A new finite difference scheme for a dissipative cubic nonlinear Schrödinger equation

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(Received 11 March 2010; revised manuscript received 22 October 2010)

This paper considers the one-dimensional dissipative cubic nonlinear Schrödinger equation with zero Dirichlet boundary conditions on a bounded domain. The equation is discretized in time by a linear implicit three-level central difference scheme, which has analogous discrete conservation laws of charge and energy. The convergence with two orders and the stability of the scheme are analysed using a priori estimates. Numerical tests show that the three-level scheme is more efficient.

Keywords: dissipative cubic nonlinear Schrödinger equation, three-level finite difference, convergence and stability analysis

PACS: 02.60.–x **DOI:** 10.1088/1674-1056/20/3/030204

1. Introduction

The cubic nonlinear Schrödinger equation with a dissipative term is

$$\mathrm{i}\frac{\partial u}{\partial t}+\frac{\partial^2 u}{\partial x^2}+\mathrm{i}\nu u+2|u|^2u=0, \tag{1}$$

$$u(x,0)=\phi(x), \tag{2}$$

where u is a complex-valued function and ν is a small coefficient. The $\phi(x)$ is a sufficiently smooth function, which vanishes for sufficiently large $|x|$. In this equation, the second term and the third term describe the dispersion and the dissipation, respectively. The last one is the nonlinear term.

When ν equals zero, Eq. (1) is well known as the cubic nonlinear Schrödinger equation (CNLS). There have been many papers studying the analytical and the numerical solutions of this equation.[1–9] If the dissipative term ν is not zero, the equation appears in many physical areas, for example, the propagation of light pulses in absorptive nonlinear optical fibres.[4] Because of the dissipative term, Zhang *et al.* pointed out that Eq. (1) was analytically intractable.[3] Although Demiray presented a traveling wave solution in Ref. [5], he admitted that it was not a solution in the classical sense. In order to understand the dynamics of complicated perturbed solition models, one has to use numerical methods.

Delfour *et al.* proposed the first conservative difference scheme in Ref. [1], but the numerical scheme produced sawteeth oscillations. Peranich modified the finite difference scheme,[6] which eliminated the sawteeth oscillations. Recently, Dai and Nassar introduced a transformation of variables to eliminate the dissipative term and then presented conservative schemes.[7]

However, the conservative schemes in previous papers are fully nonlinear implicit schemes. One has to solve the nonlinear equations by iteration at each time step. Therefore, they require much more computer time and computer memory. In this paper, we propose a three-level conservative scheme, which is linearly implicit. So we only need to solve linear equations at each time step. However, this scheme is not self-contained because the second grid function values have to be provided by another difference scheme, such as the Crank–Nicholson scheme.

Firstly, applying the transformation $u=w\,\mathrm{e}^{-\nu t}$ to Eq. (1), we have

$$\mathrm{i}\frac{\partial w}{\partial t}+\frac{\partial^2 w}{\partial x^2}+2\,\mathrm{e}^{-2\nu t}|w|^2w=0, \tag{3}$$

$$w(x,0)=\phi(x). \tag{4}$$

The conservation laws for Eq. (3) are

$$\int_R |w|^2\,\mathrm{d}x=\int_R |\phi(x)|^2\,\mathrm{d}x=\text{const}, \tag{5}$$

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$$\int_R |w_x|^2 \mathrm{d}x - \mathrm{e}^{-2\nu t}\int_R |w|^4 \mathrm{d}x - 2\int_0^t \left(\nu\,\mathrm{e}^{-2\nu s}\int_R |w|^4 \mathrm{d}x\right)\mathrm{d}s = \int_R |\phi_x|^2 \mathrm{d}x - \int_R |\phi(x)|^4 \mathrm{d}x. \quad (6)$$

Replacing w by $u\,\mathrm{e}^{\nu t}$, we obtain the conservation laws for u as follows:

$$\int_R |u|^2 \mathrm{d}x = \int_R |\phi(x)|^2 \mathrm{d}x \cdot \mathrm{e}^{-2\nu t}, \quad (7)$$

$$\mathrm{e}^{2\nu t}\int_R |u_x|^2 \mathrm{d}x - \mathrm{e}^{2\nu t}\int_R |u|^4 \mathrm{d}x - 2\int_0^t \left(\nu\,\mathrm{e}^{2\nu s}\int_R |u|^4 \mathrm{d}x\right)\mathrm{d}s = \int_R |\phi_x|^2 \mathrm{d}x - \int_R |\phi(x)|^4 \mathrm{d}x. \quad (8)$$

In what follows, we present a two-order three-level finite difference scheme and prove that this scheme satisfies the discrete conservation laws. Theorem 1 in Section 2 reveals that the numerical charge has an exponential decrease, as proposed in Ref. [7]. In Section 3 the a priori estimates estimates will be given, then the convergence and the stability can be proved with the help of a priori estimates. It is shown that our scheme rigorously has accuracy of the second order in space and time. Section 4 is devoted to the numerical tests of our scheme for different dissipation coefficients ν. It shows that the new scheme is faster and more accurate than other difference schemes.

2. Numerical scheme and conservation laws

Let ω_h, ω_τ be a family of uniform spatial and temporal discrete grids, respectively,

$$\omega_h = \{x_j | x_j = jh, j = 0, 1, 2, \ldots, J\},$$
$$\omega_\tau = \{t_n | t_n = n\tau, n = 0, 1, \ldots, \},$$

where h, τ are spatial and temporal steps respectively. The W_j^n, V_j^n are the discrete functions defined on $\omega_h \times \omega_\tau$. The definitions of difference derivatives, inner products and norms of discrete functions are as follows:

$$(W_j^n)_x = (W_{j+1}^n - W_j^n)/h,$$
$$(W_j^n)_{\bar{x}} = (W_j^n - W_{j-1}^n)/h,$$
$$(W_j^n)_t = (W_j^{n+1} - W_j^n)/\tau,$$
$$(W^n, V^n) = h\sum_j W_j^n \overline{V_j^n},$$
$$\|W^n\|^2 = (W^n, W^n),$$
$$\|W^n\|_\infty = \sup_j |W_j^n|.$$

We propose the following conservative numerical scheme for Eqs. (3) and (4):

$$\mathrm{i}\frac{W_j^{n+1} - W_j^{n-1}}{2\tau} + \frac{1}{2}\left[(W_j^{n+1})_{x\bar{x}} + (W_j^{n-1})_{x\bar{x}}\right] + \mathrm{e}^{-2\nu t_n}|W_j^n|^2(W_j^{n+1} + W_j^{n-1}) = 0, \quad (9)$$

$$W_j^0 = \phi(x_j). \quad (10)$$

The numerical solution is obtained through $U_j^{n+1} = W_j^{n+1}\exp(-\nu t_{n+1})$. The discrete conservation laws are presented in the following theorem.

Theorem 1 For all $n \geq 0$, Eq. (9) satisfies

$$\begin{aligned} &\frac{1}{2}(\|W^{n+1}\|^2 + \|W^n\|^2) \\ &= \frac{1}{2}(\|W^n\|^2 + \|W^{n-1}\|^2) \\ &= \vdots \\ &= \frac{1}{2}(\|W^1\|^2 + \|W^0\|^2) \\ &= \text{const}, \end{aligned} \quad (11)$$

$$\begin{aligned} &h\,\mathrm{e}^{-2\nu t_n}\sum_j |W_j^n|^2|W_j^{n+1}|^2 \\ &- \frac{1}{2}(\|W_x^{n+1}\|^2 + \|W_x^n\|^2) \\ &+ h\sum_{k=1}^{n}\sum_j \mathrm{e}^{-2\nu t_{k-1}}(1 - \mathrm{e}^{-2\nu\tau})|W_j^k|^2|W_j^{k-1}|^2 \\ &= h\sum_j |W_j^1|^2|W_j^0|^2 - \frac{1}{2}(\|W_x^1\|^2 + \|W_x^0\|^2), \end{aligned} \quad (12)$$

if one notice that $1 - \mathrm{e}^{-2\nu\tau} = 2\nu\tau + O(\tau^2)$.

Proof To begin with, we multiply Eq. (9) by $\overline{W_j^{n+1}} + \overline{W_j^{n-1}}$ and sums over j, which is to take the inner product of Eq. (9) with $W^{n+1} + W^{n-1}$, then take the imaginary part. The first term on the left is

$$\begin{aligned} &\mathrm{Im}\left(\mathrm{i}\frac{W^{n+1} - W^{n-1}}{2\tau}, W^{n+1} + W^{n-1}\right) \\ &= \frac{1}{2\tau}(\|W^{n+1}\|^2 - \|W^{n-1}\|^2). \end{aligned}$$

By the summation-by-part formula, the second term becomes

$$\begin{aligned} &\frac{h}{2}\mathrm{Im}\sum_j \left[(W_j^{n+1})_{x\bar{x}} + (W_j^{n-1})_{x\bar{x}}\right]\left(\overline{W_j^{n+1} + W_j^{n-1}}\right) \\ &= -\frac{h}{2}\mathrm{Im}\sum_j \left[(W_j^{n+1})_x + (W_j^{n-1})_x\right]\left(\overline{W_j^{n+1} + W_j^{n-1}}\right)_x \end{aligned}$$

$$=-\frac{h}{2}\mathrm{Im}\sum_j|(W_j^{n+1})_x+(W_j^{n-1})_x|^2=0.$$

Explicitly, the third term becomes

$$\begin{aligned}&\frac{h}{2}\,\mathrm{e}^{-2\nu t_{n+1/2}}\mathrm{Im}\sum_j(|W_j^{n+1}|^2+|W_j^n|^2)\\&\cdot(W_j^{n+1}+W_j^n)(\overline{W_j^{n+1}+W_j^n})\\=&\frac{h}{2}\,\mathrm{e}^{-2\nu t_{n+1/2}}\mathrm{Im}\sum_j(|W_j^{n+1}|^2+|W_j^n|^2)\\&\cdot|W_j^{n+1}+W_j^n|^2\\=&\,0.\end{aligned}$$

Thus we obtain $(1/2\tau)(\|W^{n+1}\|^2-\|W^{n-1}\|^2)=0$, hence prove the first part of Theorem 1.

To obtain the second conservation law Eq. (12), we multiply Eq. (9) by $\overline{W_j^{n+1}}-\overline{W_j^{n-1}}$, then sums over j and take the real part. Explicitly, the first term on the left is

$$\mathrm{Re}\sum_j\frac{\mathrm{i}}{2\tau}(W_j^{n+1}-W_j^{n-1})(\overline{W_j^{n+1}}-\overline{W_j^{n-1}})=0,$$

the second term becomes

$$\begin{aligned}&\frac{h}{2}\mathrm{Re}\sum_j\left[(W_j^{n+1})_{x\bar{x}}+(W_j^{n-1})_{x\bar{x}}\right](\overline{W_j^{n+1}}-\overline{W_j^{n-1}})\\=&-\frac{h}{2}\mathrm{Re}\sum_j\left[(W_j^{n+1})_x+(W_j^{n-1})_x\right](\overline{W_j^{n+1}}-\overline{W_j^{n-1}})_x\\=&-\frac{h}{2}\sum_j\left[|(W_j^{n+1})_x|^2-|(W_j^{n-1})_x|^2\right],\end{aligned}$$

the third term becomes

$$\begin{aligned}&h\,\mathrm{e}^{-2\nu t_n}\mathrm{Re}\sum_j|W_j^n|^2(W_j^{n+1}+W_j^{n-1})\\&\cdot(\overline{W_j^{n+1}}-\overline{W_j^{n-1}})\\=&\,h\,\mathrm{e}^{-2\nu t_n}\sum_j|W_j^n|^2(|W_j^{n+1}|^2-|W_j^{n-1}|^2).\end{aligned}$$

Adding three terms on the left side up, we obtain

$$\begin{aligned}&h\,\mathrm{e}^{-2\nu t_n}\sum_j|W_j^n|^2(|W_j^{n+1}|^2-|W_j^{n-1}|^2)\\&-\frac{h}{2}\sum_j\left[|(W_j^{n+1})_x|^2-|(W_j^{n-1})_x|^2\right]=0.\end{aligned}$$

The above equation leads to

$$\begin{aligned}&h\,\mathrm{e}^{-2\nu t_n}\sum_j|W_j^{n+1}|^2|W_j^n|^2\\&-\frac{h}{2}\sum_j\left[|(W_j^{n+1})_x|^2+|(W_j^n)_x|^2\right]\\&+h\,\mathrm{e}^{-2\nu t_{n-1}}(1-\mathrm{e}^{-2\nu\tau})\sum_j|W_j^n|^2|W_j^{n-1}|^2\\=&\,h\,\mathrm{e}^{-2\nu t_{n-1}}\sum_j|W_j^n|^2|W_j^{n-1}|^2\\&-\frac{h}{2}\sum_j[|(W_j^n)_x|^2+|(W_j^{n-1})_x|^2].\end{aligned}$$

Using the above equation, we obtain Eq. (12).

3. Convergence and stability analysis

We first give the a priori estimates from the above discrete conservative law.

Lemma Assume that $w(x,t)$ and $\phi(x)$ are smooth enough, the solution of numerical scheme Eq. (9) satisfies: $\|W^n\|\le C,\|W^n\|_\infty\le C$. Here C is a constant and depends only on the initial data W_i^0 and W_i^1.

Proof It is easy to obtain $\|W^n\|\le C$ from conservation law Eq. (11). From Eq. (12) we obtain

$$\begin{aligned}&\frac{1}{2}(\|W_x^{n+1}\|^2+\|W_x^n\|^2)\\\le&\,|E_0|+h\sum_j|W_j^n|^2|W_j^{n+1}|^2\\&+h\sum_{k=1}^n\sum_j(1-\mathrm{e}^{-2\nu\tau})|W_j^k|^2|W_j^{k-1}|^2,\end{aligned}$$

where $|E_0|=(1/2)(\|W_x^1\|^2+\|W_x^0\|^2)-h\sum_j|W_j^1|^2\cdot|W_j^0|^2$. Since $\nu\ll1$ and τ is small enough, we can easily obtain $1-\mathrm{e}^{-2\nu\tau}\le4\nu\tau$. Then

$$\begin{aligned}&\|W_x^{n+1}\|^2+\|W_x^n\|^2\\\le&\,2|E_0|+2h\sum_j|W_j^n|^2|W_j^{n+1}|^2\\&+8\nu\tau h\sum_{k=1}^n\sum_j|W_j^k|^2|W_j^{k-1}|^2.\end{aligned}\tag{13}$$

By discrete Sobolev inequality,

$$\|W^k\|_\infty\le\varepsilon\|W_x^k\|+C\|W^k\|,$$

$$\begin{aligned}&h\sum_j|W_j^k|^2|W_j^{k-1}|^2\\\le&\,\|W^k\|_\infty^2\|W_j^{k-1}\|^2\\\le&\,C^2(\varepsilon\|W_x^k\|+C\|W^k\|)^2\\\le&\,\varepsilon\|W_x^k\|^2+C\|W^k\|^2.\end{aligned}\tag{14}$$

Substituting the above inequality into inequality (13), we obtain

$$\|W_x^n\|^2\le2|E_0|+2(\varepsilon\|W_x^n\|^2+C\|W^n\|^2)$$

$$+ 8\nu\tau \sum_{k=1}^{n} (\varepsilon \|W_x^k\|^2 + C\|W^k\|^2).$$

Let ε be small enough and $1 - 2\varepsilon - 8\nu\tau\varepsilon \equiv \Delta > 0$, then

$$\begin{aligned} &(1 - 2\varepsilon - 8\nu\tau)\|W_x^n\|^2 \\ &\leq 2|E_0| + C \max_{1\leq k\leq n-1} \|W^k\|^2 + C\tau \sum_{k=1}^{n-1} \|W_x^k\|^2. \end{aligned} \tag{15}$$

Applying Gronwall inequality, we obtain

$$\|W_x^n\|^2 \leq \frac{1}{\Delta}(2|E_0| + C \max_{1\leq k\leq n-1} \|W^k\|^2) \exp\left(\frac{Cn\tau}{\Delta}\right).$$

We obtain $\|W_x^n\| \leq C$, and further obtain $\|W^n\|_\infty \leq C$ through inequality (14).

The a priori estimates are essential to prove the convergence and the stability of the numerical scheme. Next we shall give the convergence analysis.

Theorem 2 Assuming that $w(x,t)$ and $\phi(x)$ is smooth enough, we find that the Eq. (9) numerical scheme has a unique solution and the convergence error is of order $O(\tau^2 + h^2)$.

Proof Assuming that $w_j^n = w(x_j, t_n)$ is the exact solution, we find that the truncation error

$$\begin{aligned} r_j^n = {} & \mathrm{i}\frac{w_j^{n+1} - w_j^{n-1}}{2\tau} + \frac{1}{2}\left[(w_j^{n+1})_{x\bar{x}} + (w_j^{n-1})_{x\bar{x}}\right] \\ & + \mathrm{e}^{-2\nu t_n}|w_j^n|^2(w_j^{n+1} + w_j^{n-1}). \end{aligned} \tag{16}$$

It is obvious that the truncation error r_j^n is of order $O(\tau^2 + h^2)$. Let $e_j^n = w_j^n - W_j^n$ denote the error. Subtracted out Eq. (9), the error function satisfies the following equation:

$$\begin{aligned} r_j^n = {} & \mathrm{i}\frac{e_j^{n+1} - e_j^{n-1}}{2\tau} + \frac{1}{2}[(e_j^{n+1})_{x\bar{x}} + (e_j^{n-1})_{x\bar{x}}] \\ & + \mathrm{e}^{-2\nu t_n}[|w_j^n|^2(w_j^{n+1} + w_j^{n-1}) \\ & - |W_j^n|^2(W_j^{n+1} + W_j^{n-1})] \\ = {} & \mathrm{i}\frac{e_j^{n+1} - e_j^{n-1}}{2\tau} + \frac{1}{2}[(e_j^{n+1})_{x\bar{x}} + (e_j^{n-1})_{x\bar{x}}] \\ & + \mathrm{e}^{-2\nu t_n}[|w_j^n|^2(e_j^{n+1} + e_j^{n-1}) \\ & + (|w_j^n|^2 - |W_j^n|^2)(W_j^{n+1} + W_j^{n-1})]. \end{aligned} \tag{17}$$

Similar to the proof of Theorem 1, we take the inner product of Eq. (17) with $e^{n+1} + e^n$, the imaginary part is

$$\begin{aligned} & \mathrm{Im}(r^n, e^{n+1} + e^{n-1}) \\ = {} & \frac{1}{2\tau}(\|e^{n+1}\|^2 - \|e^{n-1}\|^2) \\ & + h\,\mathrm{e}^{-2\nu t_n}\mathrm{Im}\sum_j (|w_j^n|^2 - |W_j^n|^2) \\ & \cdot (W_j^{n+1} + W_j^{n-1})(\overline{e_j^{n+1} + e_j^n}). \end{aligned} \tag{18}$$

By Theorem 1 and Lemma, we obtain

$$\begin{aligned} & |w_j^n|^2 - |W_j^n|^2 \\ = {} & (|w_j^n| - |W_j^n|)(|w_j^n| + |W_j^n|) \\ \leq {} & (|w_j^n| + |W_j^n|)|w_j^n - W_j^n| \leq C|e_j^n|. \end{aligned} \tag{19}$$

Combining Lemma, inequality (19), and Hölder inequality, we obtain

$$\begin{aligned} & \frac{1}{2\tau}(\|e^{n+1}\|^2 - \|e^{n-1}\|^2) \\ \leq {} & \|r^n\|^2 + C(\|e^{n+1}\|^2 + \|e^n\|^2 + \|e^{n-1}\|^2). \end{aligned}$$

Multiplying by 2τ and summing over n, we obtain

$$\begin{aligned} & \|e^{n+1}\|^2 + \|e^n\|^2 \\ \leq {} & \|e^0\|^2 + \|e^1\|^2 + 2\tau \sum_{m=1}^{n} \|r^m\|^2 + C\tau \sum_{m=1}^{n+1} \|e^m\|^2, \end{aligned}$$

and

$$\begin{aligned} \|e^{n+1}\|^2 \leq {} & \frac{1}{\Delta}\left(\|e^0\|^2 + \|e^1\|^2 + 2\tau \sum_{m=1}^{n} \|r^m\|^2\right) \\ & \cdot \exp\left(\frac{Cn\tau}{\Delta}\right), \end{aligned}$$

by using Gronwall inequality, where $\Delta \equiv 1 - C\tau$. Since $e^0 = 0$, $\|r^m\| = O(\tau^2 + h^2)$. If we choose appropriate numerical solution to make $\|e^1\| = O(\tau^2 + h^2)$, then $\|e^{n+1}\| = O(\tau^2 + h^2)$.

It is easy to obtain the stability analysis in a way similar to the proof of Theorem 2.

4. Numerical tests

In this section we will apply the scheme of Eqs. (9) and (10) to solve Eqs. (1) and (2). We first consider the propagation of a soliton

$$\mathrm{i}\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + \mathrm{i}\nu u + 2|u|^2 u = 0, \tag{20}$$

with initial condition $\phi(x) = \exp(2\mathrm{i}x)\mathrm{sech}(x + 5)$. In order to obtain a numerical solution, we use an artificial boundary condition $u(-30, t) = u(30, t) = 0$. However, the scheme is not self-contained, the value W_j^1 must be provided by another implicit scheme. Here we choose the Crank–Nicolson scheme. We choose spatial step $h = 0.1$ and temporal step $\tau = 0.01$, respectively. The numerical results are presented over the spatial interval $[-30, 30]$ and the time interval $0 < t < 6$. The analytical solution in the averaged sense is $u(x,t) = a(t)\mathrm{sech}(\zeta)\exp\{\mathrm{i}[2x - \Omega(t)]\}$,

where $a(t) = \mathrm{e}^{-2\nu t}$, $\zeta = a(t)(x + 5 - 4t)$, $\Omega(t) = 4t + (1/4\nu)(1 - \mathrm{e}^{-4\nu t})$.[5] The error in the maximum norm is $\| e \|_\infty = \max_j |u(x_j, T) - U_j^N| = 7.71 \times 10^{-2}$.

With $\nu = 0.1$, there is a strong dissipative term resulting in amplitude decay of the soliton, as shown in Fig. 1(a). When $\nu = 0.05$, the amplitude decay is not obvious as shown in Fig. 1(b). We calculate the numerical charge and analytical charge from $h\sum_j |U_j^n|^2$ and $\int_R |u|^2 \mathrm{d}x = \int_R |\phi(x)|^2 \mathrm{d}x \cdot \mathrm{e}^{-2\nu t}$ respectively. The results show that the numerical charge fits well the analytical charge (see Fig. 2). In terms of computer time, the classical Crank–Nicolson scheme takes about 34.7 s on a PC, while the new scheme only needs 11.0 s. It shows that our scheme is faster than other conservative schemes.

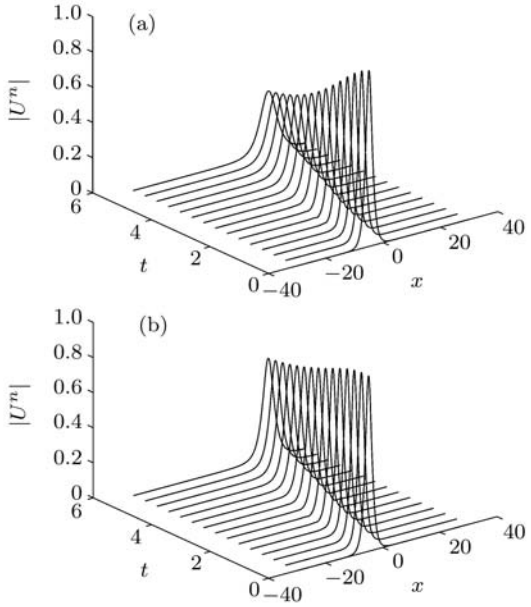


Fig. 1. Propagation of a single soliton. (a) $\nu = 0.1$, (b) $\nu = 0.05$.

Then we consider Eq. (20) with initial condition $\phi(x) = \mathrm{sech}(x - 10)\exp(-\mathrm{i}(2x - 20)) + \mathrm{sech}(x + 10)\exp(\mathrm{i}(2x - 20))$. The initial function includes a pair of solitons, which is appropriate for simulations of a soliton collision. As time goes on, the two solitons collide but recover their shapes afterwards, as shown in Fig. 3 with $\nu = 0.1$ and $\nu = 0.05$, respectively.

5. Conclusion

In this paper, we have presented a new finite difference scheme for the dissipative CNLS. The new scheme is a simple and easy programme for a conservative scheme. The L_2-norm error estimates show that this scheme has two-order accuracy in time and space. The numerical results show that the numerical charge and energy agree well with the analytical ones. Furthermore, this scheme can be generalized to the case of high order nonlinearity.[8,9] For example, the problem of an optical soliton governed by the cubic-quintic nonlinear Schrödinger equation will be our next work.

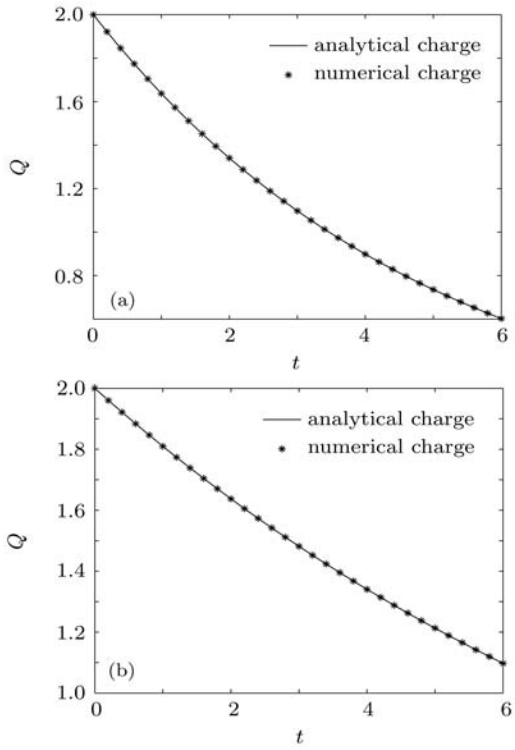


Fig. 2. Numerical and analytical charge. (a) $\nu = 0.1$, (b) $\nu = 0.05$.

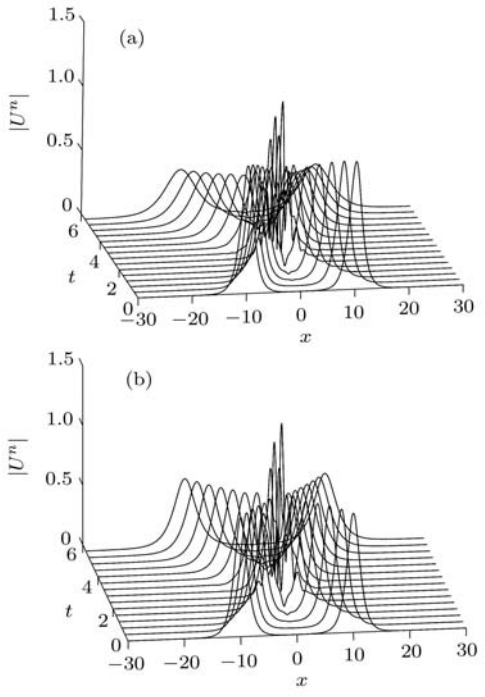


Fig. 3. Interaction of two solitons. (a) $\nu = 0.1$, (b) $\nu = 0.05$.

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