Propagation and interaction of ion–acoustic solitary waves in a quantum electron–positron–ion plasma*

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(Received 24 July 2010; revised manuscript received 13 September 2010)

This paper discusses the existence of ion–acoustic solitary waves and their interaction in a dense quantum electron–positron–ion plasma by using the quantum hydrodynamic equations. The extended Poincaré–Lighthill–Kuo perturbation method is used to derive the Korteweg-de Vries equations for quantum ion–acoustic solitary waves in this plasma. The effects of the ratio of positrons to ions unperturbation number density p and the quantum diffraction parameter $H_{\rm e}$ ($H_{\rm p}$) on the newly formed wave during interaction, and the phase shift of the colliding solitary waves are studied. It is found that the interaction between two solitary waves fits linear superposition principle and these plasma parameters have significantly influence on the newly formed wave and phase shift of the colliding solitary waves. The investigations should be useful for understanding the propagation and interaction of ion–acoustic solitary waves in dense astrophysical plasmas (such as white dwarfs) as well as in intense laser-solid matter interaction experiments.

Keywords: quantum plasma, ion–acoustic solitary wave, Korteweg-de Vries equation, interaction
PACS: 52.27.–h, 52.35.Fp, 52.35.Mw **DOI:** 10.1088/1674-1056/20/2/025202

1. Introduction

Nonlinear electrostatic pulses (solitary waves) in multi-component plasmas have attracted considerable interest during the past years. In contrast to the usual plasma consisting of positive ions and electrons, the nonlinear waves behave differently in plasmas having an additional component of positrons.[1] Electron–positron–ion (EPI) plasmas are rather widespread in nature. As for astrophysical plasmas, such a plasma plays an important role in understanding the plasmas in the active galatic nuclei,[2] in the pulsar magnetospheres,[3] in the solar atmosphere,[4,5] and even in the early universe.[6−8] As for laboratory plasmas, it is well known that the short relativistically strong laser pulse in matter can be accompanied by the EPI plasma due to photoproduction of pairs during photon scattering by nuclei.[9−11]

The ion–acoustic wave (IAW) is an ion timescale phenomenon and this mode does not exist in electron–positron plasmas. The IAW in three-component EPI plasmas have been extensively studied and there are a lot of research works on this fundamental mode in both the magnetized and unmagnetized cases. It is well known that in a frame moving with the linear phase velocity (in the long-wavelength limit), the Korteweg-de Vries (KdV) equation is the simplest form of a nonlinear equation that yields a solitary wave structure. The formation of KdV solitary waves in plasmas is due to a delicate balance between nonlinearity and dispersion. In a homogeneous plasma, the behaviour of one-dimensional solitary waves is governed by the usual form of the KdV equation. Among the nonlinear structures, ion–acoustic solitary wave (IASW) represents the most important aspect of nonlinear phenomena in modern plasma research, and there are numerous investigations have been carried out on this fundamental mode in various EPI plasma systems.[12−16]

A quantum plasma, whose constituents are the electrons, positrons, and ions, has the properties of high plasma particle number densities and low temperatures, in contrast to the classical plasma. The latter has the characters of high temperatures and low particle number densities. When an EPI plasma is cooled to extremely low temperatures, the thermal de-Broglie wavelength of the charged particles is equal to or larger than the average interparticle distance. In

*Project supported by the Research Foundation for Young Teachers of Hexi University, China (Grant No. QN-201004).
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such a situation, ultracold EPI plasma behaves like a Fermi gas and quantum mechanical effects begin to play an important role in the behaviour of charged particles.[17−19] In quantum plasmas, the dispersion caused by strong density correlation due to quantum fluctuations plays a significant role in understanding the collective effect in plasmas.

Because of the importance of quantum mechanical effects on the collective interactions in superdense astrophysical environments (such as white dwarfs and neutron stars),[20] in microelectronic devices,[21] in high-intense laser-solid density experiments,[22] and in nonlinear quantum optics,[23,24] many authors have studied the quantum acoustic solitons in different quantum EPI plasma systems by using the quantum hydrodynamic (QHD) model. The QHD model is an extension of the classical fluid model in a plasma. It is composed of a set of equations that describe the transport of charge density, momentum, and energy in a charged particle system. The difference between QHD model and classical model is that an additional term, the so-called "Bohm potential" is introduced in the equation of motion for the charged particles. By employing QHD model, Sabry *et al.*[25] studied the nonlinear IASW in quantum EPI magnetoplasmas. Mushtaq *et al.*[26] examined the characteristics and stability of IASW with transverse perturbations in ultracold quantum magnetospheric EPI plasma. Ali *et al.*[27] investigated the linear and nonlinear properties of the IASW in a quantum EPI plasma. In these studies, not much attention has been paid to the study of the basic physics of interaction between solitary waves in such a plasma system. Therefore, in this paper, the head-on collision between two IASWs in a three-component quantum plasma consisting of electrons, positrons and ions is investigated by using the extended Poincaré–Lighthill–Kuo (PLK) method.[28−30]

This paper is orgainzed as follows: In Section 2, the basic set of fluid equations governing the dynamics of the nonlinear IASWs are presented, and two KdV equations describing the system are derived. In Section 3, we give a discussion. The results are summarized in the last section.

2. Basic set of equations and interaction between two IASWs

As mentioned, we consider the propagation of IASWs in a three-species unmagnetized, collisionless quantum plasma system composed of electrons, positrons, and ions. The electrons and positrons are considered inertialess by assuming that the phase velocity of the wave is much less than the Fermi velocities of electrons and positrons, respectively. We also assume that the ions are cold, and electrons and positrons for this one-dimensional zero-temperature Fermi gas obey the following pressure law[31]

$$p_j = \frac{m_j v_{\mathrm{F}j}^2}{3n_{j0}^2} n_j^3, \tag{1}$$

where $v_{\mathrm{F}j} = \sqrt{2k_{\mathrm{B}}T_{\mathrm{F}j}/m_j}$ with $j = \mathrm{e}, \mathrm{p}$ are the electron and positron Fermi thermal speeds, respectively; k_{B} is Boltzmann constant, m_j is particle mass, and $T_{\mathrm{F}j}$ is the particle Fermi temperature; n_j is the particle number density, and n_{j0} is the equilibrium number density. Furthermore, at equilibrium, the charge neutrality condition is $n_{\mathrm{e}0} = n_{\mathrm{i}0} + n_{\mathrm{p}0}$.

The basic set of normalized QHD equations describing the nonlinear dynamics of IASWs in the EPI quantum plasma system are given as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0, \tag{2}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{3}$$

$$\frac{H_{\mathrm{e}}^2}{2} \frac{\partial}{\partial x} \left(\frac{1}{\sqrt{n_{\mathrm{e}}}} \frac{\partial^2 \sqrt{n_{\mathrm{e}}}}{\partial x^2} \right) - n_{\mathrm{e}} \frac{\partial n_{\mathrm{e}}}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{4}$$

$$\frac{H_{\mathrm{p}}^2}{2} \frac{\partial}{\partial x} \left(\frac{1}{\sqrt{n_{\mathrm{p}}}} \frac{\partial^2 \sqrt{n_{\mathrm{p}}}}{\partial x^2} \right) - \sigma n_{\mathrm{p}} \frac{\partial n_{\mathrm{p}}}{\partial x} = \frac{\partial \phi}{\partial x}, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1+p) n_{\mathrm{e}} - p n_{\mathrm{p}} - n_{\mathrm{i}}. \tag{6}$$

In the above equations, the quantities n_j ($j = \mathrm{i}, \mathrm{e}, \mathrm{p}$) are the ion, electron, and positron number densities, normalized by the unperturbed number densities n_{j0}, respectively. The ion fluid velocity u_{i} and the electrostatic wave potential ϕ are normalized by the quantum ion–acoustic speed $c_{\mathrm{si}} = (2k_{\mathrm{B}}T_{\mathrm{Fe}}/m_{\mathrm{i}})^{1/2}$ and $2k_{\mathrm{B}}T_{\mathrm{Fe}}/e$, respectively. The space coordinate x is normalized by the ratio of ion plasma frequency $\omega_{\mathrm{pi}} = \sqrt{4\pi n_{\mathrm{i}0} e^2/m_{\mathrm{i}}}$ to the quantum ion–acoustic speed $c_{\mathrm{si}} = (2k_{\mathrm{B}}T_{\mathrm{Fe}}/m_{\mathrm{i}})^{1/2}$, and the time coordinate t is normalized by the ion plasma period $\omega_{\mathrm{pi}}^{-1} = \sqrt{m_{\mathrm{i}}/4\pi\eta_{\mathrm{i}0}e^2}$, respectively; where m_{i} is the mass of ions, $\sigma = T_{\mathrm{Fp}}/T_{\mathrm{Fe}}$ and $p = n_{\mathrm{p}0}/n_{\mathrm{i}0}$ are the ratio of positron to electron Fermi temperature and the ratio of positron to ion unperturbed number density, respectively. Since from the standard definition of Fermi temperature we have $k_{\mathrm{B}}T_{\mathrm{F}j} = \hbar^2(3\pi^2)^{2/3} n_{j0}^{2/3}/2m_j$, σ is related to p through the relation $\sigma = [\,p/(1+p)]^{2/3}$.

The nondimensional quantum diffraction parameter H_j ($j = \mathrm{e}, \mathrm{p}$) is defined as

$$H_j = \sqrt{\hbar^2 \omega_{\mathrm{pi}}^2 / m_{\mathrm{i}} m_j c_{\mathrm{si}}^4}. \tag{7}$$

There is no quantum diffraction term in ion momentum equation (2), therefore, ions are treated classically in this plasma system.

Integrating Eqs. (4) and (5) once under the boundary conditions $n_{\mathrm{e}} = 1, n_{\mathrm{p}} = 1$, and ϕ=0 at $\pm\infty$, we have

$$\frac{H_{\mathrm{e}}^2}{2}\left(\frac{1}{\sqrt{n_{\mathrm{e}}}}\frac{\partial^2\sqrt{n_{\mathrm{e}}}}{\partial x^2}\right) + \frac{1}{2}(1 - n_{\mathrm{e}}^2) + \phi = 0, \tag{8}$$

$$\frac{H_{\mathrm{p}}^2}{2}\left(\frac{1}{\sqrt{n_{\mathrm{p}}}}\frac{\partial^2\sqrt{n_{\mathrm{p}}}}{\partial x^2}\right) + \frac{\sigma}{2}(1 - n_{\mathrm{p}}^2) - \phi = 0. \tag{9}$$

We assume that there are two solitary waves A and B in the plasma. They are, asymptotically, far apart in the initial state and propagate toward each other. After some time they meet, collide, and then depart. We also assume that the solitary waves have small amplitudes $\sim \epsilon$ (where ϵ is a small parameter characterizing the strength of the nonlinearity) and the interaction between the two waves are weak. Hence, we expect that the collision will be quasielastic. In order to analyse the effects of collision, we employ an extended PLK perturbation method.[32,33] This method is a combination of the standard reductive perturbation method[34,35] with the technique of strained coordinates. According to this method, we introduce the stretched coordinates

$$\begin{aligned}\xi &= \epsilon(x - ct) + \epsilon^2 P^{(0)}(\eta, \tau) \\ &\quad + \epsilon^3 P^{(1)}(\xi, \eta, \tau) + \ldots, \end{aligned} \tag{10}$$

$$\begin{aligned}\eta &= \epsilon(x + ct) + \epsilon^2 Q^{(0)}(\xi, \tau) \\ &\quad + \epsilon^3 Q^{(1)}(\xi, \eta, \tau) + \ldots, \end{aligned} \tag{11}$$

$$\tau = \epsilon^3 t, \tag{12}$$

where ξ and η denote the trajectories of the two solitary waves propagating toward to each other. The wave velocities c, the variables $P^{(j)}$ and $Q^{(j)}$ ($j = 0, 1, \ldots$) will be determined later.

The plasma parameters are expanded around the equilibrium values in powers of ϵ in the following forms:

$$n_{\mathrm{i}} = 1 + \epsilon^2 n_{\mathrm{i}1} + \epsilon^3 n_{\mathrm{i}2} + \epsilon^4 n_{\mathrm{i}3} + \ldots, \tag{13}$$

$$n_{\mathrm{e}} = 1 + \epsilon^2 n_{\mathrm{e}1} + \epsilon^3 n_{\mathrm{e}2} + \epsilon^4 n_{\mathrm{e}3} + \ldots, \tag{14}$$

$$n_{\mathrm{p}} = 1 + \epsilon^2 n_{\mathrm{p}1} + \epsilon^3 n_{\mathrm{p}2} + \epsilon^4 n_{\mathrm{p}3} + \ldots, \tag{15}$$

$$u_{\mathrm{i}} = \epsilon^2 u_{\mathrm{i}1} + \epsilon^3 u_{\mathrm{i}2} + \epsilon^4 u_{\mathrm{i}3} + \ldots, \tag{16}$$

$$\phi = \epsilon^2 \phi_1 + \epsilon^3 \phi_2 + \epsilon^4 \phi_3 + \ldots. \tag{17}$$

Then, substituting Eqs. (10)–(17) into Eqs. (2), (3), and (6)–(9), and by equating the powers of ϵ, we obtain a set of coupled equations in different orders of ϵ. To the leading order, we have

$$c\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right) n_{\mathrm{i}1} + \left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right) u_{\mathrm{i}1} = 0, \tag{18}$$

$$c\left(-\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right) u_{\mathrm{i}1} + \left(\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}\right) \phi_1 = 0, \tag{19}$$

$$n_{\mathrm{i}1} + p n_{\mathrm{p}1} - (1 + p) n_{\mathrm{e}1} = 0, \tag{20}$$

$$\sigma n_{\mathrm{p}1} + \phi_1 = 0, \tag{21}$$

$$n_{\mathrm{e}1} - \phi_1 = 0. \tag{22}$$

The solutions of Eqs. (18)$\sim$(22) read

$$\phi_1 = \psi_{\mathrm{A}}(\xi, \tau) + \psi_{\mathrm{B}}(\eta, \tau), \tag{23}$$

$$n_{i1} = \frac{1}{c^2}[\psi_{\mathrm{A}}(\xi, \tau) + \psi_{\mathrm{B}}(\eta, \tau)], \tag{24}$$

$$n_{e1} = \psi_{\mathrm{A}}(\xi, \tau) + \psi_{\mathrm{B}}(\eta, \tau), \tag{25}$$

$$n_{p1} = -\frac{1}{\sigma}[\psi_{\mathrm{A}}(\xi, \tau) + \psi_{\mathrm{B}}(\eta, \tau)], \tag{26}$$

$$u_{i1} = \frac{1}{c}[\psi_{\mathrm{A}}(\xi, \tau) - \psi_{\mathrm{B}}(\eta, \tau)], \tag{27}$$

and with the solvability condition [i.e., the condition to obtain a uniquely defined $n_{\mathrm{i}1}$, $n_{\mathrm{e}1}$, $n_{\mathrm{p}1}$ and $u_{\mathrm{i}1}$ from Eqs. (18)–(22) when ϕ_1 is given by Eq. (23)], the wave velocity c can be obtained to be

$$c^2 = \frac{\sigma}{p + \sigma(1 + p)}. \tag{28}$$

The unknown functions ψ_{A} and ψ_{B} will be obtained at higher order. Equations (23)–(27) indicate that, at the leading order, we have two waves, one of which, $\psi_{\mathrm{A}}(\xi, \tau)$, is traveling to the right, and the other one, $\psi_{\mathrm{B}}(\eta, \tau)$, is traveling to the left. At the next order, we have the similar relations for ϕ_2, $n_{\mathrm{i}2}$, $n_{\mathrm{e}2}$, $n_{\mathrm{p}2}$, and $u_{\mathrm{i}2}$.

At the next higher order, we have

$$\begin{aligned} & 2cu_{i3} \\ &= -\int\left(\frac{\partial\psi_{\mathrm{A}}}{\partial\tau} + \alpha\psi_{\mathrm{A}}\frac{\partial\psi_{\mathrm{A}}}{\partial\xi} + \beta\frac{\partial^3\psi_{\mathrm{A}}}{\partial\xi^3}\right)\mathrm{d}\eta \\ &\quad + \int\left(\frac{\partial\psi_{\mathrm{B}}}{\partial\tau} + \alpha\psi_{\mathrm{B}}\frac{\partial\psi_{\mathrm{B}}}{\partial\eta} + \beta\frac{\partial^3\psi_{\mathrm{B}}}{\partial\eta^3}\right)\mathrm{d}\xi \\ &\quad - \iint\left(2\frac{\partial P^{(0)}}{\partial\eta} - \gamma\psi_{\mathrm{B}}\right)\frac{\partial^2\psi_{\mathrm{A}}}{\partial\xi^2}\,\mathrm{d}\xi\,\mathrm{d}\eta \\ &\quad + \iint\left(2\frac{\partial Q^{(0)}}{\partial\xi} - \gamma\psi_{\mathrm{A}}\right)\frac{\partial^2\psi_{\mathrm{B}}}{\partial\eta^2}\,\mathrm{d}\xi\,\mathrm{d}\eta, \end{aligned} \tag{29}$$

where

$$\alpha = \frac{c^2}{2}\left(\frac{3}{c^4} + 1 + p - \frac{p}{\sigma^2}\right), \tag{30}$$

$$\beta = \frac{c^2}{2}\left(1 - \frac{H_{\rm e}^2}{4}(1+p) - \frac{pH_{\rm p}^2}{4\sigma^2}\right), \quad (31)$$

$$\gamma = \frac{c^2}{2}\left(\frac{1}{c^4} - 1 - p + \frac{p}{\sigma^2}\right). \quad (32)$$

The first and second terms of Eq. (29) will be proportional to $\eta(\xi)$ because the integrated function is independent of $\eta(\xi)$. Thus the first two terms of Eq. (29) are all secular terms, which must be eliminated in order to avoid spurious resonances. Hence we have

$$\frac{\partial \psi_{\rm A}}{\partial \tau} + \alpha\psi_{\rm A}\frac{\partial \psi_{\rm A}}{\partial \xi} + \beta\frac{\partial^3 \psi_{\rm A}}{\partial \xi^3} = 0, \quad (33)$$

$$\frac{\partial \psi_{\rm B}}{\partial \tau} + \alpha\psi_{\rm B}\frac{\partial \psi_{\rm B}}{\partial \eta} + \beta\frac{\partial^3 \psi_{\rm B}}{\partial \eta^3} = 0. \quad (34)$$

Equations (33) and (34) are the two-side traveling wave KdV equations in the reference frames of ξ and η, respectively. Their corresponding stationary solutions have the following form:

$$\psi_{\rm A} = \phi_1 {\rm sech}^2\left[\left(\frac{\alpha\phi_1}{12\beta}\right)^{1/2}\left(\xi - \frac{1}{3}\alpha\phi_1\tau\right)\right], \quad (35)$$

$$\psi_{\rm B} = \phi_2 {\rm sech}^2\left[\left(\frac{\alpha\phi_2}{12\beta}\right)^{1/2}\left(\eta - \frac{1}{3}\alpha\phi_2\tau\right)\right], \quad (36)$$

where ϕ_1 and ϕ_2 are the amplitude of the two solitary waves A and B at their initial position, respectively.

The third and fourth terms of Eq. (29) are not secular terms in this order, but they will become secular in the next order.[36,37] Hence we have

$$2\frac{\partial P^{(0)}}{\partial \eta} = \gamma\psi_{\rm B}, \quad (37)$$

$$2\frac{\partial Q^{(0)}}{\partial \xi} = \gamma\psi_{\rm A}. \quad (38)$$

So the leading phase changes $P^{(0)}$ and $Q^{(0)}$ due to the interaction can be calculated from Eqs. (37) and (38)

$$P^{(0)} = \frac{\gamma}{2}\int_{-\infty}^{\eta}\psi_{\rm B}(\eta',\tau){\rm d}\eta' = \frac{\gamma}{2}\left(\frac{12\beta\phi_2}{\alpha}\right)^{1/2}\left\{\tanh\left[\left(\frac{\alpha\phi_2}{12\beta}\right)^{1/2}\left(\eta - \frac{1}{3}\alpha\phi_2\tau\right)\right] + 1\right\}, \quad (39)$$

$$Q^{(0)} = \frac{\gamma}{2}\int_{+\infty}^{\xi}\psi_{\rm A}(\xi',\tau){\rm d}\xi' = \frac{\gamma}{2}\left(\frac{12\beta\phi_1}{\alpha}\right)^{1/2}\left\{\tanh\left[\left(\frac{\alpha\phi_1}{12\beta}\right)^{1/2}\left(\xi - \frac{1}{3}\alpha\phi_1\tau\right)\right] - 1\right\}. \quad (40)$$

Thus, up to $O(\epsilon^2)$, the trajectories of the two solitary waves for weak head-on interactions are

$$\xi = \epsilon(x - ct) + \epsilon^2\frac{\gamma}{2}\left(\frac{12\beta\phi_2}{\alpha}\right)^{1/2}\left\{\tanh\left[\left(\frac{\alpha\phi_2}{12\beta}\right)^{1/2}\left(\eta - \frac{1}{3}\alpha\phi_2\tau\right)\right] + 1\right\} + O(\epsilon^3), \quad (41)$$

$$\eta = \epsilon(x + ct) + \epsilon^2\frac{\gamma}{2}\left(\frac{12\beta\phi_1}{\alpha}\right)^{1/2}\left\{\tanh\left[\left(\frac{\alpha\phi_1}{12\beta}\right)^{1/2}\left(\xi - \frac{1}{3}\alpha\phi_1\tau\right)\right] - 1\right\} + O(\epsilon^3). \quad (42)$$

To obtain the phase shifts after a head-on interaction of the two solitary waves, we assume that the solitary waves A and B are, asymptotically, far apart from each other at the initial time ($t = -\infty$), i.e., solitary wave A is at $\xi = 0$, $\eta = -\infty$ and solitary wave B is at $\eta = 0$, $\xi = +\infty$. After the collision ($t = +\infty$), solitary wave B is far to the left of solitary wave A, i.e., solitary wave A is at $\xi = 0$, $\eta = +\infty$ and solitary wave B is at $\eta = 0$, $\xi = -\infty$. From Eqs. (41) and (42) we can obtain the corresponding phase shifts $\Delta_{\rm A}$ and $\Delta_{\rm B}$ as follows:

$$\Delta_{\rm A} = \epsilon(x - ct)|_{\xi=0,\eta=+\infty} - \epsilon(x - ct)|_{\xi=0,\eta=-\infty} = -\epsilon^2\gamma\left(\frac{12\beta\phi_2}{\alpha}\right)^{1/2}, \quad (43)$$

$$\Delta_{\rm B} = \epsilon(x + ct)|_{\eta=0,\xi=-\infty} - \epsilon(x + ct)|_{\eta=0,\xi=+\infty} = \epsilon^2\gamma\left(\frac{12\beta\phi_1}{\alpha}\right)^{1/2}. \quad (44)$$

3. Results and discussion

We have presented a study on the interaction between two IASWs with the same structure in a quantum plasma, consisting of electrons, positrons, and ions, by employing the QHD model and extended PLK perturbation method. Let us now discuss the effects of the ratio of positrons to ions unperturbation number density p and the quantum diffraction parameter $H_{\rm e}$ ($H_{\rm p}$) on the interaction of IASWs. In order to

investigate the nonlinear dynamics of IASWs in dense astrophysical environment, we have used parameters $(n_{e0}, n_{p0}, n_{i0}) \sim 10^{27}$–$10^{28}$ cm^{-3}.[27] These parameters are representative of the plasma in dense white dwarfs and satisfy the quasineutrality condition.

3.1. Head-on collision of two solitary waves

Figures 1 and 2 show the profile of head-on collision between two IASWs with the same structure. We note that there are two same-amplitude solitary waves propagating toward each other, and after some time they meet, collide, and then depart. From all these figures, we note that when two solitary waves collide head-on, a new wave is formed in the interaction area. It is shown that the amplitude of the newly formed wave is about two times that of the colliding solitary waves. That is to say, the interaction between the two solitary waves fits linear superposition principle in this quantum system.

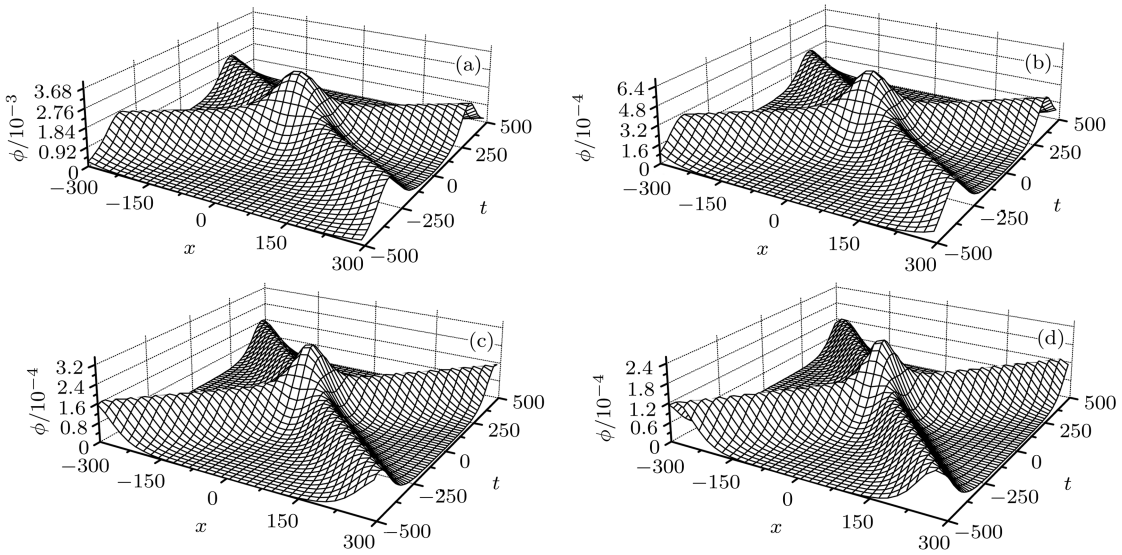


Fig. 1. Profile of interaction between two same structure IASWs for various values of p: (a) $p = 0.01$; (b) $p = 0.1$; (c) $p = 0.5$; (d) $p = 0.8$, here $H_e(H_p) = 0.01$. Recall that σ is related to p through the relation $\sigma = [\,p/(1+p)]^{2/3}$.

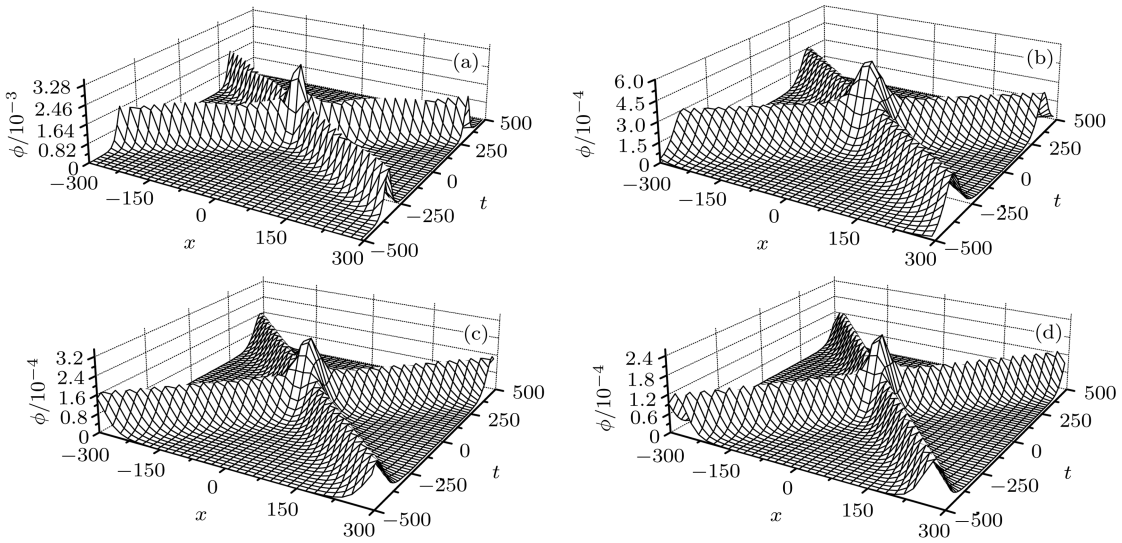


Fig. 2. Profile of interaction between two same-structure IASWs for various values of p: (a) $p = 0.01$; (b) $p = 0.1$; (c) $p = 0.5$; (d) $p = 0.8$, here $H_e(H_p) = 0.8$.

Figure 1 shows the effect of number density ratio p on the interaction of the solitary waves with smaller value of $H_{\rm e}(H_{\rm p}) = 0.01$. It is noted that both the amplitude and width of the newly formed wave decreases with the increase of p. Other careful investigations indicate that the effect of number density ratio on the amplitude and width of the newly formed wave is stronger when p in the range of $(0.01 \to 0.1)$, and this effect becomes weaker when p is in the range of $(0.5 \to 0.8)$. That is to say, the effect of number density ratio p on the structure of the newly formed wave is more remarkable for smaller p. Figure 2 illustrates the effect of number density ratio p on the interaction of the solitary waves with a bigger value of $H_{\rm e}(H_{\rm p}) = 0.8$. It is noted that the results shown in Fig. 2 are quite similar to that in Fig. 1. To examine the effect of quantum diffraction parameter $H_{\rm e}$ $(H_{\rm p})$ on the interaction of the solitary waves, detailed studies reveal that when $p < 0.1$, the wave amplitude and width increase with the increase of Bohm potential when $H_{\rm e}$ $(H_{\rm p})$ is in the range of $(0.01 \to 0.1)$, but the wave amplitude and width decrease with the increase of Bohm potential when $H_{\rm e}$ $(H_{\rm p})$ is in the range of $(0.1 \to 0.8)$. It is found that the effect of Bohm potential on the structure of the newly formed wave is weaker when $p > 0.1$.

Thus, we can conclude that the interaction between the two IASWs fits linear superposition principle in this quantum system, and plasma parameters p and $H_{\rm e}$ $(H_{\rm p})$ have remarkable effects on the newly formed wave.

3.2. The colliding phase shift

Since solitary wave A is traveling to the right and solitary wave B to the left, we see from Eqs. (43) and (44) that due to the collision each solitary wave has a negative phase shift in its traveling direction. In order to know how the plasma parameters influence the phase shifts, we give the results as shown in Fig. 3. In this figure, all physical quantities are dimensionless, and $\epsilon = 0.1$, $u_{0\xi} = 1$ are used. Figure 3 shows the phase shift $\Delta_{\rm B}$ of the solitary wave B against the quantum diffraction parameter $H_{\rm e}$ $(H_{\rm p})$ for various values of positron to ion number density ratio $p = 0.8, 0.5, 0.1$, and 0.01. It is clear that the phase shift decreases with the increase of Bohm potential and number density ratio. We note that the value of p has considerable effect on the colliding phase shift when $p < 0.5$, while the change of p has little effect on the phase shift when $p > 0.5$. That is to say, the effect of number density ratio p on the colliding phase shift is more pronounced for smaller p. We also note that the variation of $H_{\rm e}$ $(H_{\rm p})$ has obvious effect on the colliding phase shift when $p < 0.1$, and the change of $H_{\rm e}$ $(H_{\rm p})$ has little effect on the phase shift when $p > 0.1$. Thus, we can conclude that the effect of quantum diffraction parameter $H_{\rm e}$ $(H_{\rm p})$ on the colliding phase shift is more pronounced for smaller p.

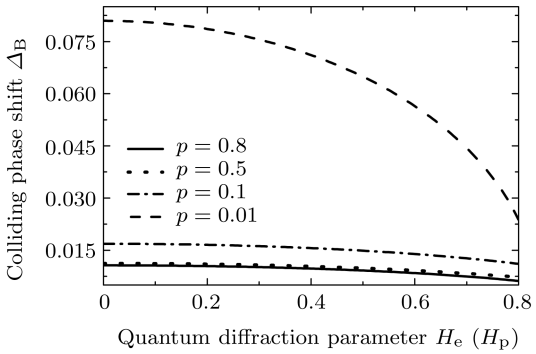


Fig. 3. Colliding phase shift $\Delta_{\rm B}$ versus the quantum diffraction parameter $H_{\rm e}$ $(H_{\rm p})$ for different values of $p(= 0.8, 0.5, 0.1, 0.01)$.

4. Conclusion

We have studied the propagation and interaction of IASWs in an unmagnetized three-component quantum electron–positron–ion plasma. The phase velocity of quantum IASW is assumed to be much smaller than the electron and positron Fermi velocities and larger than the ion Fermi velocity. Therefore we can ignore the inertia of electron and positron. By using the extended PLK perturbation method, we derive two KdV equations for IASWs in this quantum plasma system. The results show that this system admits a solution with two solitary waves; they travel toward each other. Some interesting results of this work may be summarized as below.

(i) The interaction between the two IASWs fits linear superposition principle in this quantum system.

(ii) The plasma parameters p and $H_{\rm e}$ $(H_{\rm p})$ have significantly influence on the newly formed wave and phase shift of the colliding solitary waves.

Finally, we stress that these results may be useful to the investigation of the propagation and interaction of quantum ion–acoustic solitary waves in dense astrophysical plasmas (such as white dwarfs), as well as in intense laser-solid matter interaction experiments.

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