Signal-to-noise ratio of lensless ghost interference with thermal incoherent light*

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Factors influencing the signal-to-noise ratio (SNR) of lensless ghost interference with thermal incoherent light are investigated. Our result shows that the SNR of lensless ghost interference is related to the transverse length of the object, the position of the object in the imaging system and the transverse size of the light source. Furthermore, the effects of these factors on the SNR are discussed in detail by numerical simulations.

Keywords: ghost interference, ghost imaging, signal-to-noise ratio

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1. Introduction

In recent years, much attention has been paid to ghost interference and imaging.[1−17] Usual ghost interference and imaging are the technique which allows one to perform coherent imaging with incoherent light by measuring the intensity correlation.

In 1995, ghost interference and imaging were first observed by using entangled photon pairs from spontaneous parametric down-conversion (SPDC).[1,2] Abouraddy *et al.*[3] discussed the role of entanglement in ghost imaging, and found that entanglement is a prerequisite for achieving distributed ghost imaging. However, Bennink *et al.*[4,5] showed that ghost imaging technique does not require the entanglement, and provided an experimental demonstration with a classical source. Gatti *et al.*[6] proposed theoretically that ghost imaging can be realized by using a thermal light source with classical correlation. At the same time, Cheng and Han[7] studied the incoherent coincidence imaging and its application in x-ray diffraction. Cao *et al.*[8] also discussed the geometrical optics of correlated imaging, and first pointed out that the classical thermal source behaves as a phase conjugate mirror, and hence can form a real correlated image without using a lens. Moreover, Wang and Cao,[9] and Xiong *et al.*[10] demonstrated that a thermal light source can produce subwavelength double-slit interference in joint intensity measurement. Cai and Zhu,[11] and Cai and Wang[12] investigated the ghost interference and imaging with partially coherent light. Furthermore, experimental evidences of ghost interference and imaging realized by using a true thermal light have been reported.[13−15]

As one of the most important factors to characterize an imaging system, the visibility of ghost interference has been investigated in some papers.[18−20] However, as pointed out in Refs. [21] and [22], a high visibility does not imply a good image, and what is more important than the visibility is the signal-to-noise ratio (SNR) of the image. Cheng *et al.*[23] discussed the classical noise in ghost imaging with a classical thermal light. Erkmen and Shapiro[24] derived the SNRs of three Gaussian-state ghost-imaging configurations. However, the factors influencing the SNR of lensless ghost interference have not been studied. In the present paper, our main interest focuses on the factors influencing the SNR of lensless ghost interference with thermal incoherent light. It demonstrates that the transverse length of the object, the position of the object in the imaging system and the transverse size of the light source have an influence on the SNR of ghost interference, and the effects of these factors on the SNR are investigated.

2. Model and analytical results

To investigate the SNR of lensless ghost interference with thermal incoherent light, we consider the setup as depicted in Fig. 1. The classical thermal incoherent light source, usually obtained by illuminat-

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ing a laser beam into a slowly rotating ground glass, is divided into two spatially correlated daughter light beams by beam splitter (BS). These two daughter light beams are sent through two different imaging systems, which are called the test arm and the reference arm. In the test arm, there is an object (with transmission function $t(x)$) between BS and detector D_1, and detector D_1 is a point detector in the middle of the transverse position. Here, we take a double slit with the slit width w and the distance between two slits d as an object to be imaged. In the reference arm, however, the beam propagates freely, and detector D_2 is usually a scanning point detector. The intensity correlation, which is composed of the intensity product at D_1 and D_2, is measured by an intensity correlation measurement system. To obtain the Fourier-transform image of the double slit, we set $z = z_1 + z_2$.

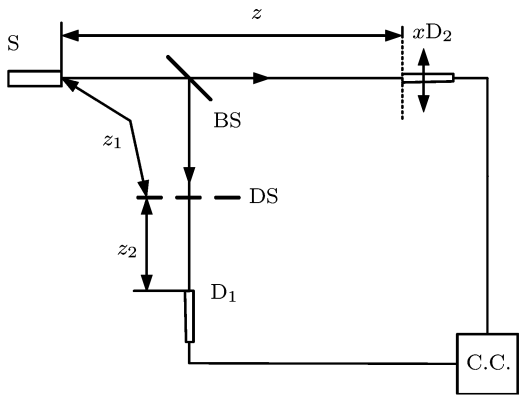


Fig. 1. Schematic of lensless ghost interference. S, thermal source; BS, beam splitter; DS, double slit; C.C., correlation calculation.

In the following, we will consider the detailed theoretical analysis of the second-order lensless ghost interference. For simplicity, only the one-dimensional case is considered. According to the scheme depicted in Fig. 1, the second-order lensless ghost interference function can be expressed as (see the Appendix)

$$\begin{aligned} g(x) &= \frac{\langle I_{\rm t} I_{\rm r}(x)\rangle}{\langle I_{\rm t}\rangle\langle I_{\rm r}(x)\rangle} \\ &= 1 + \frac{\langle \Delta I_{\rm t} \Delta I_{\rm r}(x)\rangle}{\langle I_{\rm t}\rangle\langle I_{\rm r}(x)\rangle} \\ &= 1 + \frac{l}{D}\left(1 + \frac{z_1}{z_2}\right) \\ &\quad \times \left(\sin c\left[\frac{\pi w x}{\lambda z_2}\right]\cos\left[\frac{\pi d x}{\lambda z_2}\right]\right)^2, \end{aligned} \tag{1}$$

where $\langle\cdots\rangle$ is the ensemble average, and I and ΔI are the intensity distribution and the intensity fluctuation distribution on the detection plane, respectively. In addition, $l = 2w$ is the transverse length of the double slit, D is the transverse size of the light source, and λ is the wavelength of the light source.

The first term on the right-hand side of Eq. (1) is the background term which has no interference pattern information. The second term gives the interference pattern information, so it can be considered as the signal term

$$S[\,g(x)] = \frac{\langle \Delta I_{\rm t} \Delta I_{\rm r}(x)\rangle}{\langle I_{\rm t}\rangle\langle I_{\rm r}(x)\rangle}. \tag{2}$$

Then the signal-to-background ratio (SBR) of ghost interference is

$$\begin{aligned} {\rm SBR}[\,g(x)] &= \frac{\langle \Delta I_{\rm t} \Delta I_{\rm r}(x)\rangle}{\langle I_{\rm t}\rangle\langle I_{\rm r}(x)\rangle} \\ &= \frac{l}{D}\left(1 + \frac{z_1}{z_2}\right) \\ &\quad \times \left(\sin c\left[\frac{\pi w x}{\lambda z_2}\right]\cos\left[\frac{\pi d x}{\lambda z_2}\right]\right)^2. \end{aligned} \tag{3}$$

From Eq. (3), it is found that the SBR increases as the transverse length l and the distance ratio z_1/z_2 increase, and it decreases as the transverse size of the light source D increases.

In theory the information about the ghost interference can be obtained from Eq. (1). However, in reality we could not obtain ensemble average $\langle\cdots\rangle$ in Eq. (1): we generally obtain the lensless ghost interference by calculating the normalized correlation function[13−15]

$$g'(x) = \frac{M[I_{\rm t} I_{\rm r}(x)]}{M[I_{\rm t}] M[I_{\rm r}(x)]}\,, \tag{4}$$

where

$$M[X] \equiv \frac{1}{N}\sum_{s=1}^{N} X^{(s)} \tag{5}$$

is defined as the sample average of the quantity X for the N measurements $\{X^{(s)}\}$, $s = 1, \ldots, N$.

We now calculate the noise of ghost interference. We define the noise of ghost interference as

$$\sigma[\,g(x)] = \sqrt{\Delta^2 g(x)} = \sqrt{\langle [g'(x) - g(x)]^2\rangle}\,. \tag{6}$$

In Eq. (4), the quantities $M[I_{\rm t} I_{\rm r}(x)]$, $M[I_{\rm t}]$ and $M[I_{\rm r}(x)]$ are random variables when the number of measurements N is finite, so $g'(x)$ is a function of random variables. To obtain $\Delta^2 g(x)$ in Eq. (6), we can make use of the method of error transfer function in Ref. [25]. By a not very difficult calculation, $\Delta^2 g(x)$ is found to be

$$\Delta^2 g(x) = \frac{1 + 4\mathrm{SBR}[\,g(x)] - 3\mathrm{SBR}^2[\,g(x)] + 2\mathrm{SBR}^3[\,g(x)]}{N}. \tag{7}$$

The signal-to-noise ratio (SNR) is defined as

$$\mathrm{SNR}[\,g(x)] = \frac{\mathrm{S}[\,g(x)]}{\sigma[\,g(x)]}. \tag{8}$$

Using the results of Eqs. (2) and (7), the SNR of ghost interference is calculated as

$$\mathrm{SNR}\,[\,g(x)] = \sqrt{\frac{N\mathrm{SBR}^2[\,g(x)]}{1 + 4\mathrm{SBR}[\,g(x)] - 3\mathrm{SBR}^2[\,g(x)] + 2\mathrm{SBR}^3[\,g(x)]}}. \tag{9}$$

According to Schwarz inequality[17]

$$\mathrm{SBR}[\,g(x)] = \frac{\langle \Delta I_\mathrm{t} \Delta I_\mathrm{r}(x)\rangle}{\langle I_\mathrm{t}\rangle\langle I_\mathrm{r}(x)\rangle} \leq 1. \tag{10}$$

Figure 2 presents the relationship between the SNR and the SBR when $N = 4000$ and $\mathrm{SBR}[\,g(x)] \in [0, 1]$.

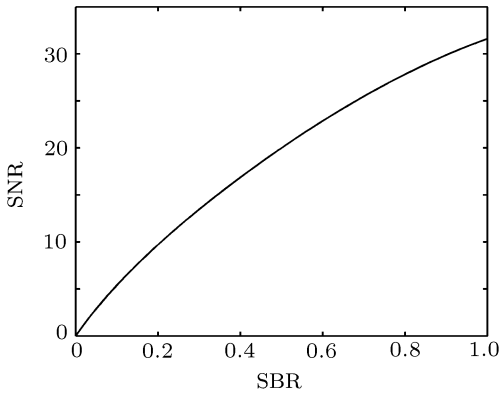


Fig. 2. Relationship between SNR and SBR.

It is shown that the increasing of the SBR results in the growth of the SNR. Then, from Eq. (3) and Fig. 2, it is obtained that the increase of the transverse length of the object l or the distance ratio z_1/z_2 leads to the increase of the SNR, whereas the increase of the transverse size of the light source D results in the decrease of the SNR.

3. Numerical stimulations

Here, we give numerical simulations to demonstrate the obtained results. Lensless ghost interference with the fixed values of $\lambda = 532$ nm, $z = 90$ mm, $d = 200$ μm and $N = 4000$, is used throughout the simulation. The transverse length of the double slit $l = 2w$, the distance ratio z_1/z_2 and the transverse size of the light source D are chosen to be variable parameters in our simulation.

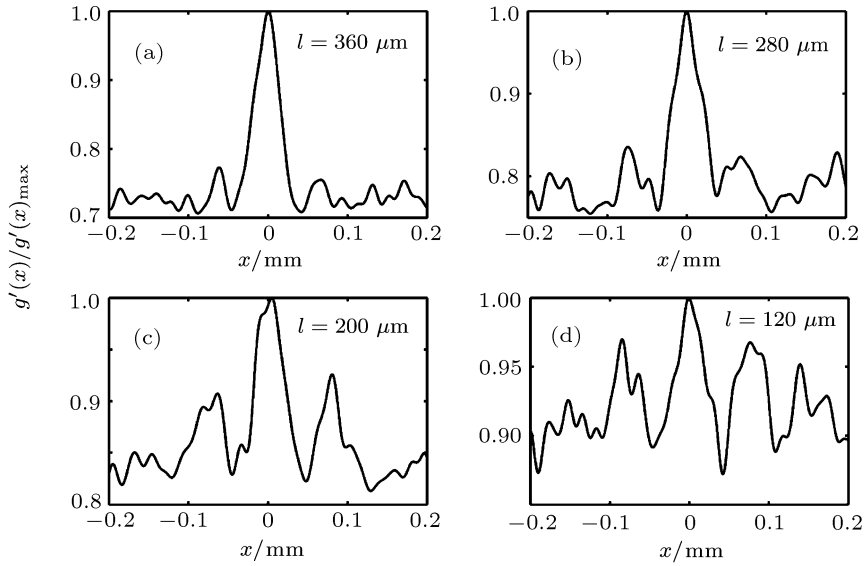


Fig. 3. Ghost interference fringes for different transverse sizes of double slit l.

Depicted in Fig. 3 are the results for $D = 3$ mm, $z_1/z_2 = 2$ and different transverse lengths of the double slit l. All the curves are normalized with a maximum value of $g'(x)$. It is noticed that lower quality image is obtained when the transmission area of the double slit is decreased. Also, in Fig. 3 it is found that the Fourier-transform image of the double slit is dependent on ratio l/d, which accords with the description of Eq. (3).

Figure 4 shows the results for $D = 2$ mm, $l = 2w = 200\,\mu$m and different values of distance ratio z_1/z_2. It is clearly indicated that when the double slit is shifted close to the thermal source, the Fourier-transform image of the double slit becomes more blurred for a given sample number. Subsequently, the corresponding diffraction pattern becomes broader, which coincides with the description of Eq. (3).

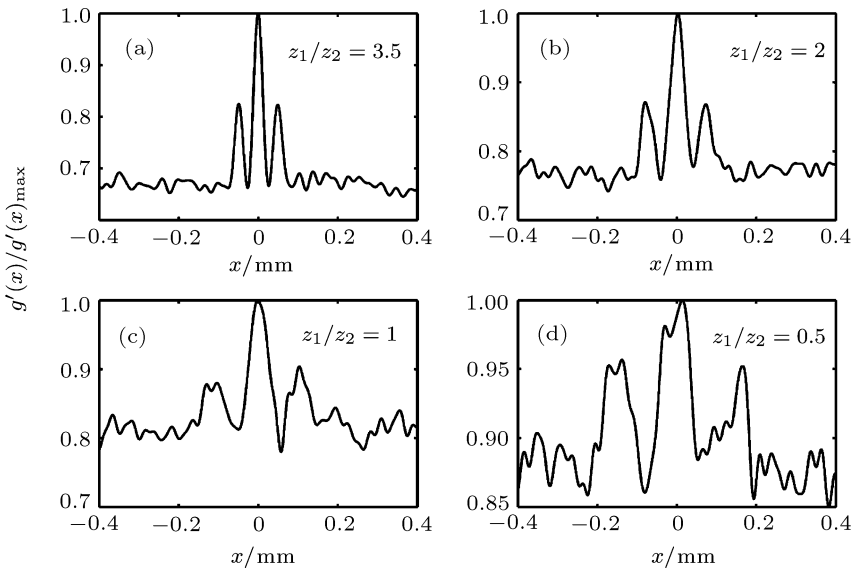


Fig. 4. Ghost interference fringes for different values of distance ratio z_1/z_2.

Figure 5 demonstrates the ghost interference with $l = 2w = 200$ μm, $z_1/z_2 = 2$, and different transverse sizes of the light source D. It is clearly seen that the image of ghost interference turns worse with source size D increasing.

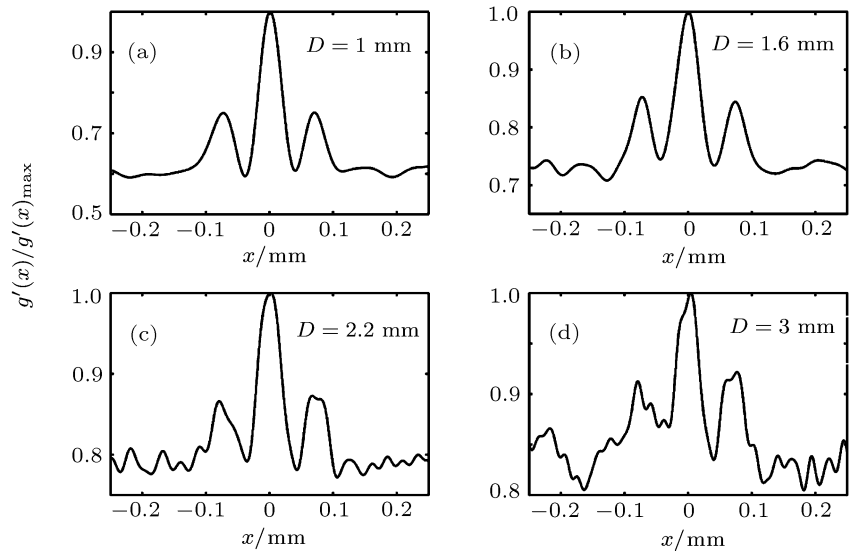


Fig. 5. Ghost interference fringes for different transverse sizes of thermal source D.

4. Conclusion

We have investigated the SNR of lensless ghost interference with thermal incoherent light, which has not been discussed before. It is shown that the quality of the interference fringes is strongly dependent on the transverse length of the double slit, the position of the double slit in the imaging system and the transverse size of the thermal source. The SNR of ghost interference can be improved when the transverse length of the double slit increases, while the Fourier-transform image of the object becomes worse when the object is shifted close to the thermal source. Moreover, the using of small transverse size of the thermal source can increase the SNR.

Appendix: The proof of Eq. (1)

The intensity correlation function in the detection plane can be expressed as follows:

$$\begin{aligned}\langle I_{\rm t}I_{\rm r}(x)\rangle =&\langle I_{\rm t}\rangle\langle I_{\rm r}(x)\rangle + \langle \Delta I_{\rm t}\Delta I_{\rm r}(x)\rangle \\ =&\int {\rm d}x_1{\rm d}x_2{\rm d}x_2'{\rm d}x_1'\langle E^*(x_1)E^*(x_2)E(x_2')E(x_1')\rangle h_1^*(0\,,\,x_1)h_2^*(x\,,\,x_2)h_2(x\,,\,x_2')h_1(0\,,\,x_1'),\end{aligned} \tag{A1}$$

where $E(x_i)$ $(i=1,2)$ is the field of the thermal source, and $h_i(x,x')$ $(i=1,2)$ is the impulse response function for path i. In particular, the impulse response functions for paths 1 and 2 are, respectively,

$$h_1(x,x') = \int {\rm d}u \frac{{\rm e}^{{\rm i}\,kz_1}}{\sqrt{{\rm i}\,\lambda z_1}} \exp\left[\frac{{\rm i}\,\pi}{\lambda z_1}(u-x')^2\right] t(u) \frac{{\rm e}^{{\rm i}\,kz_2}}{\sqrt{{\rm i}\,\lambda z_2}} \exp\left[\frac{{\rm i}\,\pi}{\lambda z_2}(x-u)^2\right], \tag{A2a}$$

$$\begin{aligned}h_2(x,x') =&\frac{{\rm e}^{{\rm i}\,kz}}{\sqrt{{\rm i}\,\lambda z}} \exp\left[\frac{{\rm i}\,\pi}{\lambda z}(x-x')^2\right] \\ =&\int {\rm d}u \frac{{\rm e}^{{\rm i}\,kz_1}}{\sqrt{{\rm i}\,\lambda z_1}} \exp\left[\frac{{\rm i}\,\pi}{\lambda z_1}(u-x')^2\right] \frac{{\rm e}^{{\rm i}\,kz_2}}{\sqrt{{\rm i}\,\lambda z_2}} \exp\left[\frac{{\rm i}\,\pi}{\lambda z_2}(x-u)^2\right].\end{aligned} \tag{A2b}$$

For any random variable, the second-order field correlation of thermal light in Eq. (A1) can be written in Gaussian statistics as

$$\langle E^*(x_1)E^*(x_2)E(x_2')E(x_1')\rangle = \langle E^*(x_1)E(x_1')\rangle\langle E^*(x_2)E(x_2')\rangle + \langle E^*(x_1)E(x_2')\rangle\langle E^*(x_2)E(x_1')\rangle. \tag{A3}$$

Here we consider the incoherent light, so $\langle E^*(x)E(x')\rangle = I\delta(x-x')$, where I is a normalized constant. Therefore in Eq. (A1),

$$\begin{aligned}\langle I_{\rm t}\rangle =&\int_{-D/2}^{D/2}\int_{-D/2}^{D/2} {\rm d}x_1{\rm d}x_1'\langle E^*(x_1)E(x_1')\rangle h_1^*(0,x_1)h_1(0,x_1') \\ =&\iint {\rm d}u{\rm d}u' t^*(u)t(u')\frac{\sin[\pi D(u-u')/(\lambda z_1)]}{\pi(u-u')}\exp\left[\frac{{\rm i}\,\pi}{\lambda z_1}(u'^2-u^2)\right] \\ &\times\frac{1}{\lambda z_2}\exp\left[\frac{{\rm i}\,\pi}{\lambda z_2}(u'^2-u^2)\right].\end{aligned} \tag{A4}$$

In Eq. (A4), $D\sim 1$ mm, $\lambda\sim 0.5\times 10^{-3}$ mm and $z_1\sim 100$ mm, so $\pi D/(\lambda z_1)\sim 50\gg 1$, then from the expression

$$\lim_{K\to\infty}\frac{\sin(Kx)}{\pi x} = \delta(x), \tag{A5}$$

we can obtain

$$\frac{\sin[\pi D(u-u')/(\lambda z_1)]}{\pi(u-u')} \approx \delta(u-u'). \tag{A6}$$

So

$$\langle I_{\rm t}\rangle = \iint {\rm d}u{\rm d}u' t(u)t(u')\delta(u-u')\exp\left[\frac{{\rm i}\,\pi}{\lambda z_1}(u'^2-u^2)\right]\frac{1}{\lambda z_2}\exp\left[\frac{{\rm i}\,\pi}{\lambda z_2}(u'^2-u^2)\right] = \frac{Il}{\lambda z_2}. \tag{A7}$$

Using the method dealing with the expression $\langle I_{\rm t}\rangle$ in Eq. (A4), we can calculate out

$$\langle I_{\rm r}(x)\rangle = \int_{-D/2}^{D/2}\int_{-D/2}^{D/2} {\rm d}x_1{\rm d}x_1'\langle E^*(x_1)E(x_1')\rangle h_2^*(x,x_1)h_2(x,x_1') = \frac{ID}{\lambda z}, \tag{A8}$$

$$\begin{aligned}\langle \Delta I_{\rm t}\Delta I_{\rm r}(x)\rangle &= \left|\langle E_{\rm t}^* E_{\rm r}(x)\rangle\right|^2 = \left|\int_{-D/2}^{D/2}\int_{-D/2}^{D/2} {\rm d}x_1{\rm d}x_1'\langle E^*(x_1)E(x_1')\rangle h_1^*(0,x_1)h_2(x,x_1')\right|^2 \\ &= \left(\frac{Il}{\lambda z_2}\sin c\left[\frac{\pi w x}{\lambda z_2}\right]\cos\left[\frac{\pi d x}{\lambda z_2}\right]\right)^2. \end{aligned}\tag{A9}$$

Then the second-order lensless ghost interference function is

$$g(x) = \frac{\langle I_{\rm t}I_{\rm r}(x)\rangle}{\langle I_{\rm t}\rangle\langle I_{\rm r}(x)\rangle} = 1 + \frac{l}{D}\left(1+\frac{z_1}{z_2}\right)\left(\sin c\left[\frac{\pi w x}{\lambda z_2}\right]\cos\left[\frac{\pi d x}{\lambda z_2}\right]\right)^2. \tag{A10}$$

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