(1-\dot{\tau})^{1/2}P^{1/2}AH_{\mathrm{c}}(e(t-\tau(t))) - (1-\dot{\tau})^{-1/2}P^{1/2}e(t)\right] \\ & + (1-\dot{\tau})^{-1}e^{\mathrm{T}}(t)\boldsymbol{P}e(t) \le (1-\alpha)^{-1}e^{\mathrm{T}}(t)\boldsymbol{P}e(t). \end{aligned} \tag{27}$$

Therefore, with inequalities (26) and (27), expression (25) can be described as

$$\begin{aligned}\Phi(t) \leq\ & e^{\mathrm{T}}(t) K_1^{\mathrm{T}} \boldsymbol{P} e(t) + e^{\mathrm{T}}(t) \boldsymbol{P} K_1 e(t) + e^{\mathrm{T}}(t-\tau(t)) K_2^{\mathrm{T}} \boldsymbol{P} e(t) + e^{\mathrm{T}}(t) \boldsymbol{P} K_2 e(t-\tau(t)) \\ & + (1-\alpha)^{-1} e^{\mathrm{T}}(t) \boldsymbol{P} e(t) + \lambda_1 e^{\mathrm{T}}(t) Q^{\mathrm{T}} Q e(t) + e^{\mathrm{T}}(t) \boldsymbol{P} e(t) \\ & + \lambda_2 e^{\mathrm{T}}(t) R^{\mathrm{T}} R e(t) + \lambda_1 e^{\mathrm{T}}(t) G_1^{\mathrm{T}} G_1 e(t) + \lambda_1 e^{\mathrm{T}}(t-\tau(t)) G_2^{\mathrm{T}} G_2 e(t-\tau(t)), \end{aligned} \tag{28}$$

where $\lambda_1 = \lambda_{\max}(\boldsymbol{P})$, and $\lambda_2 = \lambda_{\max}\left(A^{\mathrm{T}} \boldsymbol{P} A\right)$. Let $\eta(t) = (e(t-\tau(t)), e(t))^{\mathrm{T}}$, we have

$$\Phi(t) \leq \eta^{\mathrm{T}}(t) \Pi \eta(t), \tag{29}$$

in which

$$\boldsymbol{\Pi} = \begin{pmatrix} \lambda_1 G_2^{\mathrm{T}} G_2 & K_2^{\mathrm{T}} \boldsymbol{P} \\ \boldsymbol{P} K_2 & K_1^{\mathrm{T}} \boldsymbol{P} + \boldsymbol{P} K_1 + \lambda_2 R^{\mathrm{T}} R + (1-\alpha)^{-1} \boldsymbol{P} + \lambda_1 Q^{\mathrm{T}} Q + \boldsymbol{P} + \lambda_1 G_1^{\mathrm{T}} G_1 \end{pmatrix}. \tag{30}$$

If the matrix

$$\boldsymbol{\Pi} = \begin{pmatrix} \lambda_1 G_2^{\mathrm{T}} G_2 & K_2^{\mathrm{T}} \boldsymbol{P} \\ \boldsymbol{P} K_2 & K_1^{\mathrm{T}} \boldsymbol{P} + \boldsymbol{P} K_1 + \lambda_2 R^{\mathrm{T}} R + (1-\alpha)^{-1} \boldsymbol{P} + \lambda_1 Q^{\mathrm{T}} Q + \boldsymbol{P} + \lambda_1 G_1^{\mathrm{T}} G_1 \end{pmatrix} < 0, \tag{31}$$

we obtain

$$\Phi(t) \leq \eta^{\mathrm{T}}(t) \boldsymbol{\Pi} \eta(t) < 0,$$

which implies that error stochastic system (10) is stochastically robust mean square asymptotically stable for all admissible uncertainties. That is to say, noise-perturbed system (4) is globally delay-dependent synchronized with system (3) under condition (31) and a positive matrix $\boldsymbol{P} > 0$.

The above result (31) is a delay-dependent global synchronous criterion in the explicit expression, which can be easily used when feedback controller (17) is given. However, it is not always easy to know the control parameter for practical problems. Therefore, the controller parameters $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ need to be found. We are now ready to give the parameterization of the controller gains in the following.

Defining $\boldsymbol{P}\boldsymbol{K}_1 = \boldsymbol{X}$ and $\boldsymbol{P}\boldsymbol{K}_2 = \boldsymbol{Y}$, in which $\boldsymbol{X}$ and $\boldsymbol{Y}$ are two matrices each with an appropriate dimension, inequality (31) can be rewrite as

$$\begin{pmatrix} \lambda_1 G_2^{\mathrm{T}} G_2 & \boldsymbol{Y}^{\mathrm{T}} \\ \boldsymbol{Y} & \boldsymbol{X}^{\mathrm{T}} + \boldsymbol{X} + \lambda_2 \left(R^{\mathrm{T}} R\right) + (1-\alpha)^{-1} \boldsymbol{P} + \lambda_1 \left(Q^{\mathrm{T}} Q\right) + \boldsymbol{P} + \lambda_1 G_1^{\mathrm{T}} G_1 \end{pmatrix} < 0. \tag{32}$$

With inequality (32), the trivial solution of error system (10) is globally asymptotically stable in the mean square via the feedback control law

$$u(t) = \boldsymbol{P}^{-1} \boldsymbol{X} e(t) + \boldsymbol{P}^{-1} \boldsymbol{Y} e(t-\tau(t)). \tag{33}$$

When we replace the matrix $\boldsymbol{R}$ in expression (32) with the matrix $\boldsymbol{B}$, a criterion

$$\begin{pmatrix} \lambda_1 G_2^{\mathrm{T}} G_2 & \boldsymbol{Y}^{\mathrm{T}} \\ \boldsymbol{Y} & \boldsymbol{X}^{\mathrm{T}} + \boldsymbol{X} + \lambda_2 \left(B^{\mathrm{T}} B\right) + (1-\alpha)^{-1} \boldsymbol{P} + \lambda_1 \left(Q^{\mathrm{T}} Q\right) + \boldsymbol{P} + \lambda_1 G_1^{\mathrm{T}} G_1 \end{pmatrix} < 0 \tag{34}$$

can be obtained for network (2) with a linear inner-coupling function and response system (4), which guarantees that they are globally synchronized based on feedback control law (33).

Furthermore, if $\boldsymbol{P} = \boldsymbol{E}$, then inequality (32) can be described as

$$\begin{pmatrix} G_2^{\mathrm{T}} G_2 & \boldsymbol{Y}^{\mathrm{T}} \\ \boldsymbol{Y} & \boldsymbol{X}^{\mathrm{T}} + \boldsymbol{X} + \lambda \left(\boldsymbol{R}^{\mathrm{T}} \boldsymbol{R}\right) + (1-\alpha)^{-1} \boldsymbol{E} + \left(Q^{\mathrm{T}} Q\right) + \boldsymbol{E} + G_1^{\mathrm{T}} G_1 \end{pmatrix} < 0, \tag{35}$$

where $\lambda = \lambda_{\max}\left(A^{\mathrm{T}} A\right)$. At the same time, the trivial solution of error system (10) is globally asymptotically stable in the mean square under the feedback controller

$$u(t) = \boldsymbol{X} e(t) + \boldsymbol{Y} e(t-\tau(t)). \tag{36}$$

4. Numerical example

In Section 3, some novel globally asymptotic mean-square synchronization criteria are given, which guarantee the global asymptotical mean-square synchronization of the drive network with the response network with stochastic disturbances. In this section, we give two examples to show the feasibility of the controller and the usefulness of the proposed technique for the investigation discussed above in this paper.

Example 1 Consider drive network (3) and noise-perturbed response network (4), and give the parameters as follows:

$$\boldsymbol{Q}=\begin{bmatrix}0.01 & 0 & 0\\ 0 & 0.02 & 0\\ 0 & 0 & 0.03\end{bmatrix},$$

$$\boldsymbol{R}=\begin{bmatrix}0.01 & 0.02 & 0.03\\ 0.03 & -0.02 & 0.03\\ 0 & 0.04 & 0\end{bmatrix},\quad \alpha=0.05,$$

$$\boldsymbol{G}_1=\begin{bmatrix}0.02 & 0.01 & 0.02\\ 0.01 & 0.02 & 0\\ 0 & 0.0 & 0.01\end{bmatrix},$$

$$\boldsymbol{G}_2=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

and

$$\boldsymbol{A}=\begin{bmatrix}0.02 & 0.01 & 0.04\\ 0.01 & 0.03 & 0\\ 0.02 & 0.03 & 0\end{bmatrix}.$$

At the same time, for the given feedback control (17), let the feedback gains be

$$\boldsymbol{K}_1=\begin{bmatrix}-1 & -2 & -3\\ -4 & -5 & -6\\ -2 & -5 & -8\end{bmatrix}$$

and

$$\boldsymbol{K}_2=\begin{bmatrix}0.1 & -0.2 & 0\\ 0 & -0.4 & 0.1\\ -0.2 & -0.3 & 0\end{bmatrix}.$$

Based on the Matlab LMI Control Toolbox to solve LMI (31), there exists a positive matrix

$$\boldsymbol{P}=\begin{bmatrix}0.0048 & 0.0095 & 0.0136\\ 0.0095 & 0.0167 & 0.0242\\ 0.0136 & 0.0242 & 0.0347\end{bmatrix},$$

which satisfies conditions (31).

Example 2 Consider drive network (3) and noise-perturbed response network (4), and give the parameters as follows:

$$\boldsymbol{Q}=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.3\end{bmatrix},$$

$$\boldsymbol{R}=\begin{bmatrix}0.1 & 0.2 & 0.3\\ 0.3 & -0.2 & 0.3\\ 0 & 0.4 & 0\end{bmatrix},\quad \alpha=0.05,$$

$$\boldsymbol{G}_1=\begin{bmatrix}0.2 & 0.1 & 0.2\\ 0.1 & 0.2 & 0\\ 0 & 0.2 & 0.1\end{bmatrix},$$

$$\boldsymbol{G}_2=\begin{bmatrix}0.2 & 0.1 & 0\\ 0.3 & 0.1 & 0.4\\ 0 & 0.4 & 0.1\end{bmatrix},$$

and

$$\boldsymbol{A}=\begin{bmatrix}0.2 & 0.1 & 0.4\\ 0.1 & 0.3 & 0\\ 0.2 & 0.3 & 0\end{bmatrix}.$$

With Matlab LMI Control Toolbox, there exist matrices

$$\boldsymbol{X}=\begin{bmatrix}-0.6413 & -0.0210 & 0.0205\\ -0.0625 & -0.8101 & 0.0050\\ -0.2500 & -0.1250 & -0.7978\end{bmatrix},$$

$$\boldsymbol{Y}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

and

$$\boldsymbol{P}=\begin{bmatrix}0.9607 & 0 & 0\\ 0 & 0.9607 & 0\\ 0 & 0 & 0.9607\end{bmatrix},$$

which meet conditions (32). Furthermore, the feedback controller can be described as

$$u(t)=\boldsymbol{P}^{-1}\boldsymbol{X}e(t)+\boldsymbol{P}^{-1}\boldsymbol{Y}e(t-\tau(t)),$$

where

$$\boldsymbol{K}_1=\boldsymbol{P}^{-1}\boldsymbol{X}=\begin{bmatrix}-0.2138 & -0.0070 & 0.0068\\ -0.0208 & -0.2700 & 0.0017\\ -0.0833 & -0.0417 & -0.2659\end{bmatrix}$$

and

$$\boldsymbol{K}_2 = \boldsymbol{P}^{-1}\boldsymbol{Y} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

5. Conclusion

Complex networks exist in many natural and man-made systems. In this paper, we deal mainly with the analysis of global delay-dependent synchronization of general delayed complex networks with stochastic disturbances, which is a zero-mean real scalar Wiener process. Compared with Refs. [22]–[28], the networks that we have discussed are continuous-time networks with time-varying delay and stochastic disturbances. By employing the stochastic Lyapunov stability theory, Itô's differential rule and the LMI optimization technique, some delay-dependent synchronous criteria are constructed such that the drive networks asymptotically synchronize with the response networks with stochastic disturbances in mean square. In the end of the paper, we give two examples to show the feasibility and the usefulness of our results.

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