Mode decoupling in solid state ring laser based on stimulated Raman effect in polar crystals

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In this paper we study the gain saturation induced mode-coupling control in solid state ring laser devices based on the stimulated Raman effect of the polar crystals in order to realize solid state ring laser gyroscopes. We theoretically investigate the mode coupling induced by gain saturation between clockwise (CW) and counterclockwise (CCW) propagating laser modes. Because the CW and CCW running waves are pumped with counter-propagating lasers respectively, the independent coexistence can be ensured.

Keywords: ring laser, stimulated Raman effect, polar crystals, gyroscope
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1. Introduction

Ring laser gyroscopes based on helium–neon amplifying medium have been widely used for rotation sensing.[1] However, due to the gaseous nature of the medium, we meet technical complications in the production of the laser gyro. Efforts have been made to develop solid state ring lasers[2−8] but some other problems arise. In a helium–neon ring laser, there exist normally two modes, i.e. clockwise (CW) mode and counterclockwise (CCW) mode. When the laser frequency is detuned from resonance with the atoms at rest, each mode saturates a distinct group of atoms moving in the opposite direction due to the Doppler frequency shift. When the detuning is big enough, the gain-induced nonlinear coupling practically vanishes.[9] However, in the case of solid-state ring laser, the atoms or ions make only small vibrations in the vicinity of their steady positions, so the gain-induced nonlinear coupling is strong. That results in unstable bidirectional lasing or even unidirectional lasing.[10] This is the so-called gain saturation induced mode coupling. Many of the investigations have so far focused on overcoming or controlling such a nonlinearity.[4−8] Employing the direction sensitivity of stimulated Brillouin scattering (SBS), fibre Brillouin laser gyroscope can solve the above problems.[11] However, fibre encounters the problem of polarization fluctuation, which leads to resonance instability.[12] To cope with this problem, the structure of the fibre ring laser gyro becomes complicated.

2. Modeling

In this paper, we propose a new gain mechanism to reduce the gain-induced mode coupling in an integrated optics ring laser gyro. The laser is gained by the stimulated Raman scattering (SRS) effect in polar crystals. Since we employ on-chip waveguides to form the ring cavity, polarization fluctuation does not exist. Though the modeling of a guided-wave gyro based on Raman effect in silicon-on-insulator (SOI) was already reported,[13] the gain saturation induced coupling has not been discussed. Our study shows that the stable coexistence of the CW and CCW modes in a solid state ring laser can be ensured by employing counter-propagating pump lasers in polar crystal waveguides. Furthermore, optical frequency bias can be realized by introducing frequency difference into the pumping lasers. That provides the possibility of detecting of small rotation.

Figure 1 shows a scheme of the ring laser setup we suggest in this paper. The ring resonator is composed of waveguides fabricated on a polar crystal (we suggest GaP in this paper). On the top of the resonator there is another waveguide to couple the pump laser in and

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the Stokes light out. Both ends of the top waveguide are antireflection coated to reduce the coupling between the counter-propagating light waves. The pump laser with intensity $I_{\rm input}$ is coupled into the ring resonator via the directional waveguide coupler. The resonator waveguides are fabricated along the GaP [100] and the [001] crystal directions. According to Saito *et al.*,[14] the Raman gain corresponding to the longitudinal optical (LO) phonon due to the pump is much higher in the backward direction than in the forward direction. Therefore the laser $I_+^{\rm input}$ offers gain for the CCW LO Stokes mode in the resonator, while the laser $I_-^{\rm input}$ offers gain for the CW LO Stokes mode. Instead of saturating two distinct groups of atoms moving in the opposite directions, which happens only in gas laser, the CW and the CCW LO Stokes modes in the ring resonator saturate $I_+^{\rm input}$ and $I_-^{\rm input}$ pump powers respectively. In this way, we realize gain saturation induced mode coupling control in solid state ring laser.

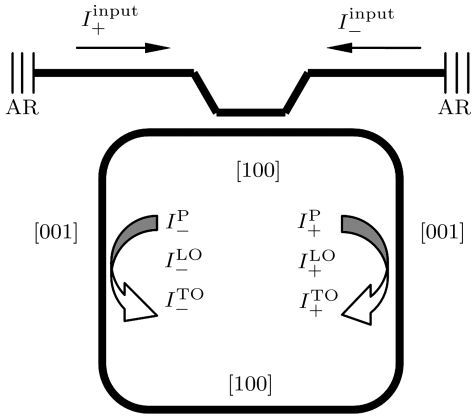


Fig. 1. Schematic diagram of a ring cavity made up from waveguides.

For simplicity, we assume that the cross-sections of these waveguides are large enough that the transversal variations of the field intensity can be neglected and they can be taken as bulk material. We suppose that the material is isotropic, with refractive index n. We express the polarization P as the sum of linear part $P^{\rm L}$ and nonlinear part $P^{\rm NL}$, and obtain the displacement

$$D = \varepsilon_0 E + P = \varepsilon_0 E + P^{\rm L} + P^{\rm NL} = n^2 \varepsilon_0 E + P^{\rm NL}. \quad (1)$$

Here E is the scalar electric field, and ε_0 is the electric permittivity in vacuum. Substituting Eq. (1) into Maxwell equations and assuming the presence of a lossy medium with conductivity σ, we have the wave equation in the waveguides

$$-c^2 \frac{\partial^2 E}{\partial s^2} + \frac{\sigma}{\varepsilon_0}\frac{\partial E}{\partial t} + n^2 \frac{\partial^2 E}{\partial t^2} = -\frac{1}{\varepsilon_0}\frac{\partial^2 P^{\rm NL}}{\partial t^2}, \quad (2)$$

which is the so-called Helmholtz equation with nonlinear polarization, where c is the velocity of light in vacuum. In a ring cavity with length L, there are normal modes, $\exp[{\rm i}\,(\beta_m s - \nu_m^c t)]$, whose frequency is ν_m^c and propagating constant is $\beta_m = 2m\pi/L$, where the mode number m is a large integer. The time dependence of the wave equation can be separated from the spatial dependence by expansion of the electric field and polarization in these normal modes, that is,

$$E = \sum_m E_m \exp({\rm i}\varphi_m) \exp[{\rm i}\,(\beta_m s - \nu_m^c t)],$$

and

$$P^{\rm NL} = \sum_m P_m^{\rm NL} \exp[{\rm i}\,(\beta_m s - \nu_m^c t)].$$

Here all these coefficients are assumed to be slowly varying. The coefficient of E is separated into amplitude term E_m and phase term $\exp({\rm i}\varphi_m)$, but that of $P^{\rm NL}$ remains complex, whose real and imaginary parts are ${\rm Re}\big(P_m^{\rm NL}\big)$ and ${\rm Im}\big(P_m^{\rm NL}\big)$ respectively. Neglecting high order differential terms and equating the real part and the imaginary part on the left-hand side of Eq. (2) to those on the right-hand side respectively, we have the self-consistency equations for a mode in the ring cavity[9]

$$\dot{E}_m + \frac{1}{2}\frac{\nu_m}{Q}E_m = -\frac{1}{2}\frac{\nu_m}{n^2\varepsilon_0}{\rm Im}\left(P_m^{\rm NL}\right), \quad (3a)$$

$$\nu_m = \Omega_m \left[n^2 + \frac{{\rm Re}\left(P_m^{\rm NL}\right)}{\varepsilon_0 E_m}\right]^{-1/2} \approx \frac{\Omega_m}{n}\left[1 - \frac{1}{2n^2\varepsilon_0 E_m}{\rm Re}\left(P_m^{\rm NL}\right)\right], \quad (3b)$$

where $\nu_m = \nu_m^c - \dot{\varphi}_m$, $\Omega_m = c\beta_m$ and Q is the quality factor of the cavity.

Next, we focus on the SRS effect, which is generated by the optical phonon. In crystals, there are two kinds of phonons, the LO phonon and the transverse optical (TO) phonon. Especially in polar crystals, e.g. GaP, these two kinds of phonons have different frequencies and different spontaneous Raman scattering efficiencies. As the stimulated Raman efficiencies are proportional to the spontaneous Raman scattering efficiencies,[15] the Raman efficiencies for LO- and TO-scatterings differ from each other. Generally, the Raman efficiency for LO-scattering is always enhanced with respect to the TO-scattering, e.g. the enhancement factor $\chi_0^{\rm LO}/\chi_0^{\rm TO}$ of GaP is 3.5.[15] Here

$\chi_0^{\rm LO}$ and $\chi_0^{\rm TO}$ are the peak values of the susceptibilities corresponding to LO and TO phonons respectively. According to the Raman selection rule,[16] the LO phonons make contributions only to the scattering in backward directions, while the TO phonons only for the scattering in forward directions for our design (see Fig. 1).

For Stokes light, the third-order Raman susceptibility depends on frequency[17]

$$\begin{aligned}&\chi^{\rm LO(TO)}(\nu^{\rm LO(TO)})\\ \approx\ &\frac{\rm i}{2}\gamma\chi_0^{\rm LO(TO)}\frac{1}{\nu_{\rm RAM}-(\nu^{\rm P}-\nu^{\rm LO(TO)})+{\rm i}\gamma/2}\\ &\times(\nu^{\rm P}-\nu^{\rm LO(TO)}\approx\nu_{\rm RAM}),\end{aligned}\tag{4}$$

where $\nu_{\rm RAM}$ is the Raman shift frequency, and γ is the Raman line width. $\chi^{\rm LO(TO)}$ reaches $\chi_0^{\rm LO(TO)}$ at resonance, that is, when the frequencies of pump ("P") and Stokes light satisfy $\nu^{\rm P}-\nu^{\rm LO(TO)}=\nu_{\rm RAM}$. As Stokes light gains, ${\rm i}\chi_0^{\rm LO(TO)}$ is positive.

In a bidirectionally pumped GaP ring Raman laser shown in Fig. 1, there are 6 running wave modes in the cavity: two pumps, two LO Stokes modes, and two TO Stokes modes. As discussed before, CW pump offers gains to both CCW LO and CW TO Stokes modes. Based on the conjugation of the third-order Raman susceptibilities for Stokes and pump light, we can write the complex polarizations of these modes as[17]

$$\begin{aligned}P_{\rm P+}^{\rm NL}&=\varepsilon_0\chi^{\rm TO}|E_+^{\rm TO}|^2E_+^{\rm P}+\varepsilon_0\chi^{\rm LO}|E_-^{\rm LO}|^2E_+^{\rm P},\\ P_{\rm LO-}^{\rm NL}&=\varepsilon_0\chi^{\rm LO}|E_+^{\rm P}|^2E_-^{\rm LO},\\ P_{\rm TO+}^{\rm NL}&=\varepsilon_0\chi^{\rm TO}|E_+^{\rm P}|^2E_+^{\rm TO},\end{aligned}\tag{5}$$

and similar equations in which the plus (or minus) is replaced by the minus (or plus) for the other three modes, CCW pump, CW LO, and CCW TO Stokes modes. Setting intensity $I=|E|^2$ and combining the Raman contributions to the complex polarization with the amplitude self-consistency equation (3a), we have

$$\begin{aligned}&\dot I_\pm^{\rm LO}\\ &=I_\pm^{\rm LO}\nu_\pm^{\rm LO}\left[-1/Q^{\rm LO}+{\rm Im}\left(-\chi^{\rm LO}\right)I_\mp^{\rm P}/n^2\right],\end{aligned}\tag{6a}$$

$$\begin{aligned}&\dot I_\pm^{\rm TO}\\ &=I_\pm^{\rm TO}\nu_\pm^{\rm TO}\left[-1/Q^{\rm TO}+{\rm Im}\left(-\chi^{\rm TO}\right)I_\pm^{\rm P}/n^2\right],\end{aligned}\tag{6b}$$

$$\begin{aligned}&\dot I_\pm^{\rm P}\\ &=\eta^{\rm P}I_\pm^{\rm input}-\eta^{\rm P}I_\pm^{\rm P}+I_\pm^{\rm P}\nu_\pm^{\rm P}[-1/Q^{\rm P}\\ &\quad-{\rm Im}\left(-\chi^{\rm LO}\right)I_\mp^{\rm LO}/n^2-{\rm Im}\left(-\chi^{\rm TO}\right)I_\pm^{\rm TO}/n^2].\end{aligned}\tag{6c}$$

Here $\eta^{\rm P}$ is a constant proportional to the coupling efficiency. These two terms containing $\eta^{\rm P}$ are the input gain and the loss introduced by the coupler. The coupling loss of Stokes light is neglected as we can design the coupling efficiency small at the frequency of Stokes light.

There are two independent sets of equations. $I_-^{\rm P}$, $I_-^{\rm TO}$, and $I_+^{\rm LO}$ belong to one set, determined by $I_-^{\rm input}$ but independent of $I_+^{\rm input}$, while $I_+^{\rm P}$, $I_+^{\rm TO}$, and $I_-^{\rm LO}$ belong to the other, determined by $I_+^{\rm input}$ but independent of $I_-^{\rm input}$. These two sets of equations are decoupled.

Examinations of Eqs. (6a) and (6b) show that the thresholds of LO and TO Stokes modes are $n^2/\left[{\rm Im}\left(-\chi^{\rm LO}\right)Q^{\rm LO}\right]$ and $n^2/\left[{\rm Im}\left(-\chi^{\rm TO}\right)Q^{\rm TO}\right]$, respectively. As the Raman efficiency for LO-scattering is always enhanced with respect to the TO-scattering,[15] it is easy to tune the pump threshold of LO Stokes mode lower than that of TO one. In this case, once the input laser is strong enough, i.e.,

$$I_\mp^{\rm input}>\frac{n^2}{Q^{\rm LO}{\rm Im}\left(-\chi^{\rm LO}\right)}\left(1+\frac{\nu_\mp^{\rm P}}{\eta^{\rm P}Q^{\rm P}}\right),\tag{7}$$

LO Stokes starts lasing but $I^{\rm TO}$ keeps zero because of its higher pump threshold as shown in Fig. 2.

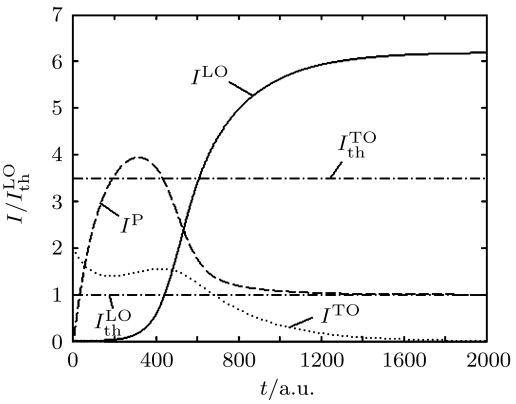


Fig. 2. Mode competition between the LO and TO scatterings. As LO-light suppresses the pump intensity to its threshold, TO-light vanishes because its pump threshold is higher than that of LO-light.

So the steady-state solutions are

$$\begin{aligned}&I_\pm^{\rm LO}\\ &=\frac{\eta^{\rm P}Q^{\rm LO}I_\mp^{\rm input}}{\nu_\mp^{\rm P}}-\frac{n^2}{Q^{\rm P}{\rm Im}\left(-\chi^{\rm LO}\right)}-\frac{n^2\eta^{\rm P}}{\nu_\mp^{\rm P}{\rm Im}\left(-\chi^{\rm LO}\right)};\\ &I_\mp^{\rm TO}=0;\quad I_\mp^{\rm P}=\frac{n^2}{Q^{\rm LO}{\rm Im}\left(-\chi^{\rm LO}\right)}.\end{aligned}\tag{8}$$

Here we see that $I^{\rm P}$ is pinned at the level of the threshold of LO scattering because of the lasing of LO Stokes light. So TO Stokes lasers would not be generated.

From the steady-state solution (8) we can see that the CW and the CCW Stokes modes are generated by different inputs. If the ring cavity is bidirectionally pumped, the two counter-propagating LO Stokes lasers can exist independently. Mode decoupling in solid state ring laser is realized.

Substituting the steady-state solutions into frequency equations (3b), we have

$$\begin{aligned}\nu_{\pm}^{\mathrm{LO}} &= \frac{\Omega_{\pm}}{n} - \frac{\Omega_{\pm}}{2nQ^{\mathrm{LO}}}\frac{\mathrm{Re}(\chi^{\mathrm{LO}})}{\mathrm{Im}(-\chi^{\mathrm{LO}})} \\ &= \frac{\Omega_{\pm}}{n} - \frac{\Omega_{\pm}}{nQ^{\mathrm{LO}}}\frac{\nu_{\mathrm{RAM}}^{\mathrm{LO}} - \nu_{\mp}^{\mathrm{P}} + \nu_{\pm}^{\mathrm{LO}}}{\gamma^{\mathrm{LO}}}. \end{aligned} \tag{9}$$

Here the possibility of a rotating frame is allowed for by taking $\Omega_+ \neq \Omega_-$. The cavity resonant frequencies of both CW and CCW modes, $\Omega_{\pm}$, are set as Ω_0 at rest. As a result of Sagnac effect, when the ring cavity is rotated, $\Omega_{\pm}$ shift to $\Omega_0 \pm \Delta\Omega$, where $\Delta\Omega$ is proportional to the rate of rotation. As $\Delta\Omega$ is much smaller than Ω_0, we make the approximations in Eq. (9) as follows:

$$\nu_{\pm}^{\mathrm{LO}} = \frac{\Omega_{\pm}}{n} - \frac{\Omega_0}{n\gamma^{\mathrm{LO}}Q^{\mathrm{LO}}}\left(\nu_{\mathrm{RAM}}^{\mathrm{LO}} - \nu_{\mp}^{\mathrm{P}} + \nu_{\pm}^{\mathrm{LO}}\right) \pm b_{\pm}\sin\Phi. \tag{10}$$

The last terms $b_{\pm}\sin\Phi$ are the locking terms caused by the backward scattering between the opposite propagating Stokes light beams, which may be omitted in the above equation. In these terms, Φ is the phase difference between the two Stokes light beams, and $b_{\pm}$ are the backscattering coefficients. Define $\kappa = \Omega_0/\left(n\gamma^{\mathrm{LO}}Q^{\mathrm{LO}}\right)$, we have

$$\begin{aligned}\nu_{\pm}^{\mathrm{LO}} &= \frac{1}{1+\kappa}\left(\frac{\Omega_{\pm}}{n} \pm b_{\pm}\sin\Phi\right) \\ &\quad + \frac{\kappa}{1+\kappa}\left(\nu_{\mp}^{\mathrm{P}} - \nu_{\mathrm{RAM}}^{\mathrm{LO}}\right). \end{aligned} \tag{11}$$

So the beat frequency of the conterpropagating LO Stokes lasers are

$$\begin{aligned}&\dot{\Phi} \\ &= \nu_{+}^{\mathrm{LO}} - \nu_{-}^{\mathrm{LO}} \\ &= \frac{1}{1+\kappa}\left[\frac{\Omega_+ - \Omega_-}{n} + \kappa\left(\nu_{-}^{\mathrm{P}} - \nu_{+}^{\mathrm{P}}\right)\right. \\ &\quad \left. + \ (b_+ + b_-)\sin\Phi\right]. \end{aligned} \tag{12}$$

This is quite similar to the normal beat frequency equation except for the additional term caused by the frequency difference between their pump lasers. We call it the frequency bias. As shown in Fig. 3, the bias term shifts the lock-in region to the vicinity of $\kappa\left(\nu_{-}^{\mathrm{P}} - \nu_{+}^{\mathrm{P}}\right)$, thus small difference between the eignfrequencies of these counter running Stokes light beams can also produce a change of the beat frequency. And this bias is realized via differentiating the two pump frequencies. This can be stable and is tunable thanks to today's mature frequency generating technology.

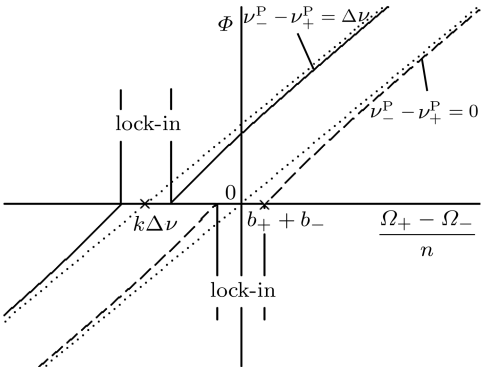


Fig. 3. Solutions of Eq. (12) in the cases of $\nu_{-}^{\mathrm{P}} - \nu_{+}^{\mathrm{P}} = 0$ (dash line) and $\nu_{-}^{\mathrm{P}} - \nu_{+}^{\mathrm{P}} = \Delta\nu$ (solid line).

It should be noticed that for simplicity some phenomena, such as the Kerr effect, are ignored in the above discussion. And the Kerr effect is the major error source of the rotation rate measurement in a gyroscope designed as above.[18] Such an error originates from the asymmetry of powers of the two input pumps. However, the error caused by such an effect can be perfectly removed in fibre SBS laser gyro by means of negative feedback or some other schemes.[19] Because of the simplicities of SRS and SBS, these solutions can be employed in our ring laser gyro, and they affect the conclusion little.

3. Conclusions

We have introduced a solid state on-chip ring laser based on the stimulated Raman effect in a polar crystal, and show its potential in rotation sensing. Mode decoupling in the ring laser is realized. Counterpropagating LO Stokes laser can exist independently in the ring cavity. By introducing a difference between the two pump frequencies, stable and tunable optical bias can be realized to overcome the ring laser lock-in effect.

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