Influence of selective atomic measurement on the entanglement properties of a two-atom outside cavity*

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Considering three two-level atoms initially in the W or Greenberger–Horne–Zeilinger (GHZ) state, one of the three atoms is put into an initially coherent light cavity and made to resonantly interact with the cavity. The two-atom entanglement evolution outside the cavity is investigated. The influences of state-selective measurement of the atom inside the cavity and strength of the light field on the two-atom entanglement evolution outside the cavity are discussed. The results obtained from the numerical method show that the two-atom entanglement outside the cavity is strengthened through state-selective measurement of the atom inside the cavity. In addition, the strength of the light field also influences the two-atom entanglement properties.

Keywords: quantum optics, two-level atom, selective atomic measurement, entanglement

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1. Introduction

Entanglement is the most fascinating feature of quantum mechanics. It provides strong tests of quantum non-locality and plays a central role in quantum information protocols, including quantum cryptography,[1] quantum teleportation,[2] quantum computations,[3] and quantum dense coding. So far, many schemes have been devoted to the generation of two-atom or multiple-atom entanglement in cavity quantum electrodynamics (cavity QED).[4−6] Therefore, characterization of entanglement for a given quantum state is a fundamental problem. There has been a lot of work on bipartite entanglement, such as von Neumann entropy,[7] distance between density operators,[8] negativity[9] and concurrence.[10] Since Gerry and Ghosh showed that squeezing could be greatly enhanced via selective atomic measurement,[11] the method of selective atomic measurement has been widely employed in quantum state engineering. For example, Yang and Guo[12] have considered a pair of two-level atoms initially in the Einstein–Podolsky–Rosen (EPR) single state, and put one of two atoms into a cavity. They drew a conclusion that the emission properties of the atom inside the cavity were greatly affected by the manipulation of the atom outside the cavity. Wu and Su[13] studied nonclassical properties in the resonant interaction of a three-level Λ-type atom with a two-mode field in coherent state. Ye[14] has shown the enhancement of squeezing in a two-photon Jaynes–Cummings model with atomic measurement. Remote control of the quantum statistical properties of a light field has been investigated by Zhou *et al.*[15] In the present paper we consider the case of three entangled two-level atoms initially in the W or GHZ state; one of them is put into an initially coherent state cavity and interacts resonantly with the coherent field. We then investigate the negative eigenvalues of partial transposition of a density matrix (which denotes the degree of entanglement between the two atoms). Finally, we discuss the influences of state-selective measurement of the atom inside the cavity and the strength of the light field on two-atom entanglement evolution outside the cavity by using numerical calculations.

2. Model

We consider the case where a two-level atom interacts resonantly with a single-mode field through single-photon transition. In the rotating-wave approximation, the Hamiltonian of the system can be written as (set $\hbar = 1$)

$$H_{\rm I} = g(as^+ + a^+ s^-), \tag{1}$$

where a^+ and a are the creation operator and the annihilation operator of the field. s^+ and s^- are the

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atomic rising operator and the atomic lowering operator, respectively, and g is the coupling coefficient between an atom and a field. Thus the time evolution unitary operator can be written as

$$U(t) = \begin{pmatrix} \cos At, & -\mathrm{i}ga\dfrac{\sin Bt}{B} \\ -\mathrm{i}ga^{+}\dfrac{\sin At}{A}, & \cos Bt \end{pmatrix}, \tag{2}$$

where $A = \sqrt{aa^{+}}g$ and $B = \sqrt{a^{+}a}g$. The state evolution of the atom–cavity system under the action of the Hamiltonian can be expressed as

$$\begin{aligned} |g\rangle|n\rangle &\rightarrow -\mathrm{i}\sin(\Omega gt)|n-1\rangle|e\rangle + \cos(\Omega gt)|n\rangle|g\rangle, \\ |e\rangle|n\rangle &\rightarrow -\mathrm{i}\sin(\Omega^{+}gt)|n+1\rangle|g\rangle \\ &\quad + \cos(\Omega^{+}gt)|n\rangle|e\rangle, \end{aligned} \tag{3}$$

where $\Omega = \sqrt{n}$ and $\Omega^{+} = \sqrt{n+1}$.

We consider the case where the three two-level atoms 1, 2 and 3 are initially in the W state, atom 3 is put into an initially coherent state cavity and interacts resonantly with the single-mode coherent field through single-photon transition. Atom 2 and atom 3 are outside the cavity. The initial state vector of the system can be written as

$$\begin{aligned} |\varphi(0)\rangle = \frac{1}{\sqrt{3}}\sum_{n=0}^{\infty} C_n(&|egg\rangle_{123} + |geg\rangle_{123} \\ &+ |gge\rangle_{123})|n\rangle, \end{aligned} \tag{4}$$

where $C_n = \exp(-|\alpha|^2/2)\alpha^n/\sqrt{n!}$, $\alpha = |\alpha|\exp(\mathrm{i}\beta)$, $|\alpha| = \sqrt{n_0}$, n_0 is the initial average photon number and β is the phase of the coherent field. For simplicity, set $\beta = 0$. Subscript i denotes the i-th atomic state. Then the state vector of the system at time t can be written as

$$\begin{aligned} |\varphi(t)\rangle = \frac{1}{\sqrt{3}}\sum_{n=0}^{\infty} C_n((&|eg\rangle_{12} + |ge\rangle_{12}) \\ &\times (-\mathrm{i}\sin(\Omega gt)|n-1\rangle|e\rangle_3 \\ &+ \cos(\Omega gt)|n\rangle|g\rangle_3) + |gg\rangle_{12} \\ &\times (-\mathrm{i}\sin(\Omega^{+}gt)|n+1\rangle|g\rangle_3 \\ &+ \cos(\Omega^{+}gt)|n\rangle|e\rangle_3)). \end{aligned} \tag{5}$$

Through tracing over the state of field and the state of atom 3, the density matrix of atoms 1 and 2 can be written as

$$\boldsymbol{\rho} = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

On the other hand, if the three two-level atoms 1, 2 and 3 are initially in the GHZ state, atom 3 is put into an initially coherent state cavity and interacts resonantly with the single-mode coherent field through single-photon transition. The initial state vector of the system is

$$|\varphi(0)\rangle = \frac{1}{\sqrt{2}}\sum_{n=0}^{\infty} C_n(|eee\rangle_{123} + |ggg\rangle_{123})|n\rangle. \tag{7}$$

The state vector of the system at time t is

$$\begin{aligned} |\varphi(t)\rangle = \frac{1}{\sqrt{2}}\sum_{n=0}^{\infty} C_n(&|gg\rangle_{12} \\ &\times (-\mathrm{i}\sin(\Omega gt)|n-1\rangle|e\rangle_3 \\ &+ \cos(\Omega gt)|n\rangle|g\rangle_3) + |ee\rangle_{12} \\ &\times (-\mathrm{i}\sin(\Omega^{+}gt)|n+1\rangle|g\rangle_3 \\ &+ \cos(\Omega^{+}gt)|n\rangle|e\rangle_3)). \end{aligned} \tag{8}$$

The density matrix of atoms 1 and 2 can be written as

$$\boldsymbol{\rho} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

3. Entanglement between two atoms outside cavity

In order to discuss the entanglement dynamics in the above system, we adopt the negative eigenvalues of the partial transposition of density matrix $\boldsymbol{\rho}$ to quantify the degree of entanglement. The state is separable if the eigenvalues of the partial transposition of density operator $\boldsymbol{\rho}$ are positive. However, the state is entangled if one of eigenvalues of $\boldsymbol{\rho}^{\mathrm{T}}$, where $\boldsymbol{\rho}^{\mathrm{T}}$ is the partial transposition of density matrix $\boldsymbol{\rho}$, is negative. For a two-qubit system described by the density operator, the negativity is defined by[16]

$$N = -2\sum_{i}\mu_i, \tag{10}$$

where μ_i are the negative eigenvalues of ρ^{T}. When $N = 0$, the two atoms are separable, and $N = 1$, they are in maximal entanglement.

We can make a partial transposition of $\boldsymbol{\rho}$ for the second atom and work out the partial transposition

reduced density matrix as follows:

$$\boldsymbol{\rho}^{\mathrm{T}} = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{31} & \rho_{32} \\ \rho_{21} & \rho_{22} & \rho_{41} & \rho_{42} \\ \rho_{13} & \rho_{14} & \rho_{33} & \rho_{34} \\ \rho_{23} & \rho_{24} & \rho_{43} & \rho_{44} \end{bmatrix}. \tag{11}$$

3.1. Entanglement between two atoms when three atoms are initially in W state

Using Eq. (6), when direct selective measurement is not performed. the degree of entanglement between the two atoms can be written as

$$N = \frac{1}{3}(\sqrt{5} - 1). \tag{12}$$

On the other hand, if atom 3 is selectively measured in state $|g\rangle$, the state vector of the system collapses onto the state

$$\begin{aligned} |\boldsymbol{\varphi}(t)\rangle \;=\; & F_1 \sum_{n=0}^{\infty} C_n((|eg\rangle_{12} + |ge\rangle_{12}) \cos(\Omega g t)\,|n\rangle \\ & -\, \mathrm{i} \sin(\Omega^{+} g t)\,|gg\rangle_{12}\,|n+1\rangle), \end{aligned} \tag{13}$$

where F_1 is a normalization factor, it can be expressed as

$$F_1^{-2} = \sum_{n=0}^{\infty} |C_n|^2 (2\cos^2(\Omega g t) + \sin^2(\Omega^{+} g t)).$$

The reduced density matrix of two atoms can be written as

$$\boldsymbol{\rho} = F_1^2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A & A & B \\ 0 & A & A & B \\ 0 & B^* & B^* & D \end{pmatrix}, \tag{14}$$

where

$$A = \sum_{n=0}^{\infty} |C_n|^2 \cos^2(\Omega g t),$$

$$B = \frac{\mathrm{i}}{2} \sum_{n=0}^{\infty} C_{n+1} C_n^* \sin(2\Omega^{+} g t),$$

and

$$D = \sum_{n=0}^{\infty} |C_n|^2 \sin^2(\Omega^{+} g t).$$

The partial transposition of density matrix $\boldsymbol{\rho}$ can be written as

$$\boldsymbol{\rho}^{\mathrm{T}} = F_1^2 \begin{pmatrix} 0 & 0 & 0 & A \\ 0 & A & 0 & B^* \\ 0 & 0 & A & B \\ A & B & B^* & D \end{pmatrix}. \tag{15}$$

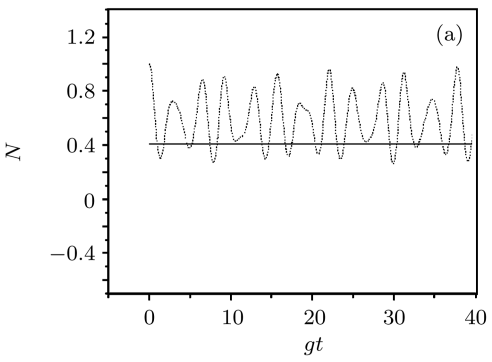
The eigenvalues of ρ^{T} are given by

$$\lambda_1 = AF^2 > 0,$$

λ_2, λ_3 and λ_4 are the solutions of the following equation:

$$\lambda^3 + p\lambda^2 + q\lambda + r = 0, \tag{16}$$

where $p = -(A + D)F_1^2$, $q = (AD - 2|B|^2 - A^2)F_1^4$, and $r = A^3 F_1^6$. Figure 1 shows the numerical results of the entanglement time evolution of the two atoms. In Fig. 1, the coordinates represent the degree of entanglement between the two atoms N, the x-coordinates refer to the scaled time gt, with $|\alpha|^2 = 0.5$ (a), 2 (b), 5 (c), and 10 (d). In Fig.1 the solid curve represents the result without performing the direct selective measurement, and the dashed curve refers to the result with performing the direct selective measurement. Figure 1 shows the influence of state-selective measurement on the atom inside the cavity and the strength of the light field on the entanglement between two the atoms outside the cavity. The result shows that for different values of $|\alpha|^2$ the degree of entanglement between the two atoms N oscillates irregularly. When $|\alpha|^2$ is small, the average value of N is larger than that without performing the direct selective measurement. With $|\alpha|^2$ value increasing, the average value of N decreases. When $|\alpha|^2$ is large enough, we can see that the degree of entanglement between the two atoms, N, displays the collapse-revival phenomena, and its oscillation period increases as $|\alpha|^2$ increases. The numerical results of the entanglement time evolution of the two atoms show that the entanglement between two atoms is strengthened by state-selective measurement of the atom inside the cavity and by selecting weak light field. Further investigation shows that the result for atom 3 selectively measured in state $|e\rangle$ is in agreement with that obtained from state $|g\rangle$.



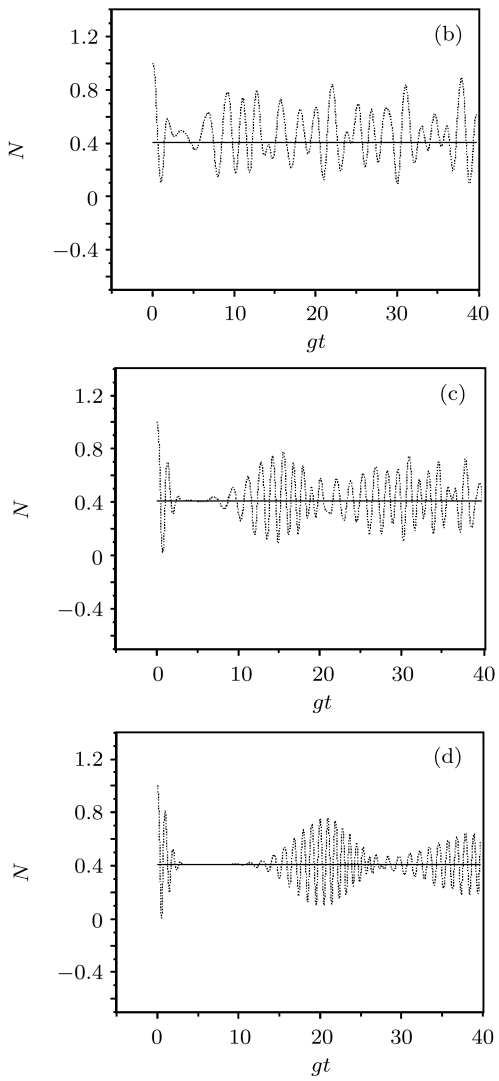


Fig. 1. Time evolutions of two-atom entanglement N with performing direct selective measurement with $|\alpha|^2 = 0.5$ (a), 2(b), 5(c) and 10 (d) and atom 3 detected in state $|g\rangle$.

3.2. Entanglement between two atoms when three atoms are initially in GHZ state

Using Eq. (9), we can obtain that the degree of entanglement between the two atoms, N, equals zero. If the atom 3 is selectively measured in state $|g\rangle$, the state vector of the system collapses onto the state

$$|\varphi(t)\rangle = F_2 \sum_{n=0}^{\infty} C_n(\cos(\Omega gt)\,|gg\rangle_{12}\,|n\rangle - \mathrm{i}\sin(\Omega^+ gt)\,|ee\rangle_{12}\,|n+1\rangle), \quad (17)$$

where F_2 is a normalization factor and can be expressed as

$$F_2^{-2} = \sum_{n=0}^{\infty} |C_n|^2(\cos^2(\Omega gt) + \sin^2(\Omega^+ gt)).$$

The reduced density matrix of the two atoms can be written as

$$\boldsymbol{\rho} = F_2^2 \begin{pmatrix} E & 0 & 0 & G \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ G^* & 0 & 0 & H \end{pmatrix}, \quad (18)$$

where

$$E = \sum_{n=0}^{\infty} |C_n|^2 \sin^2(\Omega^+ gt),$$

$$G = -\frac{\mathrm{i}}{2} \sum_{n=0}^{\infty} C_n C_{n+1}^* \sin(2\Omega^+ gt),$$

and

$$H = \sum_{n=0}^{\infty} |C_n|^2 \cos^2(\Omega gt).$$

The degree of entanglement between the two atoms can be written as

$$N = 2\,|G|\,F_2^2. \quad (19)$$

Figure 2 shows the numerical results of the entanglement time evolution of the two atoms. In Fig. 2, the ordinates represent the degree of entanglement between the two atoms, N, the x-coordinates refer to the scaled time gt. Panel (a) is for $|\alpha|^2$=1, panel (b) for $|\alpha|^2 = 5$, panel (c) for$|\alpha|^2 = 10$, and panel (d) for $|\alpha|^2 = 20$. Figure 2 shows the influences of state-selective measurement of the atom inside the cavity and the strength of light field on the entanglement between two atoms outside the cavity. The result indicates that for different $|\alpha|^2$ values, the degree of entanglement between the two atoms, N, oscillates irregularly. The two atoms are alternately in an entangled state and a disentangled state. As $|\alpha|^2$ value increases, the oscillation amplitude of N decreases and the oscillation frequency of N increases. When the $|\alpha|^2$ value is large enough, we can see that the degree of entanglement between the two atoms N shows the death of the atom-atom entanglement and the region of entanglement death increases as $|\alpha|^2$ value increases. The numerical results of the entanglement time evolution of the two atoms show that the entanglement between two atoms is strengthened by state-selective measurement of the atom inside the cavity and also by selecting a weak light field. Further investigation shows that the results obtained by selectively measuring state $|e\rangle$ of atom 3, is in agreement with that obtained from state $|g\rangle$.

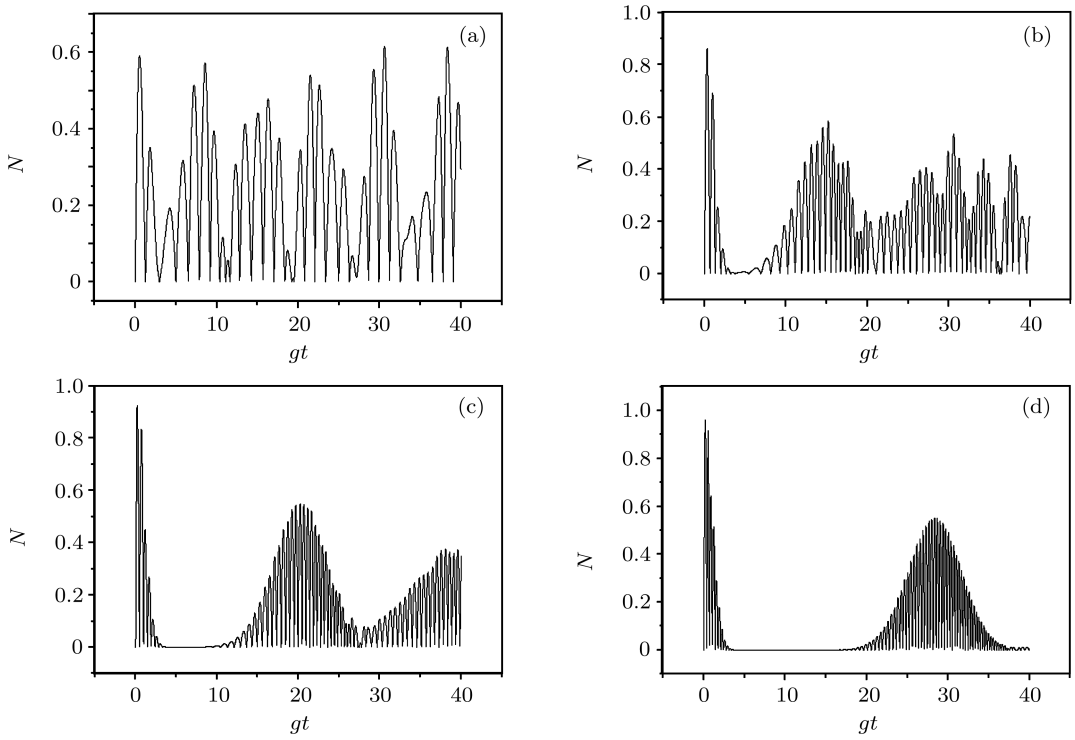


Fig. 2. Time evolutions of two-atom entanglement N with performing direct selective measurement with $|\alpha|^2 = 1$ (a), 5 (b), 10 (c) and 20 (d), and atom 3 detected in state $|g\rangle$.

4. Conclusions

Considering three two-level atoms initially in the W or GHZ state, one of the three atoms is put into an initially coherent light cavity and made to resonantly interact with the cavity. The quantum entanglement between the two atoms is studied by using the negative eigenvalues of the partial transposition of density matrix. The influences of state-selective measurement on the atom inside the cavity and the strength of the light field on the two-atom entanglement evolution outside the cavity are discussed. When three atoms are initially in the W state, the results obtained using the numerical method show that the two-atom entanglement outside the cavity is strengthened by state-selective measurement of the atom inside the cavity and by selecting a weak light field. Additionally, when three atoms are initially in the GHZ state, the results show that the two-atom entanglement outside the cavity indicates the death of the atom–atom entanglement and the region of entanglement death increases as parameter $|\alpha|^2$ value increases. Our investigation results will be helpful for the understanding of the atom–atom entanglement and the control of quantum entanglement in quantum computation and quantum information.

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