Properties of hyperon stars rotating at Keplerian frequency*

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(Received 8 October 2009; revised manuscript received 28 September 2010)

The structure and properties of a Keplerian rotating hyperon star with an equation of state (EOS) investigated using the relativistic σ–ω–ρ model are examined by employing an accurate numerical scheme. It is shown that there is a clear rotating effect on the structure and properties, and that hyperon star matter cannot support a star with a mass larger than 1.9 $M_\odot$, even a star rotating at the fastest allowed frequency. The constraints of the two known fastest rotating frequencies (716 Hz and 1122 Hz) on the mass and radius of a hyperon star are also explored. Furthermore, our results indicate that the imprint of the rapid rotation of a hyperon star on the moment of inertia is clear; the backward equatorial redshift, the forward equatorial redshift and the polar redshift can be distinguished clearly, the forward equatorial redshift is always negative; and its figuration is far from a spherical symmetric shape.

Keywords: equation of state, hyperon star, moment of inertia, redshift

PACS: 97.10.Kc, 26.60.+c, 97.60.Jd **DOI:** 10.1088/1674-1056/20/2/029701

1. Introduction

One of the most important purposes of exploring neutron stars is to understand the physics of super dense matter.[1−4] Accurate observations on the properties of neutron stars are required, especially on the globe properties, such as stellar mass and radius. However, until now, the masses of neutron stars have only been observed for binary systems and only mass measurements taken in double neutron star binaries have a relatively high level of accuracy. Typical neutron star mass measurements are presented in Fig. 1. Another crucial property in determining the equation of state (EOS) of neutron star matter is the stellar radius. However, the measurement of the radius is hampered by the technique and the fact that all neutron stars have small sizes ($\sim$ 10 km) and are too far away (at least about a thousand light years). Therefore, synthetical study of the other observable properties, such as redshift, moment of inertia, gravitational radiation, etc. is necessary.

As we know, spin frequency is one of the key factors in determining the structure and properties of neutron stars. During recent years, more and more millisecond pulsars have been observed,[5,6] and even evidence of a sub-millisecond pulsar was reported.[7] In Table 1 a dozen of the known fastest rotating pulsars are listed. The rapid rotation of a neutron star not only influences the structural properties, such as mass and radius, but also obviously affects other observable properties, such as redshift, moment of inertia and gravitational radiation.[8−16] These facts emphasize the necessity for studying the equilibrium models of rapidly rotating neutron stars in general relativity. Such models ought to provide the solutions of Einstein field equations for the static axisymmetry gravitational field, coupled to the equation of hydrostatic equilibrium, and must be solved numerically. In fact, models for dealing with rotating neutron stars have been constructed during the past few decades by several groups,[8−14] among which the scheme of Hartle,[8] and Hartle and Thorne,[9] is an approximate method for studying slowly rotating neutron stars and the others are accurate schemes to deal with rapidly rotating neutron stars. Based on the rapidly rotating models constructed in Refs. [12]–[14], one efficacious code RNS was developed and made available to the public by Stergioulas and Friedman.[14]

*Project supported by the National Natural Science Foundation of China (Grant No. 10947023), and the Fundamental Research Funds for the Central University, China (Grant No. 2009ZM0193).

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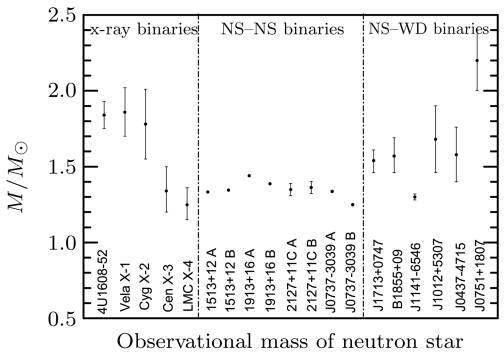


Fig. 1. Typical measurements of neutron star (NS) masses in x-ray binaries, NS–NS binaries and NS–white dwarf (WD) binaries, where 4U 1608-52 is obtained from Ref. [17], Cen X-3 and LMC X-4 are obtained from Ref. [18] and the others are taken from Ref. [4] and the references therein.

The conversion of some nucleons into hyperons will occur through weak interaction and is active in dense matter because the Fermi energy of the highest-lying nucleons will exceed the mass of the hyperons. Since the conversion relieves the Fermi pressure exerted by the baryons, the EOS will be softened and the maximum mass of the neutron star sequence associated with this kind of EOS is lowered. Neglect of the hyperons therefore always leads to an overestimated maximum possible mass. It has long been recognized that hyperons are present in the dense cores of neutron stars,[20−23] and the properties of the corresponding neutron star (normally called hyperon star) have been investigated deeply, but most of them are for the static stars.[24−28] As more and more rapidly rotating neutron stars are observed, in order to explain the observational data and to affirm if there are indeed hyperons in neutron stars, it is very important to probe the rotating effects on the hyperon star properties. The main aim of this article is to investigate the effect of the rapid rotation on the structure and properties of hyperon stars.

The rest of the present paper is organized as follows. In Section 2 we outline the EOS of hyperon star matter. We then briefly recall the structure equations of static and rotating neutron stars in Section 3. The numerical results are discussed in Section 4. Finally, a short summary is given in Section 5.

Table 1. Spin frequencies of the known fastest-spinning pulsars. The data of XET J1739-285 are taken from Ref. [7], which is not definitely confirmed to be a neutron star; the data of 4U 1608-52 are taken from Ref. [19] and the others are cited from Ref. [5].

pulsar	f/Hz	pulsar	f/Hz	pulsar	f/Hz
XET J1739-285	1122	4U 1608-52	619	J0034-0534	533
J1748-2446ad	716	J1748-2446O	596	J1748-2446Y	488
B1937+21	642	J1748-2446P	578	J1748-2446V	483
B1957+20	622	J1843-1113	542	B0021-72J	476

2. Equation of state of hyperon star matter

The relativistic σ–ω–ρ model is one of the efficacious models for dealing with super dense matter. In this work, we will employ the relativistic σ–ω–ρ model to investigate hyperon star matter. The Lagrangian density of this model is

$$\begin{aligned} L = & \sum_B \bar{\psi}_B(\mathrm{i}\gamma_\mu\partial^\mu + m_B - g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu \\ & - \frac{1}{2}g_{\rho B}\gamma_\mu\boldsymbol{\tau}.\boldsymbol{\rho}^\mu)\psi_B + \frac{1}{2}(\partial\sigma)^2 \\ & - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - U(\sigma) \\ & - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}.\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu.\boldsymbol{\rho}^\mu \\ & + \sum_l \bar{\psi}_l(\mathrm{i}\gamma_\mu\partial^\mu - m_l)\psi_l, \end{aligned} \tag{1}$$

in which $U(\sigma) = a\sigma + c\sigma^3/3! + d\sigma^4/4!$; $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$; $\boldsymbol{\rho}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu$; ψ_B is the field operator of baryon B (B = n, p, Λ, Σ, Ξ, Δ); ψ_l is the field operator of lepton l (l = e, μ); σ, ω^μ and $\boldsymbol{\rho}^\mu$ are the field operators of σ-, ω-, ρ-mesons, respectively; $g_{\sigma B}$, $g_{\omega B}$ and $g_{\rho B}$ are the coupling constants between σ-, ω-, ρ-meson and baryon, respectively; $m_B, m_l, m_i, (i = \sigma, \omega, \rho)$ are the masses of the baryon, lepton and meson, respectively; $\boldsymbol{\tau}$ is the isospin operator. In general, the coupling constants between the σ (ω or ρ) meson and nucleon (neutron and proton) are equal ($g_{\sigma\mathrm{n}} = g_{\sigma\mathrm{p}}$, $g_{\omega\mathrm{n}} = g_{\omega\mathrm{p}}$) and determined by the saturated property and symmetry energy of the nuclear matter. As for leptons, we assume that they are free Fermi gas. From the Lagrangian density described by Eq. (1), the EOS of the hyperon star matter can be obtained. In the numerical calculation, the following

parameters are adopted:[29] $a = -2.1 \times 10^7$ MeV^3, $c = 0.97M_{\rm n}$, $d = 1277$, $g_{\rm s} = 6.73$, $g_{\rm v} = 8.59$, $M_{\rm n} = 938$ MeV, $m_\omega = 783$ MeV, $m_\sigma = 550$ MeV, $m_\rho = 770$ MeV and $K = 224$ MeV. Like the parameters in other models, the parameters adopted here are obtained by fitting and reproducing the saturated properties and the symmetric energy of the nuclear matter. Unlike other models, the linear term in the self-interaction of the sigma meson is considered here. The coefficient of this term can move the "tachyon pole" in the sigma meson for a density even higher than that found in the inner core of the neutron star. Therefore it can avoid the negative mass of the nucleon in the numerical calculation. The EOS for the neutron star matter described by this model is softer than that described by other models in the high-density region as more hyperons appear.

For a density below about 0.07 fm^{-3} (1 fm = 10^{-15} m), the results of Refs. [30] and [31] are employed. For comparison, the widely used APR EOS,[32] which consists of neutrons, protons, electrons and muons, is also employed. The EOSs considered in this paper are displayed in Fig. 2, where the EOS of hyperon star matter is denoted as HS.

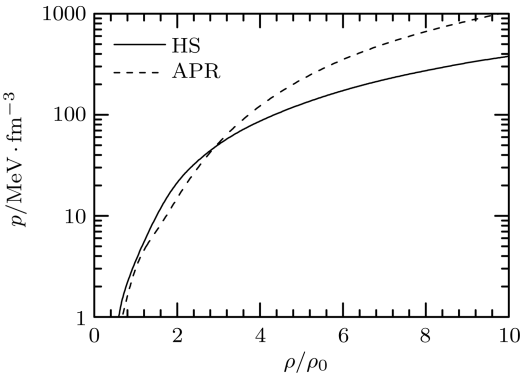


Fig. 2. Equations of state, where energy density ρ is in units of saturation nuclear density ρ_0.

3. Structure equations of static and rotating neutron stars

Both the static and rotating neutron star structure equations, which are deduced in the framework of general relativity, are briefly reviewed below.

3.1. Static neutron stars

Supposing that the matter of a neutron star can be treated as a perfect fluid, its energy–momentum tensor can be expressed as

$$T^{\alpha\beta} = pg^{\alpha\beta} + (p+\rho)u^\alpha u^\beta, \tag{2}$$

where $\alpha, \beta = 0, 1, 2, 3$, and u^α is the four-velocity satisfying $u^\alpha u_\alpha = -1$. A static spherically symmetric neutron star is described by the following metric:

$$-{\rm d}s^2 = -{\rm e}^{2\Phi}{\rm d}t^2 + {\rm e}^{2\Lambda}{\rm d}r^2 + r^2({\rm d}\theta^2 + \sin^2\theta{\rm d}\phi^2), \tag{3}$$

where Φ and Λ are the functions of radius r only. According to the Einstein field equation

$$R^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}R = 8\pi T^{\alpha\beta} \tag{4}$$

and combining it with Eqs. (2) and (3), one can obtain the static neutron star structure equation, namely, a familiar Tolman–Oppenheimer–Volkoff (TOV) equation:[33,34]

$$\frac{{\rm d}p}{{\rm d}r} = -\frac{(p+\rho)[m(r) + 4\pi r^3 p]}{r[r - 2m(r)]}, \tag{5}$$

where

$$m(r) = \int_0^r 4\pi r^2 \rho {\rm d}r, \tag{6}$$

In order to numerically calculate the structure of a static neutron star, the EOS of the neutron star matter needs to be provided.

3.2. Rotating neutron stars

In order to deal with a rapidly rotating neutron star in the framework of general relativity, the following assumptions are made:[8] (i) the matter can be described as a perfect fluid and its energy–momentum tensor is also described by Eq. (2); (ii) the matter distribution and the spacetime are axisymmetric and in a stationary state; (iii) the matter has no meridional motion and the only motion is a circular rotation, where the angular velocity Ω is constant to a distant observer at rest. Under these assumptions, the metric of a rapidly rotating neutron star can be expressed as

$$\begin{aligned} -{\rm d}s^2 = &-{\rm e}^{2\nu}{\rm d}t^2 + {\rm e}^{2\mu}({\rm d}r^2 + r^2{\rm d}\theta^2) \\ &+ {\rm e}^{2\psi}r^2\sin^2\theta({\rm d}\phi - \omega{\rm d}t)^2, \end{aligned} \tag{7}$$

where the metric functions $(\nu, \mu, \omega, \psi\ )$ are functions of r and θ only. To the rotating stars, the four-velocity u^α in the energy–momentum tensor is given by[35]

$$u^\alpha = \frac{{\rm e}^{-\nu}}{\sqrt{1-v^2}}(1, 0, 0, \Omega), \tag{8}$$

where the proper velocity v with respect to a local zero angular momentum observer is defined as

$$v \equiv r\sin\theta\,{\rm e}^{\psi-\nu}(\Omega - \omega). \tag{9}$$

By using the metric of Eq. (7) and the energy momentum tensor based on Eqs. (2), (8) and (9), the Einstein field equation can be expressed only by metric functions, the energy density and pressure of the matter, and the proper velocity. For detailed expressions, please refer to Ref. [36].

It is easy to understand that for a given EOS, fixed central density $\rho_{\rm c}$ and spin frequency Ω, the structure of a rotating neutron star should be uniquely determined. However, it is not easy to obtain the unique structure because of the dependence of the metric on the polar coordinate θ due to rotational deformation. In what follows we briefly outline the numerical method for calculating the structure of a rotating neutron star. Equilibrium neutron star models must satisfy Einstein field equations and the equation of hydrostatic equilibrium. The equation of hydrostatic equilibrium can be derived from the equations of motion and takes the following form:[35]

$$\frac{1}{p+\rho}\nabla p + \nabla\nu - \frac{1}{2}\nabla\ln(1-v^2) = 0. \tag{10}$$

Employing appropriate boundary conditions, and combining them with a selected EOS and Einstein field equations, the equation of hydrostatic equilibrium of the rotating neutron star can be solved numerically. Several available numerical codes for obtaining the relativistic rotating neutron star structures and properties have been successfully developed; for the introduction and the comparison of these codes please refer to Ref. [35]. In the present work, the RNS code based on Refs. [12]–[14] is employed to compute the structure and the properties of rapidly rotating neutron stars.

Aside from mass and radius, there are several other interesting global quantities which characterize rotating neutron stars. They are moment of inertia, redshift and eccentricity. The moment of inertia is defined by

$$I = \frac{J}{\Omega}, \tag{11}$$

where Ω is the star's angular velocity and J is the angular momentum, which can be calculated from[35]

$$J = 2\pi \int \mathrm{e}^{2\mu+2\psi}\frac{(p+\rho)v}{1-v^2}r^3\sin^2\theta\,\mathrm{d}r\,\mathrm{d}\theta. \tag{12}$$

For a rotational frequency much lower than the Kepler frequency, approximating to the first order terms in ω and Ω, the moment of inertia can be expressed as[37]

$$I = \frac{8\pi}{3}\int_0^R r^4\,\mathrm{e}^{-\Phi}\frac{p+\rho}{\sqrt{1-\dfrac{2m(r)}{r}}}\frac{\bar{\omega}}{\Omega}\,\mathrm{d}r, \tag{13}$$

where $\mathrm{e}^{-\Phi}$ is the metric function of the corresponding static neutron star with the same central density and deviations from spherical symmetry are neglected. Here, one available empirical equation[38]

$$I \approx 0.237\ MR^2\left[1+4.2\frac{M\ km}{M_\odot\ R}+90\left(\frac{M\ km}{M_\odot\ R}\right)^4\right] \tag{14}$$

is employed in our calculation for the slow-rotation neutron star.

For a rapidly rotating neutron star, the "redshift" of the radiation results from two mechanisms: the gravitational redshift and the Doppler shift (which is a red shift in the backward direction and a blue shift in the forward direction). For the radiation emitted tangential to the surface at the equator, the forward and the backward redshift are defined by[35]

$$z_{\rm f} = \left(\frac{1-v_e}{1+v_e}\right)^{1/2}\frac{\mathrm{e}^{-\nu_e}}{1+R_e\omega_e\,\mathrm{e}^{(\nu_e-\psi_e)/2}}-1, \tag{15}$$

$$z_{\rm b} = \left(\frac{1+v_e}{1-v_e}\right)^{1/2}\frac{\mathrm{e}^{-\nu_e}}{1-R_e\omega_e\,\mathrm{e}^{(\nu_e-\psi_e)/2}}-1, \tag{16}$$

respectively, where the subscript e denotes the value in the equatorial surface. At the polar, there is only gravitational redshift, which is determined by

$$z_p = \sqrt{|g_{tt}|}-1 = \mathrm{e}^{-\nu_p}-1, \tag{17}$$

where the subscript p denotes the value in the polar surface.

The eccentricity e is defined by[11]

$$e = \left[1-\left(\frac{R_p}{R_e}\right)^2\right]^{1/2}, \tag{18}$$

which is consistent with the traditional definition of an ellipsoid configuration.

4. Numerical results and discussion

The structure and properties of static and rapidly rotating hyperon stars are calculated by employing the RNS code.

As mentioned in Section 3, the configuration of a neutron star is uniquely specified if the central density and the rotating frequency are fixed. In order to probe the rotating effect, we calculate the following two typical cases: static models and maximum rotating (at

Keplerian frequency) models. The masses and the radii of these two models, each as a function of the central density, are shown in Figs. 3 and 4, respectively.

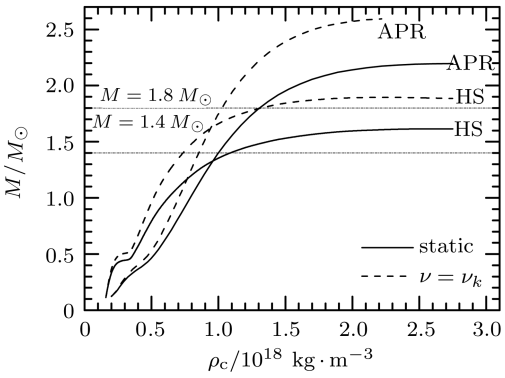


Fig. 3. Curves for the masses of a hyperon star (with HS EOS) and a neutron star (with APR EOS) versus central density ρ_c, where static (solid lines) and Keplerian (dashed lines) cases are plotted.

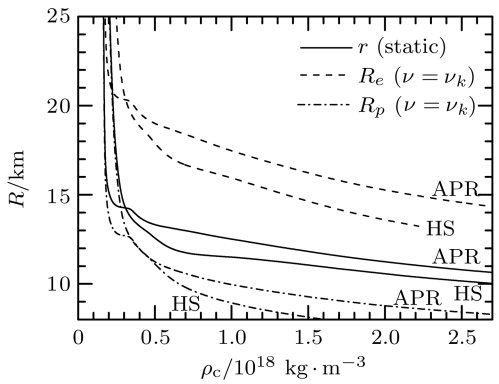


Fig. 4. Radii of a hyperon star and a neutron star versus central density ρ_c. Both the static and Keplerian models are plotted, where R_e and R_p denote the radii for the equator and the polar of the Keplerian cases, respectively.

For the EOS considered here, it is shown that the maximum mass point corresponds to a central density around ten times the saturation nuclear density, and the Keplerian rotation increases the maximum mass to as high as 18% compared with the static models. A more direct illustration of the rotating effect on mass and radius is presented in Fig. 5, where the radius of the rotating star is the radius at the equator. Comparing this result with that shown in Fig. 1, it is easy to see that hyperon star matter can support all double neutron star binaries, but cannot support high mass neutron stars with a mass larger than 1.9 $M_\odot$. Presently, the definitely known fastest rotating neutron star is J1748-2446ad, which has a frequency of 716 Hz,[5]. The burst oscillation frequency of 1122 Hz for the x-ray transient XTE J1739-285 suggests that it may contain the fastest rotating neutron star yet found.[7] It is interesting to study how these observational frequencies constrain the properties and the EOS of dense matter.

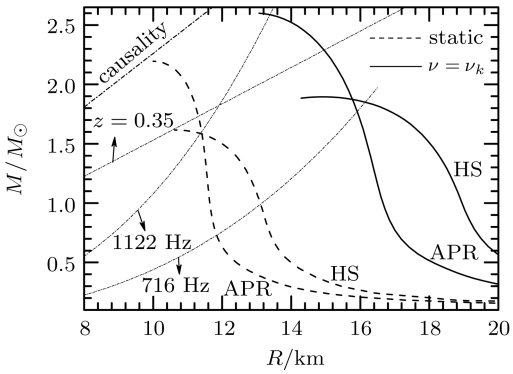


Fig. 5. Mass–radius (equatorial radius) relations (1). The heavy lines denote the relations of the static (dashed lines) and the Keplerian rotating (solid lines) hyperon stars and neutron stars. The static star sequences are constrained by the rotation speed limit (set as the Keplerian frequency) 716 Hz of J1748-2446ad and 1122 Hz of XTE J1739-285 are plotted according to Ref. [39], which gives the upper limits of the radii supported by the static stars if they are driven to rotate at such frequencies. A possible redshift measurement of $z = 0.35$[40] is also plotted. For further explanations see the text.

Very recent research by Haense *et al.*[39] can help us obtain the constraint. They modified the empirical formula for Keplerian frequency proposed by Lattimer and Prakash[4] and derived a set of exact empirical formulae. They express the Keplerian frequency and equatorial radius of Keplerian configuration in terms of mass and radius as

$$\nu_{\rm k} \approx C \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{10\ {\rm km}}\right)^{-3/2}, \qquad (19)$$

and

$$R_{\rm k} \approx aR. \qquad (20)$$

respectively, where M is the mass of the rotating star and R is the radius of the non-rotating star with mass M, $C = 1.08$ kHz, and $a = 1.5$. These relations give rise to the rotating frequency limit and its equatorial radius based on the fixed mass M and radius R of non-rotating neutron stars. In Fig. 5, constraints of the rotation speed limits 716 Hz of J1748-2446ad and 1122 Hz of XTE J1739-285 are plotted according to Haensel *et al.*[39] These constraints give rise to the upper limits of the radii for the static stars if they are driven to rotate at such fixed frequencies, that is, a stable neutron star rotating at such a frequency

can only be supported in the upper region of the line. It should be pointed out that this limit only applies for static (non-rotating) neutron stars. One can see that a static hyperon star can support rotations at frequencies of 716 Hz and 1122 Hz. For the hyperon stars and neutron stars considered here, in the mass range $0.5\ M_\odot < M < 0.9\ M_{\max}$, we also have $R_{\rm K} \approx 1.5\ R$ at a fixed mass ($R_{\rm K}$ is the radius of the Keplerian rotating star), which accords with the result of Ref. [39] very well. In Fig. 6, we display the mass–radius relations in another way, where the stellar sequences of $\nu = 716$ Hz and 1122 Hz are for a fixed rotating frequency and stable hyperon stars and neutron stars. A stable rotating hyperon star (or neutron star) with these fixed spin frequencies can exist only between the max-mass point (the cross point on the upper dot line, see Fig. 6) and the mass-shedding point (the cross point on the $\nu = \nu_k$ line, see Fig. 6).

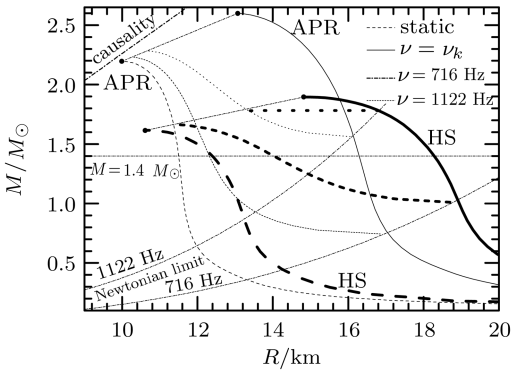


Fig. 6. Mass–radius (equatorial radius) relations (2). The heavy lines are for hyperon stars (with HS EOS) and the thin lines (except for the short dashed lines) are for neutron stars (with APR EOS), where the curves marked with the stellar sequences of $\nu = 716$ Hz and 1122 Hz show the mass–radius relations for these fixed rotating frequencies. The Newtonian limit lines of $\nu = 716$ Hz and 1122 Hz give the upper limits of the radii for the rapid rotating stars with such fixed frequencies in Newtonian theory, which is shown to accord well with the general relativistic result. For further explanations see the text.

The range of the allowed mass values supported by the HS EOS for the hyperon stars rotating at 716 Hz is much wider than that at 1122 Hz. A clearer presentation of the stellar masses supported by a given EOS for the hyperon stars or neutron stars rotating at a fixed frequency (period) can be found in Fig. 7. In this figure, we display the theoretically permitted smallest periods as a function of stellar mass, where the periods of the two known fastest-spinning pulsars are drawn. In this figure, it is shown that the smallest allowed stellar mass supported by a given EOS changes very slowly for a neutron star rotating at a frequency smaller than 500 Hz ($P > 2$ ms), while it changes very quickly for that at a frequency larger than 1000 Hz ($P < 1$ ms). From this figure, one can see that hyperon stars or neutron stars with masses smaller than 1.5 $M_\odot$ cannot sustain a rotation at a frequency as fast as 1122 Hz and it is nearly impossible to find a pulsar that has both a mass smaller than 1.4 $M_\odot$ and a spin period smaller than 1 ms. Combining Fig. 7 with Fig. 2, we can see that at a fixed stellar mass, a softer EOS can support a smaller rotating period because this kind of model is more centrally condensed and has a smaller stellar radius.

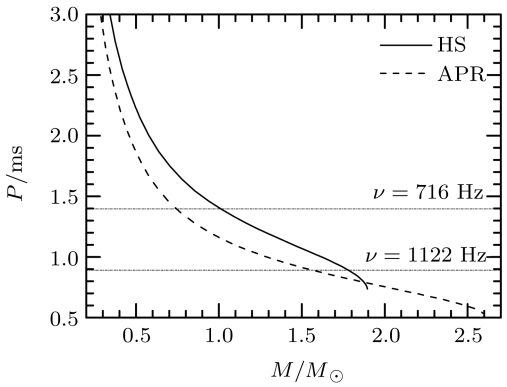


Fig. 7. Periods of the Keplerian rotating hyperon stars and neutron stars versus stellar masses.

One of the important properties for hyperon star (or neutron star) investigations is the moment of inertia, which is crucial for describing the rotating energy and probing the gravitational radiation of the deformation of the neutron star.[41] The moments of inertia of slowly rotating and rapidly rotating (at Keplerian frequency) hyperon stars and neutron stars each as a function of stellar mass are presented in Fig. 8, where the moments of inertia of the slowly rotating star are calculated based on the empirical formula obtained by Lattimer and Schutz.[38] It is shown that the Keplerian rotation will produce a clear effect on the moment of inertia of a hyperon star, especially at the max-mass point of the static case. The moment of inertia of the Keplerian rotating hyperon star is always larger than that of the slowly rotating star, the largest increase in amplitude can be up to 60% for the same mass. In addition, the static (or Keplerian rotating) neutron star described by EOS APR has a far larger max moment of inertia than that of the corresponding hyperon star.

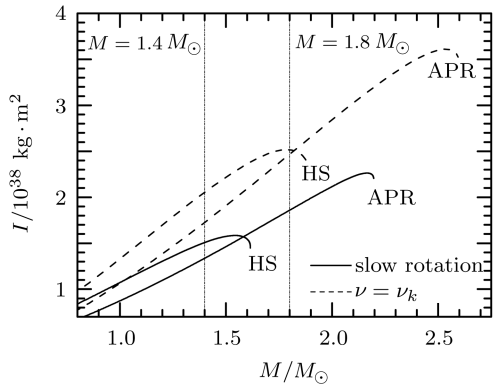


Fig. 8. Moments of inertia of slowly rotating and Keplerian rotating hyperon stars and neutron stars versus stellar mass.

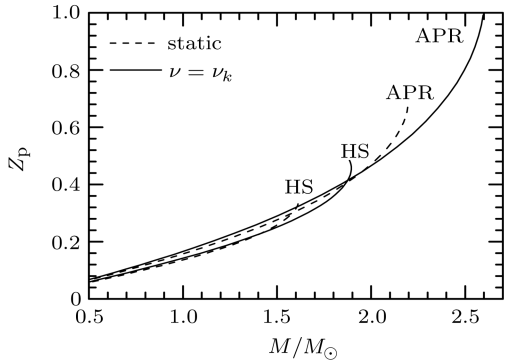


Fig. 9. Curves for polar redshift Z_p of the static and the Keplerian rotating hyperon stars and neutron stars versus stellar mass.

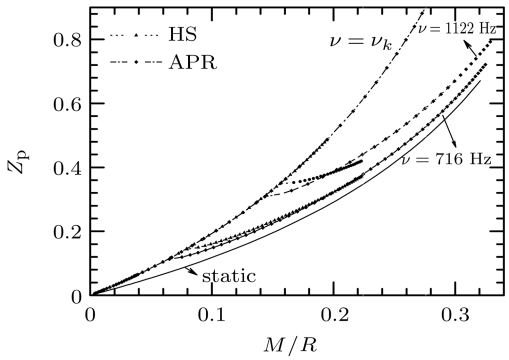


Fig. 10. Curves for polar redshift Z_p of the static and the rotating (at $\nu = 716$ Hz, 1122 Hz and Keplerian frequency, respectively) hyperon stars and neutron stars versus compactness M/R.

Another important property is the redshift.[42,43] Its measurement can provide a strict constraint on the EOS of the compact star matter. For a static hyperon star (or neutron star), there is only gravitational redshift; but for a rapidly rotating star, the radiations from different parts of the star have different redshifts. If Earth is in the polar plane of the neutron star, we will detect a polar redshift Z_p;[42] if the detected radiation is in a direction tangential to the surface at the equator, we will obtain totally different redshifts: the forward redshift Z_f and the backward redshift Z_b because of the Doppler shift.[11,42]

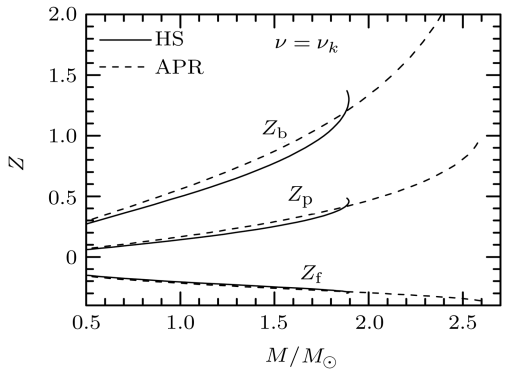


Fig. 11. Curves for polar redshift Z_p, backward equatorial redshifts Z_b and forward equatorial redshifts Z_f for hyperon stars and neutron stars rotating at Keplerian frequencies versus stellar mass M.

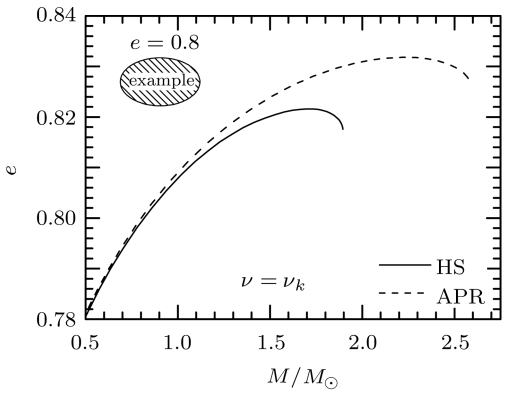


Fig. 12. Curves for eccentricity ($e = [1 - (R_p/R_e)^2]^{1/2}$) of the Keplerian rotating hyperon stars and neutron stars versus stellar mass M.

In Figs. 9–11, the redshifts of hyperon stars and neutron stars are displayed. The polar redshifts as a function of stellar mass are plotted in Fig. 9. It is shown in this figure that at a fixed stellar mass, the rotating effects on redshift are not notable except for that in the region near the max-mass point. This is because the rapid rotation results in a marked deformation and the radius at polar plane is smaller than that of the static case. Measurement of the gravitational redshift for the active x-ray burster EXO 0748–676 with 45-Hz spin frequency[40] indicates its redshift as $z \approx 0.35$. Our result shows that both the static star and the rapidly rotating hyperon star can support the measurement of redshift $z \approx 0.35$, see Figs. 5 and 9. The polar redshifts versus compactness M/R are displayed in Fig. 10, where the polar redshifts of hyperon stars and neutron stars rotating at the two known fastest rotating frequencies are also presented. It is shown that at a fixed compactness, the rotating

effect on the polar redshift is remarkable and the polar redshifts of static and Keplerian rotating neutron stars are only dependent on compactness. In Fig. 11, we plot the curves for backward equatorial redshift $Z_{\rm b}$ and forward equatorial redshift $Z_{\rm f}$ versus stellar mass M for hyperon stars and neutron stars rotating at Keplerian frequencies. For comparison, the curves for polar redshift $Z_{\rm p}$ versus stellar mass M are also plotted. It is shown that at a fixed stellar mass, the redshifts $Z_{\rm b}$, $Z_{\rm f}$ and $Z_{\rm p}$ can clearly be distinguished from each other, and as the stellar mass increases, the difference becomes more distinct. Another interesting result is that for both hyperon stars and neutron stars rotating at Keplerian frequency, the values of forward equatorial redshift $Z_{\rm f}$ are always negative, that is, in the forward equatorial direction the shift of the radiation is toward the blue region.

In order to show the deformation caused by the rapid rotation, the eccentricities of hyperon stars and neutron stars rotating at Keplerian frequency each as a function of stellar mass are displayed in Fig. 12. It is shown that the maximum deformation (the deformation at the Keplerian frequency) supported by the star increases as stellar mass increases, except for that in the region near the max-mass point. For a Keplerian rotating hyperon star with mass $M = 1.4\ M_{\odot}$, its eccentricity is about 0.818, which is slightly smaller than that of the neutron star described by APR EOS. Such a large eccentricity means that the configuration of a rapidly rotating neutron star can be far from the spherical symmetric shape.

5. Conclusion

In this paper, the structure and properties of rapidly rotating hyperon stars are examined. The results show that there is a clear rotating effect on the mass and radius of a hyperon star at a fixed central density, and as the hyperons emerging in the hyperon star lead to softening the EOS of the stellar matter, hyperon stars cannot support a compact star with a mass larger than $M = 1.9\ M_{\odot}$; even stars rotating at the allowed fastest frequency. We also investigated the constraints of the two known fastest rotating frequencies on the properties of hyperon stars. Our results show that it is hard to find a pulsar that has both a mass smaller than $1.4\ M_{\odot}$ and a spin period smaller than 1 ms.

We also investigated the moments of inertia, the redshifts and the eccentricities of hyperon stars. Our results show that the moment of inertia of the Keplerian rotating hyperon star is far larger than that of the slow rotating star, the largest increase in amplitude can be up to 60% for a fixed stellar mass. The calculation of the redshifts show that the polar redshifts of static and Keplerian rotating hyperon stars and neutron stars are dependent only on stellar compactness. For a hyperon star rotating at its Keplerian frequency, the backward equatorial redshifts, the forward equatorial redshifts and the polar redshifts can clearly be distinguished from each other, and the value of forward equatorial redshift $Z_{\rm f}$ is always negative, that is, it is a blue-shift. Our results also show that the configuration of a hyperon star rotating at its Keplerian frequency is far from a spherical symmetric shape.

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