Effects of dust size distribution in ultracold quantum dusty plasmas*

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(Received 31 March 2010; revised manuscript received 18 May 2010)

The effect of dust size distribution in ultracold quantum dusty plasmas are investigated in this paper. How the dispersion relation and the propagation velocity for the quantum dusty plasma vary with the system parameters and the different dust distribution are studied. It is found that as the Fermi temperature of the dust grains increases the frequency of the wave increases for large wave number dust acoustic wave. The quantum parameter of $H_{\rm d}$ also increases the frequency of the large wave number dust acoustic wave. It is also found that the frequency ω_0 and the propagation velocity v_0 of quantum dust acoustic waves all increase as the total number density increases. They are greater for unusual dusty plasmas than those of the usual dusty plasma.

Keywords: quantum dusty plasmas, dispersion relation, velocity of soliton

PACS: 52.35.Sb, 52.35.Vy, 52.35.Mw **DOI:** 10.1088/1674-1056/20/2/025203

1. Introduction

The dusty plasma physics has generated tremendous interest recently owing to its importance in space, astrophysics, and laboratory plasmas.[1−4] The dusty or complex plasma consists of electrons, ions, and extremely massive, highly charged micrometer-sized dust particles. The collective dusty plasma interactions have been studied both theoretically and experimentally during the past few years.[5,6] Dust acoustic waves (DAW) were first reported theoretically in unmagnetized dusty plasmas by Rao *et al.*[7] The existence of dust ion–acoustic waves (DIAW) was also discovered by Shukla and Silin at higher frequency.[8] Laboratory experiments on dusty plasmas have confirmed the existence of DAW and DIAW.[9−11]

Recently a great deal of attention has been focused on quantum effects in dusty plasmas.[12−15] When a dusty plasma is cooled down to an extremely low temperature so that the de-Broglie wavelength of the charge carriers could be comparable to the dimension of the systems, the ultracold dusty plasma behaves like a Fermi gas and quantum mechanical effects are expected to play a central role in the behaviour of charged dust particles. The thermal de-Broglie wavelength for the j-th species is $\lambda_{{\rm B}j} = \hbar/m_j V_{{\rm T}j} = a_j/\lambda_{{\rm D}j}$, where $a_j = \hbar/2q_j\sqrt{\pi m_j n_{j0}}$, $\lambda_{{\rm D}j}$ is Debye length, and q_j is the charge ($j = 1, 2, \ldots, N$), where we assume that there are N different dust particles. For classical regimes, we assume $\lambda_{\rm B} \ll \lambda_{{\rm D}j}$ and that is why the charged particles are assumed as point-like. But in quantum regimes, when $\lambda_{\rm B} \gg \lambda_{{\rm D}j}$, the quantum effects become more important and could not be ignored any more.[12−16] The important role of quantum mechanical effects in plasmas has been recognized in microelectronic devices,[17] dense astrophysical systems,[18] metallic nanostructures,[19] and laser plasmas.[20] In microelectro-mechanical systems, the presence of charged dust impurities can affect the system. Mathematically, the quantum hydrodynamic (QHD)[21] model generalizes the fluid model for plasmas, due to the inclusion of a quantum correction term, the so-called Bohm potential.

Although there are so many investigations on quantum effects about the dusty plasmas, most of them focused on the mono-sized quantum dusty plasma.[12−15] In fact, a dusty plasma consists of

*Project supported by the National Natural Science Foundation of China (Grant No. 10875098) and the Natural Science Foundation of Northwest Normal University, China (Grant Nos. NWNU-KJCXGC-03-48 and NWNU-KJCXGC-03-17).

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many different dust grains with different sizes.[22−26] It has been widely accepted that the dust size distribution can be described by a power law in space plasmas,[27,28] and given by a Gaussian distribution in laboratory plasmas.[22] But generally, the dust size distribution function does not exactly satisfy a power law distribution nor a Gaussian distribution. The distribution function depends on many factors or the environment. Therefore, it is important to investigate the role an arbitrary dust size distribution plays in quantum dusty plasmas. Recently, the dust size distribution effect in quantum dust plasmas has also been considered in a magnetized dusty plasmas.[29] The character of the nonlinear waves has been studied both theoretically and numerically.[29] However, until now the linear or nonlinear waves in unmagnetized quantum dusty plasmas taking into consideration the dust size distribution have not been reported in the literature.

In this paper, we first study the arbitrary dust size distribution effects in unmagnetized quantum dusty plasmas. We consider a polynomial-expressed distribution of dust particles in quantum dusty plasma. For dust grains with radius r in a given range $[r_{\min}, r_{\max}]$, the differential polynomial-expressed distribution function is of the form,[25]

$$n(r)\mathrm{d}r = [a_0 + a_1 r + a_2 r^2 + a_3 r^3 + \cdots]\mathrm{d}r; \quad (1)$$

it satisfies the following equation

$$N_{\mathrm{tot}} = \int_{r_{\min}}^{r_{\max}} n(r)\mathrm{d}r, \quad (2)$$

where r is the radius of dust grain, $a_0, a_1, a_2, a_3, \ldots$ are all constants, N_{tot} is the total number density of dust grains. Outside the limits $r < r_{\min}$ and $r > r_{\max}$, we use $n(r) = 0$.

In this paper, we consider the role of dust size distribution and employ the QHD model. We study the dispersion relation and propagation velocity of quantum dust acoustic waves in an unmagnetized, collisionless, utracold quantum dusty plasma. We study the quantum effects and dust size distribution effects for the dispersion relation.

In Section 2, the basic set of QHD equations are presented. In Section 3, the dispersion relation is studied. In Section 4, the propagation velocity of soliton is studied. In Section 5, the conclusion is presented.

2. Basic equation

We consider a one-dimensional QHD model to describe the dynamics of quantum dust acoustic waves in a three-component quantum dusty plasma consisting of electrons, ions and charged dust grains. We assume that the plasma particles obey the following equation[30]

$$p_s = \frac{m_s V_{\mathrm{F}_s}^2}{3n_{s0}^2} n_s^3, \quad (3)$$

where s equals e for electrons, i for ions and d for dust grains, m_s is the mass, $V_{\mathrm{F}_s} = \sqrt{2k_{\mathrm{B}}T_{\mathrm{F}s}/m_s}$ is Fermi speed, k_{B} is the Boltzmann constant, and $T_{\mathrm{F}s}$ is the Fermi temperature, n_s is the number density with its equilibrium value n_{s0}.

In the following we assume that there are N different dust grains with different sizes. We assume that the j-th ($j = 1, 2, 3, \ldots, N$) dust grain pressure in a one-dimensional zero-temperature Fermi gas obeys the following law

$$p_{\mathrm{d}j} = \frac{m_{\mathrm{d}j} V_{\mathrm{F}_{\mathrm{d}j}}^2}{3n_{\mathrm{d}j0}^2} n_{\mathrm{d}j}^3, \quad (4)$$

where $m_{\mathrm{d}j}$ ($j = 1, 2, 3, \ldots, N$) is the mass of the j-th dust grain, $V_{\mathrm{F}_{\mathrm{d}j}} = \sqrt{2k_{\mathrm{B}}T_{\mathrm{F}\mathrm{d}j}/m_{\mathrm{d}j}}$ is the j-th dust grain Fermi thermal speed (each dust grain is assumed to have equal temperature), and $T_{\mathrm{F}\mathrm{d}j}$ is the j-th dust grain Fermi temperature, $n_{\mathrm{d}j}$ is the j-th dust grain number density with its equilibrium value $n_{\mathrm{d}j0}$, $p_{\mathrm{d}j}$ is the j-th dust grain pressure.

In equilibrium, the charge neutrality condition is

$$n_{\mathrm{i}0} = \sum_{j=1}^{N} Z_{\mathrm{d}0j} n_{\mathrm{d}0j} + n_{\mathrm{e}0}, \quad (5)$$

$$n_{\mathrm{d}0j} = n_{\mathrm{d}0}(r_j)\mathrm{d}r, \quad (6)$$

where $n_{\mathrm{i}0}, n_{\mathrm{e}0}$, and $n_{\mathrm{d}0j}$ are the number densities of unperturbed ions, electrons, and the j-th dust grain, respectively. $Z_{\mathrm{d}0j}$ is the unperturbed number of charges residing on the j-th dust grain measured in units of electron charge. When N is infinity, the charge neutrality condition (5) is expressed by the following equation

$$n_{\mathrm{i}0} = \int_{r_1}^{r_2} Z_{\mathrm{d}}(r) n_{\mathrm{d}}(r)\mathrm{d}r + n_{\mathrm{e}0}. \quad (7)$$

The dynamics of the low phase velocity ($V_{\mathrm{Fd}} \ll V_{\mathrm{P}} \ll V_{\mathrm{Fe,i}}$), and low frequency ($\nu_{\mathrm{dn}} \ll | \partial_t | \ll \nu_{\mathrm{e,in}}$, $| \partial_t | \ll (V_{\mathrm{Fe,i}}^2/\nu_{\mathrm{e,in}})\partial_x^2$), where ν_{sn} is the normalized charged particle-neutral collision frequency. The basic equations in such a quantum dusty plasma with N different dust grain species is governed by the following set of equations

$$\frac{\partial u'_{\mathrm{d}j}}{\partial t'} + u'_{\mathrm{d}j}\frac{\partial u'_{\mathrm{d}j}}{\partial x'} = \frac{Z'_{\mathrm{d}j}e}{m'_{\mathrm{d}j}}\frac{\partial \varphi'}{\partial x'} - \frac{1}{m'_{\mathrm{d}j}n'_{\mathrm{d}j}}\frac{\partial p_{\mathrm{d}j}}{\partial x'}$$

$$+\frac{\hbar^2}{2m'^2_{\rm dj}}\frac{\partial}{\partial x'}\left(\frac{\dfrac{\partial^2}{\partial x'^2}\sqrt{n'_{\rm dj}}}{\sqrt{n'_{\rm dj}}}\right), \quad (8)$$

$$\frac{\partial n'_{\rm dj}}{\partial t'}+\frac{\partial}{\partial x'}(n'_{\rm dj}u'_{\rm dj})=0, \quad (9)$$

$$\frac{\partial^2\varphi'}{\partial x'^2}=4\pi e\left[\sum_{j=1}^{N}n'_{\rm dj}Z'_{\rm dj}+n'_{\rm e}-n'_{\rm i}\right], \quad (10)$$

where $u'_{\rm dj}$ is the fluid velocity in the x direction of the j-th dust grain, $Z'_{\rm dj}$ is the charge number of the j-th dust grain, φ' is the electrostatic potential. Equations (4)–(10) constitute the quantum hydrodynamic (QHD) model for a three-component e–d–i quantum dusty plasma.

We now introduce the normalizations: $u_{\rm dj}=u'_{\rm dj}/c_{\rm d}$, $n_{\rm dj0}=n'_{\rm dj0}/N_{\rm tot}$, $t=t'\omega_{\rm pd}$, $Z_{\rm dj}=Z'_{\rm dj}/\bar{z}_{\rm d0}$, $m_{\rm dj}=m'_{\rm dj}/\bar{m}_{\rm d}$, $\varphi=e\varphi'/B_0$, $n_j=n'_j/n_{j0}$, $(j={\rm e,i})$, $c_{\rm d}=\sqrt{(2\bar{z}_{\rm do}k_{\rm B}T_{\rm Fi})/\bar{m}_{\rm d}}$, $\omega_{\rm pd}=\sqrt{(4\pi\bar{z}^2_{\rm do}e^2N_{\rm tot})/\bar{m}_{\rm d}}$, $\lambda_{\rm D}=\sqrt{(2k_{\rm B}T_{\rm Fi}/4\pi\bar{z}_{\rm d0}e^2N_{\rm tot})}$ is the Debye length. Therefore we obtain the following set of dimensionless equations

$$\frac{\partial u_{\rm dj}}{\partial t}+u_{\rm dj}\frac{\partial u_{\rm dj}}{\partial x}=\frac{Z_{\rm dj}}{m_{\rm dj}}\frac{\partial\varphi}{\partial x}-\delta\frac{1}{n^2_{\rm dj0}}\frac{n_{\rm dj}}{m_{\rm dj}}\frac{\partial n_{\rm dj}}{\partial x}+\frac{H^2_{\rm d}}{2m_{\rm dj}}\frac{\partial}{\partial x}\left(\frac{\dfrac{\partial^2}{\partial x^2}\sqrt{n_{\rm dj}}}{\sqrt{n_{\rm dj}}}\right), \quad (11)$$

$$\frac{\partial n_{\rm dj}}{\partial t}+\frac{\partial}{\partial x}(n_{\rm dj}u_{\rm dj})=0, \quad (12)$$

$$\frac{\partial^2\varphi}{\partial x^2}=\sum_{j=1}^{N}n_{\rm dj}Z_{\rm dj}+\mu_{\rm e}n_{\rm e}-\nu_{\rm i}n_{\rm i}, \quad (13)$$

where $B_0=c^2_{\rm d}\bar{m}_{\rm d}/\bar{z}_{\rm d0}$, $\delta=T_{\rm Fd}/T_{\rm Fi}\bar{z}_{\rm d0}$, $H_{\rm d}=\sqrt{\hbar^2\omega^2_{\rm pd}/\overline{m}_{\rm d}c^4_{\rm d}}$, $\mu_{\rm e}=n_{\rm e0}/\bar{z}_{\rm d0}N_{\rm tot}$, $\nu_{\rm i}=n_{\rm i0}/\bar{z}_{\rm d0}N_{\rm tot}$, $\mu_{\rm e}=1/(\mu-1)$, $\nu_{\rm i}=\mu/(\mu-1)$; $n_{\rm e}$ and $n_{\rm i}$ are given as follows:[13]

$$n_{\rm e}=\left[1+\frac{2\varphi}{\sigma}+\frac{H^2_{\rm e}}{\sigma}\left(1+\frac{2\varphi}{\sigma}\right)^{-1/4}\times\frac{\partial^2}{\partial x^2}\left(1+\frac{2\varphi}{\sigma}\right)^{1/4}\right]^{1/2}, \quad (14)$$

$$n_{\rm i}=\left[1-2\varphi+H^2_{\rm i}(1-2\varphi)^{-1/4}\times\frac{\partial^2}{\partial x^2}(1-2\varphi)^{1/4}\right]^{1/2}, \quad (15)$$

where $\sigma=T_{\rm Fe}/T_{\rm Fi}$; $H_{\rm e}=\sqrt{(\bar{z}_{\rm d0}\hbar^2\omega^2_{\rm pd})/m_{\rm e}\overline{m}_{\rm d}c^4_{\rm d}}$, $H_{\rm i}=\sqrt{(\bar{z}_{\rm d0}\hbar^2\omega^2_{\rm pd})/m_{\rm i}\overline{m}_{\rm d}c^4_{\rm d}}$ are quantum parameters for electron and ion, respectively.

3. Dispersion relation

We now study the dispersion relation of the system by assuming $n_{\rm dj}=n_{\rm dj0}+\tilde{n}_j\,{\rm e}^{{\rm i}(kx-\omega_0t)}$, $u_{\rm dj}=\tilde{u}_j\,{\rm e}^{{\rm i}(kx-\omega_0t)}$, $\varphi=\tilde{\varphi}\,{\rm e}^{{\rm i}(kx-\omega_0t)}$, where k and ω_0 are the wave number and the frequency, respectively; $\tilde{n}_j\,{\rm e}^{{\rm i}(kx-\omega_0t)}$, $\tilde{u}_j\,{\rm e}^{{\rm i}(kx-\omega_0t)}$, and $\tilde{\varphi}\,{\rm e}^{{\rm i}(kx-\omega_0t)}$ are small perturbations in equilibrium state. We assume the fluid velocity and electrostatic potential equal zero in an unperturbed equilibrium state. Substituting these expansions into Eqs. (11)–(13), we obtain the dispersion relation as follows:

$$\sum_{j=1}^{N}\frac{4k^2Z^2_{\rm dj}m_{\rm dj}n_{\rm dj0}}{4k^2m_{\rm dj}\delta+k^4H^2_{\rm d}-4m^2_{\rm dj}\omega^2_0}=-\left(k^2+\mu_{\rm e}\left(\frac{1}{\sigma}-k^2\frac{H^2_{\rm e}}{4\sigma^2}\right)+\nu_{\rm i}\left(1-k^2\frac{H^2_{\rm i}}{4}\right)\right). \quad (16)$$

The charge and the mass of the j-th dust grain can be expressed as follows:[22] $Z_{\rm dj}=k_zr_j$, $m_{\rm dj}=k_{\rm m}r^3_j$, where $k_z\approx 4\pi\varepsilon_0V_0/e$, $k_{\rm m}\approx 4\pi\rho_{\rm d}/3$, ε_0 is the vacuum permittivity, V_0 is the electric surface potential at equilibrium, $\rho_{\rm d}$ is the mass density of the dust grains (assumed to be constant and equal for all grains). We assume that the dust size distribution is given by Eq. (1). Therefore, after substituting Eq. (1) into Eq. (16), we obtain

$$\int_{r_{\rm min}}^{r_{\rm max}}\frac{4k^2k^2_zk_{\rm m}(a_0r^5+a_1r^6)}{4k_{\rm m}r^3k^2\delta+k^4H^2_{\rm d}-4k^2_{\rm m}r^6\omega^2_0}\,{\rm d}r=-\left(k^2+\mu_{\rm e}\left(\frac{1}{\sigma}-k^2\frac{H^2_{\rm e}}{4\sigma^2}\right)+\nu_{\rm i}\left(1-k^2\frac{H^2_{\rm i}}{4}\right)\right). \quad (17)$$

Equation (17) is an implicit function of relation between frequency ω_0 and wave number k. We can find the dispersion relation of $\omega_0=\omega_0(k)$ by numerical simulations. In our numerical simulation, we choose the typical parameters reported previously:[14,31,32] $n_{\rm i0}=2.0\times10^{30}\ {\rm m}^{-3}$, $n_{\rm e0}=5\times10^{29}\ {\rm m}^{-3}$, $Z_{\rm d0}=10^3$, $m_{\rm i}\doteq m_{\rm p}$, $T_{\rm Fe}=100\ {\rm K}$, $m_{\rm d}=10^{-17}\ {\rm kg}$, $\rho_{\rm d}\sim1\ {\rm g/cm}^3$, $a\sim0.1$–$1\ \mu{\rm m}$.

Figure 1 plots the dispersion relation of DAW by considering the effect of dust size distribution for different values of the quantum parameter $H_{\rm d}$. We note

that the frequency of dust acoustic waves ω_0 increases as the quantum parameter $H_{\rm d}$ increases, if the wave number k is large enough. If k is small enough, the frequency is independent of the quantum parameter $H_{\rm d}$, as seen in Fig. 1.

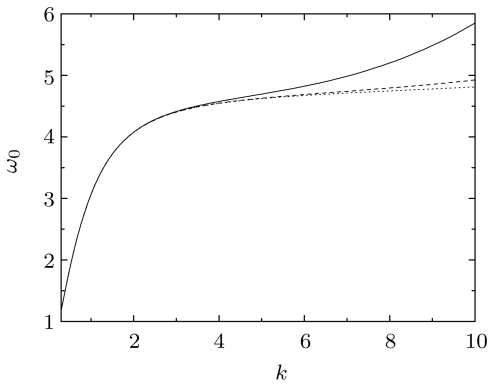


Fig. 1. Normalized dispersion relation for the quantum dust acoustic wave with $H_{\rm d} = 2.656 \times 10^{-9}$ (dotted line), $H_{\rm d} = 2.656 \times 10^{-7.5}$ (dashed line), $H_{\rm d} = 2.656 \times 10^{-7}$ (solid line). Other parameter values are $\mu = 4.0$, $\sigma = 10$, $H_{\rm i} = 6.494 \times 10^{-3}$, $H_{\rm e} = 0.2782$, $\delta = 5 \times 10^{-8}$, $k_z = 1$, $k_{\rm m} = 4$, $r_{\rm min} = 0.01$, $r_{\rm max} = 1$, $a_0 = 0.9$, $a_1 = -0.3$ and $a_2 = a_3 = \cdots = 0$.

Figure 2 illustrates the dispersion relation, $\omega_0 = \omega_0(k)$, of DAW by considering the effect of dust size distribution for different values of the Fermi temperature ratio (δ) of dust grains to ions of the dusty plasma. We also find that for the smaller k the frequency is independent of δ, while for large k the frequency is a function of δ. The frequency of the wave increases as the parameter of δ increases. This means that as the Fermi temperature of the dust grains increases the frequency of the wave increases for large wave number.

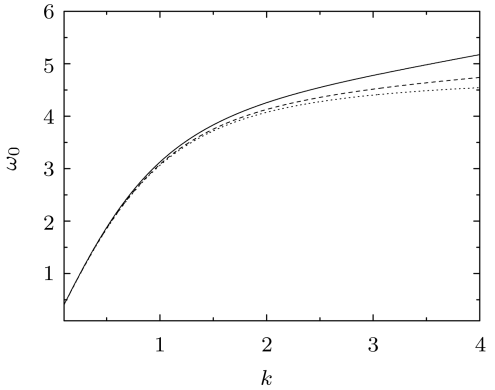


Fig. 2. Normalized dispersion relation for the quantum dust acoustic wave with $\delta = 5 \times 10^{-8}$ (dotted line), $\delta = 5 \times 10^{-7}$ (dashed line), $\delta = 5 \times 10^{-6.5}$ (solid line). $H_{\rm d} = 2.656 \times 10^{-9}$. All other parameters are the same as those in Fig. 1.

Figures 3 and 4 show how the dust distribution function affects the dispersion relation for the system. For generality and simplicity, we first let $n(r) = a_0$, $k = 1$ and find that ω_0 depends on the parameter a_0. It indicates that as a_0 increases, ω_0 increases (see Fig. 3). Since a_0 stands for the magnitude of total number density of dust particles, which should always be positive, we can conclude that the wave frequency increases as the total number density of dust particles increases.

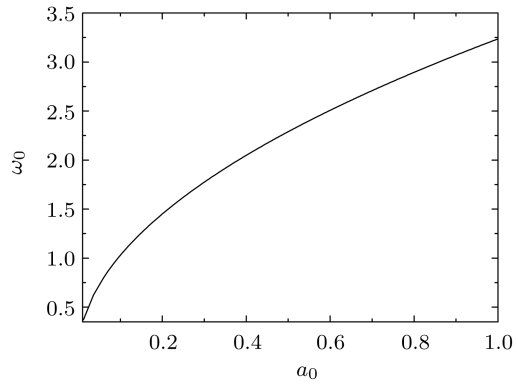


Fig. 3. The variation of ω_0 with respect to a_0 for $k = 1$, other parameter values are $\mu = 4.0$, $\sigma = 10$, $H_{\rm d} = 2.656 \times 10^{-9}$, $H_{\rm i} = 6.494 \times 10^{-3}$, $H_{\rm e} = 0.2782$, $\delta = 5 \times 10^{-8}$, $k_z = 1$, $k_{\rm m} = 4$, $r_{\rm min} = 0.01$, $r_{\rm max} = 1$.

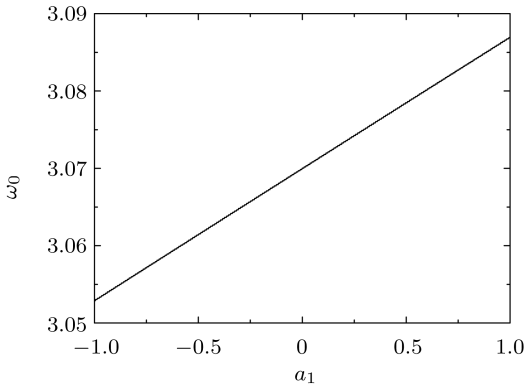


Fig. 4. The variation of ω_0 with a_1 for $k = 1$, other parameter values are $\mu = 4.0$, $\sigma = 10$, $H_{\rm d} = 2.656 \times 10^{-9}$, $H_{\rm i} = 6.494 \times 10^{-3}$, $H_{\rm e} = 0.2782$, $\delta = 5 \times 10^{-8}$, $k_z = 1$, $k_{\rm m} = 4$, $r_{\rm min} = 0.01$, $r_{\rm max} = 1$, $a_0 = 0.9$.

Second, for the same purpose, we let $n(r) = a_0 + a_1 r$, $k = 1$ and $a_0 = 0.9$. We note that ω_0 depends on the parameter of a_1, i.e. ω_0 increases as a_1 increases. As we know from Ref. [25], the number density of larger dust particles is greater than that of smaller ones for the case of $a_1 > 0$. While for the case of $a_1 < 0$, which is usually found in both space plasmas and in the laboratory, the number density of larger dust particles is smaller than that of smaller dust particles. Therefore, we conclude that for the usual dusty plasmas the wave frequency is smaller than that for the unusual case. This result is simi-

lar to that found in classical dusty plasma in which the quantum effects is negligible.[25]

4. Propagation velocity of soliton

For the small, but finite amplitude waves, we use the traditional perturbation method[33] as previously used extensively. For dusty plasmas with an arbitrary dust size distribution, we introduce the stretched variables as follows:

$$\xi = \varepsilon(x - v_0 t), \quad \tau = \varepsilon^3 t, \tag{18}$$

where ε is a small parameter characterizing the strength of nonlinearity, v_0 is the propagation velocity of soliton. The variables can be expanded as follows:

$$\begin{aligned} &n_{\mathrm{d}j} = n_{\mathrm{d}0j} + \varepsilon^2 n_{\mathrm{d}1j} + \varepsilon^4 n_{\mathrm{d}2j} + \cdots ; \\ &u_{\mathrm{d}j} = \varepsilon^2 u_{\mathrm{d}1j} + \varepsilon^4 u_{\mathrm{d}2j} + \cdots \quad (j = 1, \ldots, N); \\ &\varphi = \varepsilon^2 \varphi_1 + \varepsilon^4 \varphi_2 + \cdots . \end{aligned} \tag{19}$$

Substituting Eqs. (18) and (19) into Eqs. (11)–(13) and collecting the lowest order terms of ε, we obtain

$$n_{\mathrm{d}1j} = \frac{n_{\mathrm{d}0j}}{v_0} u_{\mathrm{d}1j}, \tag{20}$$

$$u_{\mathrm{d}1j} = \frac{Z_{\mathrm{d}j} v_0}{\delta - m_{\mathrm{d}j} v_0^2} \phi_1, \tag{21}$$

$$\sum_{j=1}^{N} n_{\mathrm{d}1j} Z_{\mathrm{d}j} = -\left(\frac{\mu_{\mathrm{e}}}{\sigma} + \nu_{\mathrm{i}}\right) \phi_1. \tag{22}$$

Using Eqs. (20)–(22), we obtain the propagation velocity of soliton as follows:

$$\sum_{j=1}^{N} \frac{Z_{\mathrm{d}j}^2 n_{\mathrm{d}0j}}{\delta - m_{\mathrm{d}j} v_0^2} = -\left(\frac{\mu_{\mathrm{e}}}{\sigma} + \nu_{\mathrm{i}}\right). \tag{23}$$

Substituting Eq. (1) into Eq. (23), we obtain

$$\int_{r_{\min}}^{r_{\max}} \frac{k_z^2 (a_0 r^2 + a_1 r^3)}{\delta - k_{\mathrm{m}} r^3 v_0^2} = -\left(\frac{\mu_{\mathrm{e}}}{\sigma} + \nu_{\mathrm{i}}\right). \tag{24}$$

Equation (24) is an implicit expression of propagation velocity of soliton. The numerical simulation results are given in the following.

Figures 5 and 6 show how the dust distribution function affects the propagation velocity of DAW. For the simple case, we let $n(r) = a_0$, and find that v_0 depends on the parameter a_0. It shows that v_0 increases as a_0 increases, as clearly seen in Fig. 5. Therefore, we conclude that the propagation velocity of DAW increases as the total number density of dust particles increases.

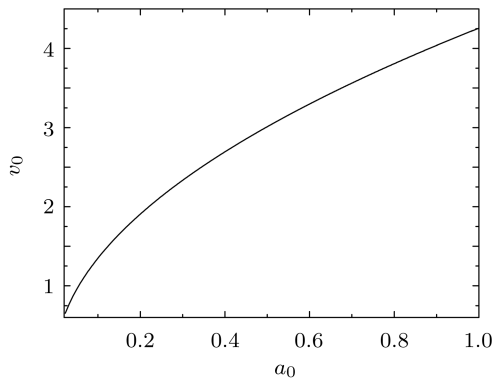


Fig. 5. The variation of v_0 with a_0; other parameter values are $\mu = 4.0$, $\sigma = 10$, $\delta = 5 \times 10^{-8}$, $k_z = 1$, $k_{\mathrm{m}} = 4$, $r_{\min} = 0.01$, $r_{\max} = 1$.

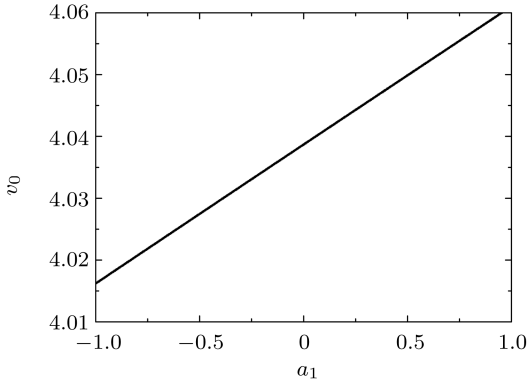


Fig. 6. The variation of v_0 with a_1; other parameter values are $\mu = 4.0$, $\sigma = 10$, $\delta = 5 \times 10^{-8}$, $k_z = 1$, $k_{\mathrm{m}} = 4$, $r_{\min} = 0.01$, $r_{\max} = 1$, $a_0 = 0.9$.

For the more complex case, we let $n(r) = a_0 + a_1 r$, and $a_0 = 0.9$. It is noted from Eq. (24) that v_0 is a function of a_1. From Fig. 6, we can see that v_0 increases as a_1 increases. So, we can conclude that for the usual dusty plasmas of $a_1 < 0$ the wave propagation velocity is smaller than that of unusual case $a_1 > 0$. This result is similar to that found in classical dusty plasma.[25]

5. Conclusion

The dispersion relation and the propagation velocity of soliton have been studied for quantum dusty plasma, taking into consideration the dust size distribution. For an arbitrary dust size distribution in quantum dusty plasmas, we choose the polynomial-expressed distribution function to investigate how the dust size distribution influences the dispersion relation and the propagation velocity of quantum dust acoustic waves.

From numerical results, we can find that the dispersion relation is affected by the Fermi temperature

ratio δ of dust grains to ions, the quantum parameter $H_{\rm d}$ and the dust size distribution in quantum dust plasmas.

We conclude that as the Fermi temperature of the dust grains increases the frequency of the wave increases for large wave number DAW. The quantum parameter of $H_{\rm d}$ also increases the frequency of the large wave number DAW. We can see that the frequency of the wave ω_0 increases as the parameter δ and the quantum parameter $H_{\rm d}$ increase if the wave number k is large enough. We find that the frequency ω_0 and the propagation velocity v_0 of quantum dust acoustic waves all increase as the total number density increases. They are greater for unusual dusty plasmas than those of the usual dusty plasma. This conclusion is similar to that found in classical dusty plasma in which the quantum effects are neglected.[25]

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