# Invisible anti-cloak with elliptic cross section using phase complement*

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(Received 23 June 2010; revised manuscript received 6 August 2010)

Based on the theory of phase complement, an anti-cloak with circular cross section can be made invisible to an object outside its domain. As the cloak with elliptic cross section is more effective to make objects invisible than that with circular cross section, a scaled coordinate system is proposed to design equivalent materials of invisible anti-cloak with elliptic cross section using phase complement. The cloaks with conventional dielectric and double negative parameters are both simulated with the geometrical transformations. The results show that the cloak with elliptic cross section through phase complement can effectively hide the outside objects.

**Keywords:** invisible anti-cloak, phase complement, electromagnetic radiation

**PACS:** 41.20Jb, 02.70.Dh **DOI:** 10.1088/1674-1056/20/2/024101

## 1. Introduction

The design of electromagnetically invisible anti-cloak has attracted much attention in the recent years. Pendry *et al.*[1] and Cummer *et al.*[2] proposed the coordinate transformation concept to control the spatial electromagnetic fields, and designed an invisible cloak that could perfectly hide arbitrary objects inside the cloaked domain in 2006. Since then, many kinds of cylindrical cloaks have been widely investigated due to their invisible properties.[3−9] These studies have shown that the electromagnetic field is guided around the object and cannot penetrate into the cloak, but the object cannot "see" out according to the principle of reversibility of light. Chen *et al.*[10] found a theoretical way to allow some light to penetrate through the invisible anti-cloak in 2008. Then the inside one could see out, but the object still have to be inside the cloak.

Recently, combining the optical transformation with the complementary medium, Yang *et al.*[11] and Lai *et al.*[12] designed a new kind of invisible anti-cloak,[11,12] which can blind objects at a distance outside the cloaking shell. Different from the traditional perfect cloak, this invisible anti-cloak with complementary medium allows the inside object to "see" out but to be invisible from outside. This theory opens a new branch of metamaterials applied to invisible anti-cloaks. This type of complement cloak is designed as a cylindrical shape which is relatively easier to analyse, but it is very inconvenient for long or huge objects to be invisible due to the curvature of cylinder. A new shape of cloak with elliptic cross section is presented in this article.

There is wider space outside the elliptical shell than a circular one, so when elliptical cross section is employed, objects with long and huge shape can be hidden outside the cloak effectively. A scaled coordinate system which can avoid the complicated calculation in the Jacobian transformation matrix is introduced in this paper. To deduce this new complement cloak for elliptic cross section, which is more effective for hiding various shapes of objects, the procedure developed earlier in Ref. [12] is applied. Both the cloaks of conventional dielectric and the double negative cloaks are simulated respectively under incident cylindrical wave. Note that the two kinds of elliptical cloaks can effectively hide the outside objects.

## 2. Geometrical transformations for elliptical cross section

It is suggested that the computation of the Jacobian transformation matrix $\boldsymbol{J}$ is necessary for changing the coordinate system.[6] According to the form

*Project supported by the National Natural Science Foundation of China (Grant No. 60871070), and the Fundamental Research Funds for the Central Universities, China.

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invariances of Maxwell's equations in the initial space and the transformed space, the permittivity and permeability tensors of the materials in the transformed space can be obtained as

$$\overline{\overline{\varepsilon'}} = \varepsilon \boldsymbol{T}^{-1}, \quad \overline{\overline{\mu'}} = \mu \boldsymbol{T}^{-1}, \tag{1}$$

where $\boldsymbol{T} = \boldsymbol{J}^{\mathrm{T}}\boldsymbol{J}/\det(\boldsymbol{J})$.

To build the elliptic cross section of which the plane is scaled by a factor $k$ along the $y$ coordinate, the Jacobian transformation matrix becomes

$$\boldsymbol{J}_{xx'} = \boldsymbol{J}_{xx_k}\boldsymbol{J}_{x_kr}\boldsymbol{J}_{rr'}\boldsymbol{J}_{r'x'_k}\boldsymbol{J}_{x'_kx'}. \tag{2}$$

Equation (2) will be discussed for further detail.

In the scaled coordinate system $(x_k, y_k)$, the ellipse will become a circle determined by the transformation of $y = ky_k$ (while $x = x_k$) characterized by the Jacobian matrix $\boldsymbol{J}_{xx_k} = \mathbf{diag}\,(1, k, 1)$.

Using the method of building a circular complementary cloak in the scaled coordinate system,[10] the associated Jacobian matrix $\boldsymbol{J}_{x_kr}\boldsymbol{J}_{rr'}\boldsymbol{J}_{r'x'_k}$ is defined through the following three steps.

**Step 1** Transform into cylindrical coordinates, which leads to

$$\begin{aligned}\boldsymbol{J}_{x_kr}(r,\theta) &= \frac{\partial(x_k, y_k, z_k)}{\partial(r,\theta,z)} \\ &= \begin{pmatrix}\cos(\theta) & -r\sin(\theta) & 0\\ \sin(\theta) & r\cos(\theta) & 0\\ 0 & 0 & 1\end{pmatrix} \\ &= \boldsymbol{R}(\theta)\,\mathbf{diag}\,(1, r, 1),\end{aligned} \tag{3}$$

where

$$\boldsymbol{R}(\theta) = \begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$$

and

$$\mathbf{diag}\,(1, r, 1) = \begin{pmatrix}1 & 0 & 0\\ 0 & r & 0\\ 0 & 0 & 1\end{pmatrix}.$$

**Step 2** Radial contraction in cylindrical coordinates.

To build a circular complementary cloak, two geometric transformations will be used. First, consider a geometric transformation which compresses the layer of air $b < r < c$ into the layer of complementary medium $a < r' < b$, thus

$$\begin{cases} r' = (r-b)(a-b)/(c-b) + b, & b \le r \le c,\\ \theta' = \theta, & 0 < \theta < 2\pi,\\ z' = z, \end{cases} \tag{4}$$

where $a, b, c$ are the core radius, the outer radius of complementary layer, and the outer radius of the canceled air layer respectively.

The associated Jacobian matrix is

$$\boldsymbol{J}_{rr'} = \frac{\partial(r,\theta,z)}{\partial(r',\theta',z')} = \mathbf{diag}\,(\alpha, 1, 1), \tag{5}$$

where $\alpha = f'(r') = (c-b)/(a-b)$.

Second, a geometric transformation which compresses the whole system $0 < r < c$ into the layer of core material $0 < r' < a$ is considered to restore the optical path, thus

$$\begin{cases} r' = ar/c, & 0 \le r \le c,\\ \theta' = \theta, & 0 < \theta < 2\pi,\\ z' = z. \end{cases} \tag{6}$$

The associated Jacobian matrix[5] is with $\alpha = f'(r') = c/a$.

**Step 3** Transform into the Cartesian coordinate.

The associated Jacobian matrix is

$$\begin{aligned}\boldsymbol{J}_{r'x'_k}(r',\theta') &= \frac{\partial(r',\theta',z')}{\partial(x'_k, y'_k, z'_k)} \\ &= \boldsymbol{J}_{rx}^{\mathrm{T}} \\ &= \mathrm{diag}\left(1, \frac{1}{r'}, 1\right)\boldsymbol{R}(-\theta).\end{aligned} \tag{7}$$

Subsequently an inverse scaling along the $y$ axis, $y' = (1/k)\,y'_k$, is used to recuperate the initial elliptical shape and the associated Jacobian matrix is $\boldsymbol{J}_{x'_kx'}$.

Combining Eq. (1) with Eq. (2), the material properties of the complementary cloak of elliptic cross section are described by the transformation matrix as

$$\begin{aligned}&\boldsymbol{T}^{-1}\\ &= \mathbf{diag}(1, k, 1)\\ &\quad\times \boldsymbol{R}(\theta'_k)\,\mathbf{diag}\left(\frac{1}{\alpha}, \frac{r'_k}{r}, 1\right)\boldsymbol{R}(-\theta_k)\,\mathbf{diag}\left(1, \frac{1}{k^2}, 1\right)\\ &\quad\times \boldsymbol{R}(\theta_k)\,\mathbf{diag}\left(\frac{1}{\alpha}, \frac{r'_k}{r}, 1\right)\boldsymbol{R}(-\theta'_k)\,\mathbf{diag}(1, k, 1)\frac{\alpha r}{r'_k},\end{aligned} \tag{8}$$

where

$$\theta_k = \theta'_k = 2\arctan\left(y'\Big/\left(x'/k + \sqrt{(x'/k)^2 + y'^2}\right)\right),$$

$$r'_k = \sqrt{(x'/k)^2 + y'^2},$$

and

$$\alpha = f'(r').$$

It is supposed that an object with permittivity $\varepsilon_0$ and permeability $\mu_0$ is located outside the complementary cloak shell and in the canceled air layer. In order to make the object invisible,[12] an "image" object with permittivity $\varepsilon_0' = \varepsilon_0 \boldsymbol{T}^{-1}$ and permeability $\mu_0' = \mu_0 \boldsymbol{T}^{-1}$ should be arranged in the complementary medium layer so as to counteract the object optically.

## 3. Results and discussion

Since the proposed cloak has no symmetry in any direction, a cylindrical wave is adopted to study its interaction with electromagnetic waves from different orientations by using the finite-element method (FEM). Then the elliptical cylindrical cloak is reduced into a two-dimensional (2D) one, and the FEM simulation is made in the area with $5\lambda_0$ along the $x$ direction and $5\lambda_0$ along the $y$ direction. The cell size is $\delta = \lambda_0/10$. A line source (considering the $p$ polarization) is located at the position $(4, -4)$ with the wavelength $\lambda_0 = 2$. Perfectly matched layers[13] is used to model the outgoing waves.

Consider an elliptical cloak with the following parameters: axis ratio $k = 0.5$, $a = 1$, $b = 2$, and $c = 3$. In the scaled coordinate system, there are two geometric transformations: one is the layer of air at $2 < r < 3$, compressed into the layer of complementary medium of $1 < r' < 2$, by choosing linear transformation mapping of $r = 4 - r'$ and $\alpha = -1$; the other is the whole system at $0 < r < 3$, transformed into the layer of core material at $0 < r' < 1$, by choosing linear transformation mapping of $r = 3r'$ and $\alpha = 3$.

This result is given in Fig. 1(a). It is obvious that the cylindrical wave is perfectly guided through the cloak. Since the impedance match cannot be perfectly satisfied at the elliptical boundary, one sparkling spot which indicates the scattering is observed in Fig. 1(a). And we consider that the scattering can be eliminated by finer mesh generation. Set the axis ratio $k = 1$, and keep other factors unchanged, the elliptical cross section will become circular cross section as shown in Fig. 1(b).

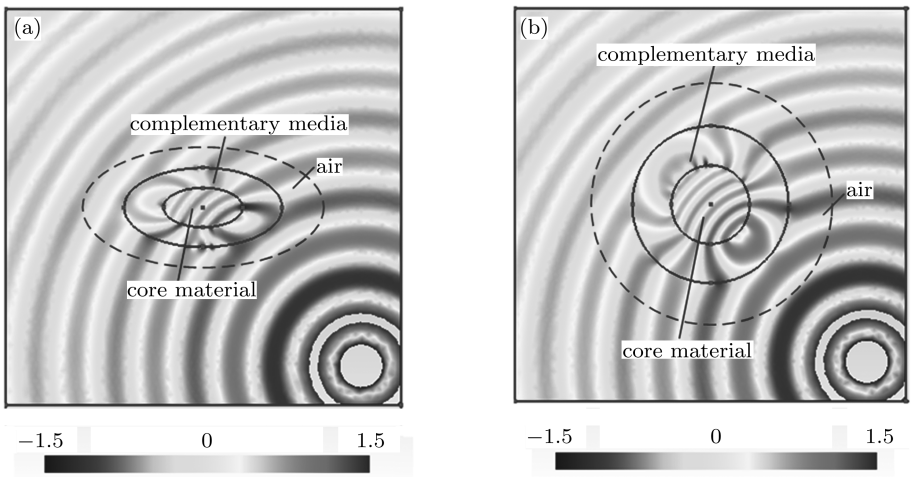



**Fig. 1.** Electric-field distributions in complementary cloaks with (a) elliptical and (b) circular cross sections.

In the complementary cloak with elliptical cross section, we add an elliptical shell with $\varepsilon_0 = 2$ and $\mu_0 = 1$, the major axis is bounded between $2.4 < |x'| < 2.6$ while the minor axis is restricted between $1.2 < |y'| < 1.3$. In this case, the "image" shell is with $\varepsilon_0' = 2\boldsymbol{T}^{-1}$ and $\mu_0' = \boldsymbol{T}^{-1}$, and the major axis bounded between $1.4 < |x'| < 1.6$ and the minor axis bounded between $0.7 < |y'| < 0.8$ are settled in the complementary shell. Figure 2(a) shows the electric field distributions of the elliptical shell without cloaking, while figure 2(b) displays those of the elliptical shell shielded by the corresponding "image" shell.

As shown in Fig. 2, in the left panel without the cloak, as expected, the elliptical shell makes the electric-field slightly perturbed especially in the forward direction. But after hiding by the complementary cloak in the right panel, electric-field is almost undisturbed.

Finally, considering another elliptical shell with $\varepsilon_0 = -1$ and $\mu_0 = -1$, the location as in Fig. 2 is unchanged. In order to make the shell invisible, the corresponding "image" shell with $\varepsilon_0' = -\boldsymbol{T}^{-1}$ and $\mu_0' = -\boldsymbol{T}^{-1}$ is pro-

posed. Figure 3(a) shows the elliptical shell that is directly illuminated without cloak electric-field distributions and figure 3(b) shows the one after cloaking electric-field distributions.

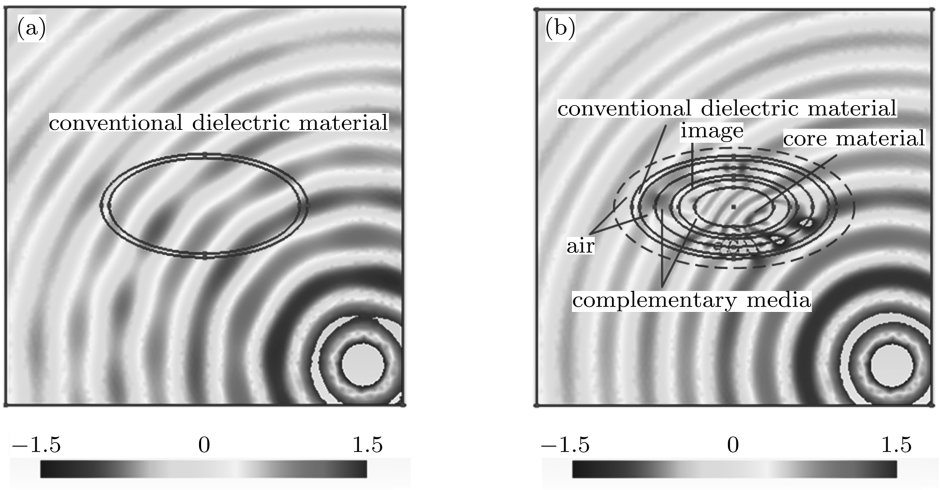



**Fig. 2.** Electric-field distributions under a cylindrical source. Panel (a) shows elliptical shell with $\varepsilon_0 = 2$ and $\mu_0 = 1$, directly illuminated without cloak; panel (b) displays elliptical shell shielded with elliptical complementary cloak and the corresponding "image".

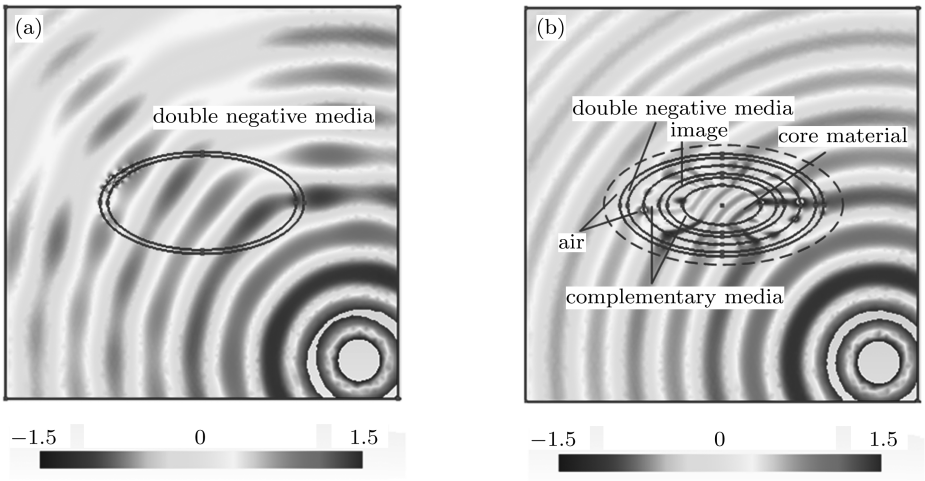



**Fig. 3.** Electric-field distributions under a cylindrical source. Panel (a) shows the elliptical shell with $\varepsilon_0 = -1$ and $\mu_0 = -1$, directly illuminated without cloak; panel (b) displays the elliptical shell is shielded with elliptical complementary cloak and the corresponding "image".

From Fig. 3(a) it can be found that the elliptical shell without the cloak makes the electric-field slightly perturbed especially in the forward direction. But after hiding the cloak it is similar to Fig. 2(b). The cylindrical wave is perfectly guided through the cloaked object without any obvious scattering. Note that the elliptical shell is always in the layer of air.

## 4. Conclusion

In order to hide long and thin objects, the circular cross section of complementary anti-cloak is transformed into the elliptic one in this article. Owing to the elliptic cross section with an asymmetric shape, the scaled coordinate systems where the ellipses are mapped into circles are introduced. Numerical simulations show that the complementary cloak of the conventional dielectric material and those of the double negative medium with elliptic cross section both can make objects invisible. The presented method is for the two-dimensional case, but it can be readily extended to the three-dimensional case.

# References


[1] Pendry J B, Schurig D and Smith D R 2006 *Science* **312** 1780

[2] Cummer S A, Popa B I, Schurig D, Smith D R and Pendry J B 2006 *Phys. Rev.* E **74** 036621

[3] Ma H, Qu S B, Xu Z and Wang J F 2009 *Chin. Phys.* B **18** 1123

[4] Ma H, Qu S B, Xu Z, Zhang J Q and Wang J F 2009 *Chin. Phys.* B **18** 179

[5] Chen H Y and Chan C T 2007 *Appl. Phys. Lett.* **90** 241105

[6] Nicolet A, Zolla F and Guenneau S 2008 *IEEE Trans. Mag.* **44** 1150

[7] Ma H, Qu S B, Xu Z, Zhang J Q and Wang J F 2009 *Chin. Phys.* B **18** 1850

[8] Wu Q, Zhang K, Meng F Y and Li L W 2009 *Acta Phys. Sin.* **58** 1619 (in Chinese)

[9] Ma H, Qu S B, Xu Z, Zhang J Q and Wang J F 2009 *Chin. Phys.* B **18** 1025

[10] Chen H Y, Luo X D, Ma H R and Chan C T 2008 *Opt. Express* **16** 14603

[11] Yang T, Chen H Y, Luo X D and Ma H R 2008 *Opt. Express* **16** 18545

[12] Lai Y, Chen H Y, Zhang Z Q and Chan C T 2009 *Phys. Rev. Lett.* **102** 093901

[13] Berenger J P 1994 *J. Comput. Phys.* **114** 185