Thermopower in parallel double quantum dots with Rashba spin–orbit interaction*

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(Received 22 June 2010; revised manuscript received 8 September 2010)

Based on the Green's function technique and the equation of motion approach, this paper theoretically studies the thermoelectric effect in parallel coupled double quantum dots (DQDs), in which Rashba spin–orbit interaction is taken into account. Rashba spin–orbit interaction contributions, even in a magnetic field, are exhibited obviously in the double quantum dots system for the thermoelectric effect. The periodic oscillation of thermopower can be controlled by tunning the Rashba spin–orbit interaction induced phase. The interesting spin-dependent thermoelectric effects will arise which has important influence on thermoelectric properties of the studied system.

Keywords: quantum dot, thermopower, Rashba spin–orbit interaction, spin-dependent inter-dot coupling

PACS: 73.21.La, 72.25.Dc, 72.20.Pa **DOI:** 10.1088/1674-1056/20/2/027301

1. Introduction

Recently the thermoelectrical effects in the mesoscopic system, especially for the quantum dot (QD) system, have gained renewed interest for its properties of enhancing the efficiency of thermal and electrical conversion. Beenakker and Staring[1] have mentioned that the thermopower is a far more sensitive probe of the transport mechanisms in QD than the conductance. Kim *et al.*[2−4] reported the thermoelectric effects of phase-coherent Aharnov–Bohm (AB) interferometer with an embedded QD in the Kondo regime. They showed that the thermopower is one of the most sensitive to the shape or the particle-hole asymmetry of the transmission probability and is an appropriate experimental probe to investigate the Fano interference and the AB effect. Krawiec *et al.* discussed the thermoelectric properties of Kondo correlated QD coupled to two ferromagnetic electrodes,[5,6] two normal leads,[7] one normal and one superconducting lead,[8] respectively, and showed that the thermoelectric properties are obviously related to the spin of electron, asymmetrical in the coupling strength between the dot and the lead and the lead material. Employing the x-boson approach for the impurity Anderson model, Francoa *et al.* studied thermoelectric properties of two parallel coupled QDs connected to two paramagnetic conducting leads. The results showed that the energy level of the QD can change the sign of the thermopower, which is associated with an oscillatory behaviour and a suppression of the thermopower magnitude at low temperatures.[9,10] Zhang[11] studied thermoelectric properties of DQD systems and focused on the interplay between the Fano effect and electronic correlations. Generally, the thermopower is a more functional tool to study the energy transport than that of conductance.

In addition, the Rashba spin–orbit interaction (RSOI) is an important mechanism that influences the electron state in low dimensional structure. With the help of RSOI, the spin polarization becomes infinity, and more effective than injecting spin currents from ferromagnet into semiconductor in a two-terminal geometry.[12,13] Datta and Das[14] proposed

*Project supported by the Scientific Research Fund of Heilongjiang Provincial Education Department of China (Grant No. 11551145).

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a promising future theory about a spin field effect transistor, one can use the RSOI to control the spin-polarized distributions of carriers. In recent years, an AB ring device, in which one or two QDs having RSOI are located in its arms, is proposed to realize the spin-polarized transport.[15] The QD ring has already been realized in experiments and was used to investigate many important transport phenomena, such as the Fano and Kondo effects,[16−22] and the spin-dependent transport.[23] When the RSOI in the QDs is taken into consideration, the electrons flowing through different arms of the AB ring will acquire a spin-dependent phase factor in the tunnel-coupling strength, which results in different quantum interference effect for the spin-up and spin-down electrons.

From the above works one can see that the RSOI influences obviously many properties of a nano-meter quantum system. Then the reports on the influence of the RSOI on the thermal properties of a QD system are not found according to our information. In this paper we shall present our study on the effects of the RSOI and magnetic field on the thermopower oscillation and the spin interference effects for a double semiconductor QD. The interplay of the RSOI and spin interference effect greatly influences the thermoelectric transport properties in this system. The issue includes dependence of the thermopower on the RSOI and the problem how thermopower is modulated by this interaction. Answering these questions is not only interesting for the mesoscopic physics but also for the potential application in heat and electric conversion devices.

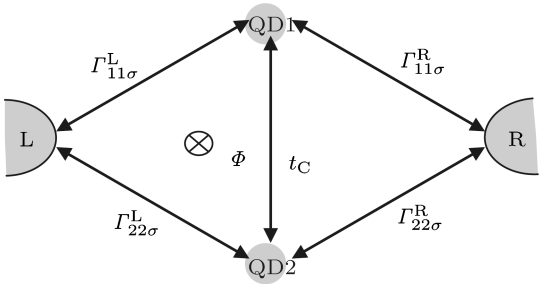


Fig. 1. The AB interferometer with two coupled QDs for each lead with temperature $T_{\rm L(R)}$ and electrochemical potential $\mu_{\rm L(R)}$. Relative to the Fermi energy in the leads the energy level positions in each dot are ε_1 and ε_2 respectively.

2. Model and formulation

We consider the parallel double QDs system connected to left and right leads, in which a QD is embedded in each arm and there is the coupling between two QDs, as depicted schematically in Fig. 1. The system is modeled by a tight-binding Hamiltonian

$$\begin{aligned} H =& \sum_{\boldsymbol{k}\alpha\sigma} \varepsilon_{\boldsymbol{k}\alpha} c^{\dagger}_{\boldsymbol{k}\alpha\sigma} c_{\boldsymbol{k}\alpha\sigma} + \sum_{i} \varepsilon_i d^{\dagger}_{i\sigma} d_{i\sigma} \\ &+ \sum_{\boldsymbol{k}\alpha i\sigma} t_{\alpha i\sigma} (c^{\dagger}_{\boldsymbol{k}\alpha\sigma} d_{i\sigma} + d^{\dagger}_{i\sigma} c_{\boldsymbol{k}\alpha\sigma}) \\ &- t_{\rm c} \sum_{\sigma} (d^{\dagger}_{1\sigma} d_{2\sigma} + d^{\dagger}_{2\sigma} d_{1\sigma}) + \sum_{i} U_i n_{i\uparrow} n_{i\downarrow}, \quad (1) \end{aligned}$$

where $\alpha = {\rm L(R)}$ and $i(=1,2)$ are the indices of lead and dot, respectively; $c^{\dagger}_{\boldsymbol{k}\alpha\sigma}(c_{\boldsymbol{k}\alpha\sigma})$ is the creation (annihilation) operator for a conduction electron with wave vector $\boldsymbol{k}$ and spin σ ($\sigma = \uparrow, \downarrow$ or ± 1, and $\bar{\sigma} = -\sigma$) in the lead α; $d^{\dagger}_{i\sigma}(d_{i\sigma})$ is the creation (annihilation) operator of the electron in the dot i, and $t_{\alpha i\sigma}$ is the hopping matrix element of an electron between the dot i and the lead α. For simplicity, the matrix element $t_{\alpha i\sigma}$ is assumed to be independent of the wave vector $\boldsymbol{k}$. The last term in Eq. (1) denotes the inter-dot coupling, in which $t_{\rm c}$ is the dot–dot tunneling coupling strength and is taken as a real parameter, U_i is the Coulomb interaction energy in QD i, $n_{i\sigma} = d^{\dagger}_{i\sigma} d_{i\sigma}$ is the particle number operator per spin σ for the dot i. The hopping matrix elements $t_{\alpha i\sigma}$ can be written in the form

$$\begin{aligned} t_{\rm L1\sigma} &= |t_{\rm L1}| {\rm e}^{{\rm i}\,\varphi/4} {\rm e}^{-{\rm i}\,\sigma\phi_{\rm R1}/2}, \\ t_{\rm R1\sigma} &= |t_{\rm R1}| {\rm e}^{-{\rm i}\,\varphi/4} {\rm e}^{{\rm i}\,\sigma\,\phi_{\rm R1}/2}, \\ t_{\rm L2\sigma} &= |t_{\rm L2}| {\rm e}^{-{\rm i}\,\varphi/4} {\rm e}^{-{\rm i}\,\sigma\phi_{\rm R2}/2}, \\ t_{\rm R2\sigma} &= |t_{\rm R2}| {\rm e}^{{\rm i}\,\varphi/4} {\rm e}^{{\rm i}\,\sigma\phi_{\rm R2}/2}, \end{aligned} \quad (2)$$

where

$$\varphi = 2\pi \Phi / \Phi_0$$

is the phase shift due to the magnetic flux threading into the AB ring, $\Phi_0 = h/c$ is the flux quantum. The phase factor $\varphi_{{\rm R}i}$ arises from the RSOI in dot i,[24,25] which is related to the structure parameters as

$$\varphi_{{\rm R}i} = \alpha L_i m^* / \hbar^2,$$

where α is the RSOI strength, L_i is the length of dot i, m^* is the electron effective mass.

In the following calculation, we define the linewidth matrix elements as

$$\Gamma^{\alpha}_{ij\sigma} = 2\pi \sum_{\boldsymbol{k}} t_{\alpha i\sigma} t^{*}_{\alpha j\sigma} \delta(\varepsilon - \varepsilon_{\boldsymbol{k}\alpha})$$

and $\Gamma = \Gamma^{\rm L} + \Gamma^{\rm R}$. The matrix $\Gamma^{\rm L(R)}_{\sigma}$ is given by

$$\Gamma_{\sigma}^{\mathrm{L(R)}}=\begin{pmatrix}\Gamma_{11}^{\mathrm{L(R)}} & \sqrt{\Gamma_{11}^{\mathrm{L(R)}}\Gamma_{22}^{\mathrm{L(R)}}}\mathrm{e}^{\pm\mathrm{i}\,\phi_{\sigma}/2}\\ \sqrt{\Gamma_{11}^{\mathrm{L(R)}}\Gamma_{22}^{\mathrm{L(R)}}}\mathrm{e}^{\mp\mathrm{i}\,\phi_{\sigma}/2} & \Gamma_{22}^{\mathrm{L(R)}}\end{pmatrix}. \tag{3}$$

where $\Gamma_{ij}^{\mathrm{L(R)}}$ is the phase-independent parts of $\Gamma_{ij\sigma}^{\mathrm{L(R)}}$, $\phi_{\sigma}=\varphi-\sigma\varphi_{\mathrm{R}}$ is the spin-dependent phase factor with $\varphi_{\mathrm{R}}=\varphi_{\mathrm{R1}}-\varphi_{\mathrm{R2}}$. It is obvious that ϕ_{σ} only depends on the difference between phase factors φ_{R1} and φ_{R2} induced by the RSOI. Then we assume that there is the RSOI in only one of two dots.

We define the retarded Green's function $G_{ij\sigma}^{\mathrm{r}}(t-t')$ and advanced Green's function $G_{ij\sigma}^{\mathrm{a}}(t-t')$ $(i,j=1,2)$ as

$$\begin{aligned}G_{ij\sigma}^{\mathrm{r(a)}}(t-t') &= \mp\mathrm{i}\,\theta(t-t')\langle\{d_{i\sigma}(t),d_{j\sigma}^{\dagger}(t')\}\rangle\\ &= \langle\langle d_{i\sigma}(t),d_{j\sigma}^{\dagger}(t')\rangle\rangle^{\mathrm{r(a)}}.\end{aligned} \tag{4}$$

The transmission spectral function for each spin component can be expressed in terms of $G_{\sigma}^{\mathrm{r(a)}}(\varepsilon)$ which is the Fourier transform of $G_{ij\sigma}^{\mathrm{a}}(t-t')$,[26]

$$T_{\sigma}(\varepsilon)=\sum_{j}T_{j}(\varepsilon), \tag{5}$$

where

$$T_{j}(\varepsilon)=\frac{e^{2}}{\hbar}\{G_{\sigma}^{\mathrm{a}}\Gamma_{\sigma}^{\mathrm{R}}G_{\sigma}^{\mathrm{r}}\Gamma_{\sigma}^{\mathrm{L}}\}_{jj}, \tag{6}$$

By employing the equation of motion of Green's function, one can obtain the matrix Green's function

$$G_{\sigma}^{\mathrm{r}}(\varepsilon)=[1-g_{\sigma}^{\mathrm{r}}(\varepsilon)\Sigma_{\sigma}^{\mathrm{r}}]^{-1}g_{\sigma}^{\mathrm{r}}(\varepsilon), \tag{7}$$

where the retarded self-energy $\Sigma_{\sigma}^{\mathrm{r}}$, originating from the dot-lead coupling, is given by $\Sigma_{ij\sigma}^{\mathrm{r}}=-\frac{\mathrm{i}}{2}(\Gamma_{ij\sigma}^{\mathrm{L}}+\Gamma_{ij\sigma}^{\mathrm{R}})=-\frac{\mathrm{i}}{2}\Gamma_{ij\sigma}$ in the wide band limit. The isolated DQD Green's function

$$g_{ii\sigma}^{\mathrm{r}}(\varepsilon)=\frac{\varepsilon-\varepsilon_{i\sigma}-U_{i}(1-\langle n_{i\bar{\sigma}}\rangle)}{(\varepsilon-\varepsilon_{i\sigma})(\varepsilon-\varepsilon_{i\sigma}-U_{i})}, \tag{8a}$$

$$g_{i\bar{i}\sigma}^{\mathrm{r}}(\varepsilon)=t_{\mathrm{c}}. \tag{8b}$$

The lesser Green's function $G_{\sigma}^{<}$ can be written as

$$G_{\sigma}^{<}(\varepsilon)=G_{\sigma}^{\mathrm{r}}(\varepsilon)\Sigma_{\sigma}^{<}G_{\sigma}^{\mathrm{a}}(\varepsilon), \tag{9}$$

where the lesser self-energy is $\Sigma_{\sigma}^{<}=\mathrm{i}(f_{\mathrm{L}}\Gamma_{\sigma}^{\mathrm{L}}+f_{\mathrm{R}}\Gamma_{\sigma}^{\mathrm{R}})$.

The spin-up and spin-down electronic occupation number $n_{i\sigma}$ in dot i can be calculated self-consistently by

$$\langle n_{i\sigma}\rangle=-\mathrm{i}\int\frac{\mathrm{d}\varepsilon}{2\pi}G_{i\sigma,i\sigma}^{<}(\varepsilon). \tag{10}$$

When a current passes through a semiconductor system, the electrical current is accompanied by a thermal current. The thermal current arises from the fact that the electrical current causes the existence of temperature difference.[27] The electric current operator can be represented as a change in the number of electrons per unit time in the lead α,[28]

$$\begin{aligned}I_{\alpha} &= -\frac{e}{\hbar}\frac{\mathrm{d}\langle -N_{\alpha}\rangle}{\mathrm{d}t}=\frac{e}{\mathrm{i}\,\hbar}\langle[H,N_{\alpha}]\rangle\\ &= \frac{\mathrm{i}\,e}{\hbar}[t_{\alpha1\sigma}G_{\boldsymbol{k}\alpha,1\sigma}^{<}+\mathrm{H.c.}]\\ &\quad+\frac{\mathrm{i}\,e}{\hbar}[t_{\alpha2\sigma}G_{\boldsymbol{k}\alpha,2\sigma}^{<}+\mathrm{H.c.}],\end{aligned} \tag{11}$$

where $N_{\alpha}=\sum\limits_{\boldsymbol{k},\sigma}c_{\boldsymbol{k}\alpha\sigma}^{\dagger}c_{\boldsymbol{k}\alpha\sigma}$ and $H_{\alpha}=\sum\limits_{\boldsymbol{k},\sigma}\varepsilon_{\boldsymbol{k}\alpha\sigma}c_{\boldsymbol{k}\alpha\sigma}^{\dagger}c_{\boldsymbol{k}\alpha\sigma}$, and Keldysh Green's functions are

$$G_{\boldsymbol{k}\alpha\sigma,i\sigma}^{<}(t,t')=\mathrm{i}\langle d_{i\sigma}^{\dagger}(t')c_{\boldsymbol{k}\alpha\sigma}(t)\rangle, \tag{12a}$$

$$G_{i\sigma,\boldsymbol{k}\alpha\sigma}^{<}(t,t')=\mathrm{i}\langle c_{\boldsymbol{k}\alpha\sigma}^{\dagger}(t')d_{i\sigma}(t)\rangle, \tag{12b}$$

The energy current flowing into the AB interferometer from the αlead can be written as

$$\begin{aligned}I_{\alpha}^{\mathrm{E}} &= -\frac{e}{\hbar}\left\langle\frac{\mathrm{d}H_{\alpha}}{\mathrm{d}t}\right\rangle=\frac{e}{\mathrm{i}\,\hbar}\langle[H,H_{\alpha}]\rangle\\ &= \frac{\mathrm{i}}{\hbar}\varepsilon_{\boldsymbol{k}\alpha}[t_{\alpha1\sigma}G_{\boldsymbol{k}\alpha,1\sigma}^{<}+\mathrm{H.c.}]\\ &\quad+\frac{\mathrm{i}}{\hbar}\varepsilon_{\boldsymbol{k}\alpha}[t_{\alpha2\sigma}G_{\boldsymbol{k}\alpha,2\sigma}^{<}+\mathrm{H.c.}].\end{aligned} \tag{13}$$

The heat current flowing from the lead α into two QDs is determined by

$$I_{\alpha}^{\mathrm{Q}}=I_{\alpha}^{\mathrm{E}}-\mu_{\alpha}I_{\alpha}=I_{\alpha1}^{\mathrm{Q}}+I_{\alpha2}^{\mathrm{Q}}, \tag{14}$$

where the local thermal current

$$\begin{aligned}I_{\mathrm{L(R)}j}^{\mathrm{Q}} &= \sum_{\sigma}\int\frac{\mathrm{d}\varepsilon}{2\pi\hbar}(\varepsilon-\mu_{\alpha})[f_{\mathrm{L(R)}}(\varepsilon)\\ &\quad-f_{\mathrm{R(L)}}(\varepsilon)]T_{j}(\varepsilon),\end{aligned} \tag{15}$$

and the local electric current

$$I_{\mathrm{L(R)}j}=e\sum_{\boldsymbol{k}\sigma}\int\frac{\mathrm{d}\varepsilon}{2\pi\hbar}[f_{\mathrm{L(R)}}(\varepsilon)-f_{\mathrm{R(L)}}(\varepsilon)]T_{j}(\varepsilon). \tag{16}$$

In the linear regime, i.e., for small voltages $eV\to0$ and small temperature gradient $\Delta T=T_{\mathrm{L}}-T_{\mathrm{R}}\to0$, $\Delta T\ll T$ where $T_{\mathrm{L(R)}}$ is the temperature of the left (right) lead and in the following calculations we assume $T_{\mathrm{L}}\approx T_{\mathrm{R}}=T$. The kinetic coefficients are defined as

$$L_{11}=\frac{T}{h}\sum_{\sigma}\int\mathrm{d}\varepsilon\left(-\frac{\partial f_{\alpha}(\varepsilon)}{\partial\varepsilon}\right)_{T}T_{\sigma}(\varepsilon), \tag{17a}$$

$$L_{12} = \frac{T^2}{h}\sum_{\sigma}\int \mathrm{d}\varepsilon\left(\frac{\partial f_\alpha(\varepsilon)}{\partial T}\right)_\varepsilon T_\sigma(\varepsilon), \tag{17b}$$

$$L_{22} = \frac{T^2}{h}\sum_{\sigma}\int \mathrm{d}\varepsilon\left(\frac{\partial f_\alpha(\varepsilon)}{\partial T}\right)_\varepsilon (\varepsilon-\mu_\alpha)^2 T_\sigma(\varepsilon). \tag{17c}$$

The equilibrium Fermi function $f_\alpha(\varepsilon)$ in the lead α in the above equations is

$$f_\alpha(\varepsilon) = \{\exp[(\varepsilon-\mu_\alpha)/K_{\mathrm{B}}T_\alpha]+1\}^{-1}, \tag{18}$$

where μ_α ($eV = \mu_{\mathrm{L}} - \mu_{\mathrm{R}}$) is the chemical potential, T_α is the temperature.

The electric conductance can be written as

$$G = -\frac{e^2}{T}L_{11}. \tag{19}$$

Under the condition of an open circuit ($I = 0$), the thermopower S of a system is represented as

$$S = -\lim_{\Delta T\to 0}\frac{\Delta V}{\Delta T}\bigg|_{I=0} = -\frac{1}{eT}\cdot\frac{L_{12}}{L_{11}}, \tag{20}$$

where ΔV is the bias voltage between two leads.

3. Numerical result and discussion

For simplicity, in the following calculations only the symmetric case is considered, i.e., $\varepsilon_1 = \varepsilon_2 = \varepsilon_0$, we set the line-width matrix elements as $\Gamma_{11}^{\alpha} = \Gamma_{22}^{\alpha} = \Gamma$ ($\alpha \in L, R$), and $e = \hbar = k_{\mathrm{B}} = 1$. We assume that the inter-dot coupling strength t_{c} is of the order of Γ, the electron–electron interaction $U_1 = U_2 = U$, the density of state (DOS) of the electron in the leads is constant under a wide band limit, and $\Gamma_{ij}^{\alpha}(\varepsilon)$ is independent of ε.

Figure 2 demonstrates the evolution of thermopower by changing the interdot coupling strength t_{c}, which can be experimentally controlled by tuning the gate voltage. With increasing t_{c}, the spin-up thermopower will recover $S > 0$ the double-peak figure. For a small t_{c}, the sign of spin-down thermopower as a function of t_{c} is shown in Fig. 2(b). The vertical dotted lines denote positions of the peak and valley, which can help us find that the position is not the same, but we can conclude that the thermopower can work in an optimal value for tuning coupling coefficient t_{c} properly.

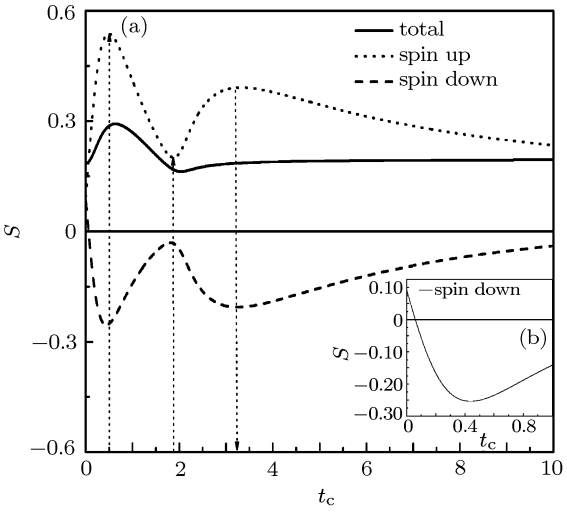


Fig. 2. Thermopower as a function of the interdot coupling strength t_{c}, where $\varphi = \pi$, $\varphi_{\mathrm{R}} = \pi/2$ and $U = 0$ [spin-up (dotted line), spin-down (dashed line), and total thermopower (solid line)].

Because of the phase locking effect, when there is no external magnetic flux or magnetic materials, the spin of the conductance for two-terminal device does not become polarized. Besides, when there is no magnetic flux and the RSOI, the thermopower remains spin unpolarized, which has already been reported in Ref. [29]. In the presence of an external magnetic field, the thermopower dips for different spins are not the same. Figure 3 shows for the spin-up, spin-down electrons and total thermopower changes with the total flux φ, for different φ_{R} with $T = \Gamma$, $\Gamma_{ij}^{\mathrm{L(R)}} = \Gamma$, $t_{\mathrm{c}} = \Gamma$. It is shown that the spin-up and spin-down electrons have different contributions to the thermopower in the presence of magnetic flux and RSOI. When $\varphi_{\mathrm{R}} = 0$ and there is the magnetic flux, we can find that the devotement of the spin-up electrons for the thermopower is the same as that of the spin-down electrons. From Fig. 3 one can see that when $\varphi_{\mathrm{R}} = 0$ three curves have the same shape, i.e. the shape of curves is independent of the value of φ_{R} even in the presence of magnetic field. But the curves for $\varphi_{\mathrm{R}} \neq 0$ shift φ_{R} in comparison with the curve for $\varphi_{\mathrm{R}} = 0$ correspondingly. When the magnetic flux exists, the different quantum interference effects of the spin-up and spin-down electrons will induce the spin-dependent phenomenon. The electric conductance made us known that the spin is also sensitive to the phase of φ_{R} caused by RSOI. It also found that the RSOI in QD can provide a new approach to manipulate quantum interference. Then when there is the

RSOI in the QD or magnetic flux in AB ring, the thermopowers should depend on the spin of carrier. The curve for the spin-up (down) electron thermopower shifts along the direction of increasing magnetic (decreasing) flux. This shift originates from the polarization of spin of electrons induced by RSOI. Then the total thermopower changing with the magnetic flux can show the polarization of the spin of transporting electrons. For example, for $\varphi_{\rm R} = \pi/2$ the function curve of total thermopower as the magnetic flux is symmetrical on $\varphi = 2\pi$. When the $\varphi_{\rm R} = \pi/2$, the peaks of the total thermopower cannot be split. Since, in this regime the contribution from the conduction electron spin moment to the transport of the whole system is negligible. The thermopower is a measure between electron and hole quasiparticle excitations. The sign of thermopower can indicate the main carrier in transporting charge and heat.

When there is no electron–electron interaction, i.e. $U = 0$, figure 3(a) shows that for the total thermopower, the interval between $0 < \varphi < 3\pi/2$ and $5\pi/2 < \varphi < 4\pi$, the sign of the thermopwer is positive. When transmission spectral weight lied in the hole channel is more than that in the electron channel, the charge and heat are carried mainly by hole channels, which is taken as holelike transport. In this case, the sign of the thermopower is positive. If $3\pi/2 < \varphi < 5\pi/2$, the sign of thermopower is negative, since charge and heat transport through the electron channels is predominant, the thermopower is negative, this transport called as electronlike transport. If the same amount of electron and heat are carried by each electron and hole channel, the sign of the electric/heat current is the same/opposite for electron and hole channel. The thermopower is zero.[2] At the position of $\varphi = 3\pi/2$ and $\varphi = 5\pi/2$, the thermopower is zero. If $0 < \varphi < 2\pi$, the contribution of the spin-up electrons for total thermopower is dominating component, while the electric current is yet polarized. If $2\pi < \varphi < 4\pi$, the contribution of the spin-down electrons for total thermopower is the dominating component.

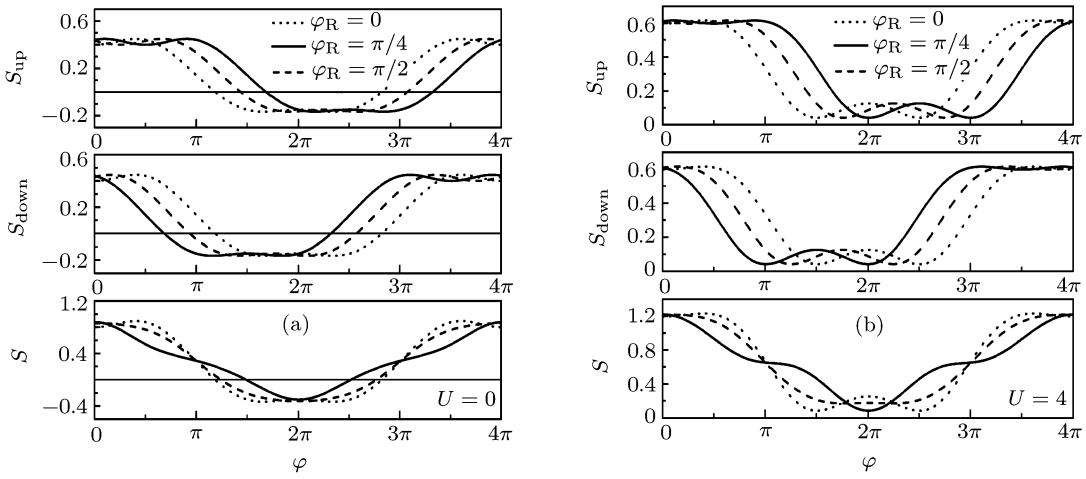


Fig. 3. The thermopower (dotted line, dashed line and solid line) as a function of the magnetic flux φ for $\varphi_{\rm R} = 0$, $\pi/4$ and $\pi/2$, respectively: (a) $U = 0$ and (b) $U = 4\Gamma$.

We discuss the many-body effect in Fig. 3(b). When the electron–electron interaction is considered, the interaction energy $U = 4\Gamma$. The similar results to the noninteracting case are shown in Fig. 3(b). Comparing Fig. 3(a) with Fig. 3(b), we point out that the sign of thermopower becomes positive in all the period. The Coulomb interaction has great effect on the thermoelectric transport of this system, when there has same Coulomb interaction in two dots, the thermopower shifts upwardly. Hence, one can deduce the sign of thermopower from the intra-dot electron–electron interaction as well as infer the conclusion that whether the contributing transport process is electron- or hole-like.

In Fig. 4 we show the temperature dependence of the total, spin-up and spin-down thermopowers for various phase factor $\varphi_{\rm R}$ induced by RSOI, with a fixed magnetic flux $\varphi = \pi$ and intradot electron–electron interaction $U = 4\Gamma$. We draw curves for $\varphi_{\rm R} = \pi/4$ and $15\pi/4$, $3\pi/4$ and $13\pi/4$, $5\pi/4$ and $11\pi/4$, $7\pi/4$ and $9\pi/4$, respectively. From Fig. 4 we can find that there is a peak in each curve which shows the maximal

thermopower and the corresponding temperature. When the RSOI exists, the temperature dependence of the spin-up thermopower and the spin-down thermopower may be very different. For example, when $\varphi_{\rm R} = 3\pi/4$ and $\varphi_{\rm R} = 13\pi/4$, the shape of the spin-up and spin-down thermopower are opposite almost in the indicating temperature scale in Fig. 4. This result shows that although the total thermopowers in two states are the same, the curve for $\varphi_{\rm R} = 3\pi/4$ shows the net spin-up thermopower and the curve for $\varphi_{\rm R} = 13\pi/4$ shows the net spin-down thermopower. When RSOI is presented inside the QD, by the coaction of RSOI and the magnetic flux threading the ring, the spin polarized thermopower and the spin flip effect will occur in the temperature dependence of thermopower curves.

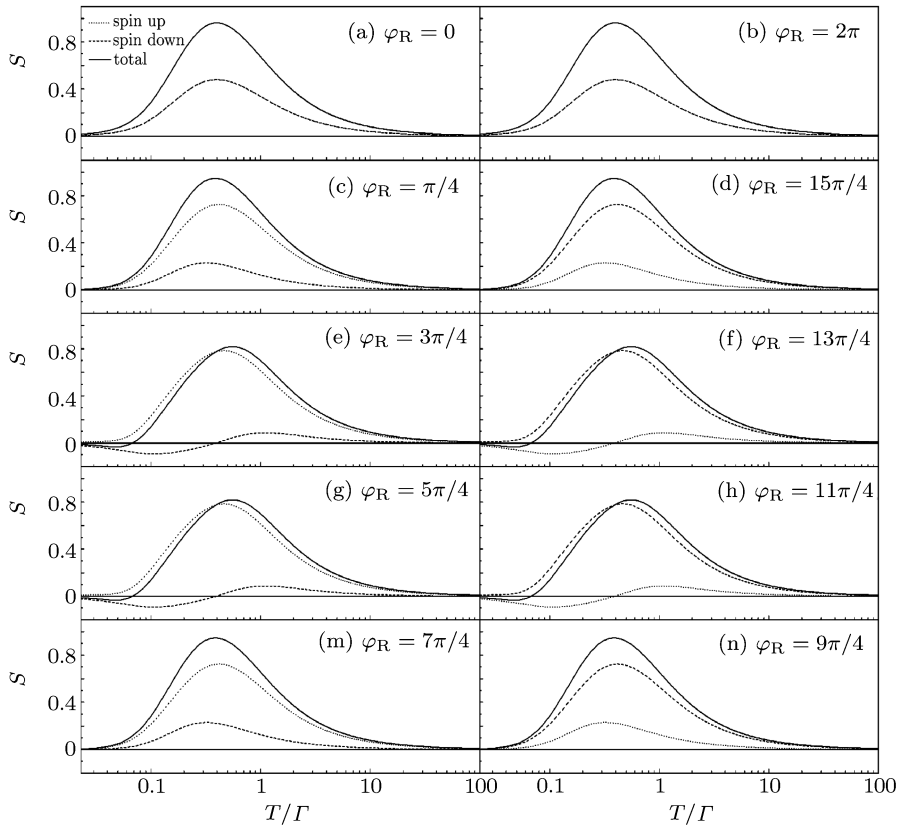


Fig. 4. The thermopowers as functions of temperature for different values of the RSOI induced phase $\varphi_{\rm R}$ for $\varphi_{\rm R} = 0$, $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$, 2π, $9\pi/4$, $11\pi/4$, $15\pi/4$ [spin-up (dotted line), spin-down (dashed line), and total thermopower (solid line)].

In order to make the physical picture clearer, we also can fix φ and turn the $\varphi_{\rm R}$. Figure 5 shows that the thermopower of each spin component as a function of the RSOI strength $\varphi_{\rm R}$ for various φ with $\Gamma_{ij}^{\rm L(R)} = \Gamma$, $t_{\rm c} = \Gamma$ and $T = \Gamma$. It is interesting to note that the thermopower oscillation is a periodic function of RSOI phase $\varphi_{\rm R}$ with a period of 4π. We can find that in the absence of magnetic field $\varphi = 0$, the function curve of the thermopower with $\varphi_{\rm R}$ is symmetrical on $\varphi_{\rm R} = 2\pi$. But the positive and negative thermopower is asymmetrical and the electron's contribution is small in comparison with hole. It has bigger maximal positive peak and the negative maximal absolute values than the thermopower for spin-up and spin-down electrons. The thermopowers for spin-up and spin-down have the same development as the spin is unpolarized. The total thermopower has the same behaviour as the spin-up and spin-down thermopower. While in the presence of magnetic field $\varphi = \pi$, the curve of the spin-up thermopower shifts rightwards π and the curve of the spin-down thermopower shifts leftwards π corresponding to the curve of $\varphi = 0$. Because spin-up and spin-down electrons experience different magnetic potentials, thus, as the magnetic field increases, the thermopowers for spin-up and spin-down electrons are different. In Fig. 5(a) we can find that when the magnetic field is considered and no intra-dot electron–electron interaction, the sign of total ther-

mopower is positive. The magnitude of thermopower does not change much with increasing the RSOI phase factor $\varphi_{\rm R}$ when $\varphi = \pi$, and the holes is the main carrier. The total thermopower curve is a relative smooth line, from this curve we can also find that the magnetic field does not play an active role in the case of existing RSOI existence.

Figure 5(b) describes the change of the thermopower with the increase of RSOI phase factor $\varphi_{\rm R}$, taking into account the electron–electron interaction and the absence of magnetic field $\varphi = 0$. We can find that although there is the electron–electron interaction, the thermopower is spin-independent. The value and the sign of thermopower are changed. In the presence of magnetic field $\varphi = \pi$, the total thermopower changes with the RSOI phase factor $\varphi_{\rm R}$ periodically, which is different from Fig. 4(a) and it is hole-like transport. For the spin-up thermopower, the maximal values appear at $\varphi = \pi/2$ and $\varphi = 3\pi/2$, the two minimal values appear at $\varphi = 5\pi/2$ and $\varphi = 7\pi/2$ respectively in a period. The spin-down thermopower shifts leftward π according to the spin-up one. It is almost always positive in the whole period, suggesting a hole nature of transport.

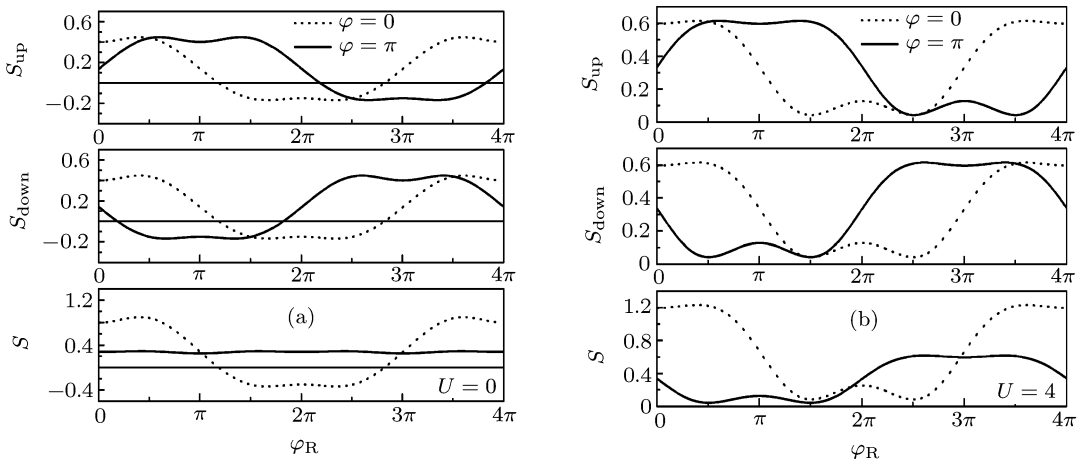


Fig. 5. Thermopower oscillation as a periodic function of $\varphi_{\rm R}$ with a period of 4π for $\Gamma_{ij}^{\rm L(R)} = \Gamma = t_{\rm c}$, $T = \Gamma$ and $\varphi = 0, \pi$ corresponding to dotted and solid lines respectively: (a) $U = 0$; (b) $U = 4\Gamma$.

We studied the thermal properties with the RSOI. It is shown that the RSOI strongly modifies thermal properties of the system. The thermoelectric figure of merit $Z = S^2G/\kappa$[5,7] is a measure of the usefulness of materials or devices for thermopower generators or cooling systems. We can also conclude that the low temperature contribution to the ZT is completely negligible, which has the same result as that given by Sakano *et al.*[30,31] As T increases, the above factors result in an optimal ZT. In this regime, the interesting practical applications should be noticed. We expect that a system with high figure of merit is a good heat-to-spin-voltage converter. We believe that in an appropriate condition, the higher thermoelectric transform efficiency in this system can be obtained.

4. Conclusions

In summary, we have investigated the spin thermopower of parallel DQD system with the RSOI and the applied magnetic field when the inter-dot interaction and the intra-dot interaction are taken into consideration. In the parallel-coupled DQD, not only the phase but also the magnitude of the effective coupling between dots and leads can be tuned by the magnetic flux, RSOI and intradot electron–electron interaction. By adjusting the height and thickness of the tunnelling barrier between two dots through gate voltages, we can tune the inter-dot coupling $t_{\rm c}$ and the dot-lead coupling $\Gamma_i^{\rm L(R)}$, which is another effective factor in changing the thermopower. It is demonstrated that sign reversal and oscillation of spin thermopower can be modulated by the interplay between the RSOI and magnetic field. With the increase of RSOI phase, the spin thermopower oscillates periodically and the intervals between adjacent peaks are uniform. The phase shift and spin flip effect can be found in our result.

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