# Reference model based consensus control of second-order multi-agent systems*

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This paper deals with the consensus problem of multi-agent systems with second-order dynamics. The objective is to design algorithms such that all agents will have same positions and velocities. First, a reference model based consensus algorithm is proposed. It is proved that the consensus can be achieved if the communication graph has a spanning tree. Different from most of the consensus algorithms proposed in the literature, the parameters of the control laws are different among agents. Therefore, each agent can design its control law independently. Secondly, it gives a consensus algorithm for the case that the velocities of the agents are not available. Thirdly, the effectiveness of the input delay and the communication delay is considered. It shows that consensus can be achieved if the input delay of every agent is smaller than a bound related to parameters in its control law. Finally, some numerical examples are given to illustrate the proposed results.

**Keywords:** consensus problems, multi-agent systems, directed graphs, second-order dynamics

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## 1. Introduction

The coordination problem of multi-agent systems has been a popular subject in systems and control in recent years. Examples of the coordination problem include flocking,[1,2] formation control,[3] synchronization,[4] Rendezvous,[5] consensus,[6,7] and so on. Significant efforts have been made to study the consensus problem because it plays an important role in some other coordination problems. The objective of the consensus problem is to reach an agreement on certain quantities of interest in a group of agents, by negotiating between neighbours.

In many researches on the consensus problems of multi-agent systems, all agents are governed by first-order dynamics (continuous or discrete-time).[8−18] Some preliminary results were reported in Refs. [8]–[10]. Many followed researches were made from various aspects such as time-delays,[13] asynchronous sampling rate,[14] reference states,[15] information quantization,[16,17] time-varying topologies,[18] and so on. In the meanwhile, there were more and more researches focused on the second-order multi-agent systems, where all the agents were governed by second-order dynamics.[19−22] Recently, a necessary and sufficient condition on the achievement of the consensus has been derived in Ref. [23]. Reference [24] gave some necessary and sufficient conditions for solving consensus problems of double-integrator dynamics via sampled control. The consensus protocols considered in Refs. [19]–[24] not only depend on the neighbours' positions but also their velocities. However, in some cases, the velocities of the agents may not be available. So it is more desirable to derive some consensus protocols that depend on the neighbors' information variables only. This problem was considered in Refs. [25] and [26] and some consensus protocols were given. The $H_\infty$ consensus problem of second-order multi-agent systems was considered in Refs. [20] and [21].

Summarizing most of the existing results we can find that, when the communication graph is directed, the consensus can be achieved for the first-order multi-agent systems if the communication graph having a spanning tree. However, the communication graph has a spanning tree is not a sufficient condition for the second-order multi-agent systems to achieve consensus, it is also needed that some parameters in the consensus algorithm satisfy certain conditions, which relates to the eigenvalues of the Laplacian matrix corresponding to the communication graph. In fact, these eigenvalues are usually not available.

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In this paper, we will consider the consensus problem of multi-agent systems with second-order dynamics. A reference model based on consensus algorithm will be given, together with the sufficient condition on the achievement of the consensus. Sometimes, the velocities of the agents are not available. To solve this problem, we will give an observer-based consensus algorithm. The effectiveness of the time-delays caused by communication network (communication delay) and the actuator (input delay) will also be considered.

The contributions of the current paper are fourfold. First, the proposed consensus algorithm can solve the consensus problem for any directed network having a spanning tree. The parameters in the algorithm do not need to satisfy any condition related to the communication network. This is different from the commonly used consensus algorithm on the second-order consensus problem. Secondly, the parameters in the algorithm can be different among agents. Therefore, each agent can design its own control law, i.e., this algorithm is a purely distributed algorithm. Thirdly, a consensus algorithm without velocity measurement is proposed for the directed network. Compared with the results in Refs. [25] and [26], the communication graph considered in this paper is more general. Finally, in the existing results on this problem, the upper bound of allowable input delay is determined by the communication network.[23] But, in this paper, the upper bound of allowable input delay for each agent is determined by its own parameters in the control law. So each agent can change its parameters to increase the allowable upper bound of its controller's input delay.

The rest of this paper is organized as follows. Section 2 formulates the consensus problem of the multi-agent systems with second-order dynamics. The main results are presented in Section 3. Some numerical examples are given in Section 4 to illustrate the proposed results and the paper is concluded in Section 5.

## 2. Problem formulation

Graph theory notions: Let $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ be a directed graph of order $n$, where $\mathscr{V} = \{1, \ldots, n\}$ is the node set, $\mathscr{E} \in \mathscr{V} \times \mathscr{V}$ is the edge set. The edge $(j, i) \in \mathscr{E}$ if and only if node $i$ can obtain the information of node $j$. In this case, we say node $j$ is the parent node and node $i$ is the child node. The neighbouring set $\mathscr{N}_i$ of agent $i$ is defined as $\mathscr{N}_i = \{j : (j, i) \in \mathscr{E}\}$. The adjacency matrix $A_d = [a_{ij}] \in R^{n\times n}$ is defined as $a_{ij} = 0$ when $i = j$; when $i \neq j$, $a_{ij} > 0$ if $(j, i) \in \mathscr{E}$, and $a_{ij} = 0$ otherwise. The Laplacian matrix $L = D - A_{\rm d}$, where

$$D = \begin{bmatrix} \sum_{j=1}^{n} a_{1j} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sum_{j=1}^{n} a_{nj} \end{bmatrix}.$$

A directed path is a sequence of edges in the form of $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots$, where $v_{i_1}, v_{i_2} \in \mathscr{V}$. A directed graph is called to have a spanning tree if there exists a node such that their exists a directed path from all other nodes to this node.

We use a directed graph to denote the communication topology among the agents. The dynamics of the second-order multi-agent systems can be described by the following equations:

$$\dot{x}_i(t) = v_i(t), \tag{1}$$

$$\dot{v}_i(t) = u_i(t), \tag{2}$$

where $x_i(t)$, $v_i(t)$ and $u_i(t) \in R^m$ are the position, velocity and control input of agent $i$, respectively.

We say that algorithm $u_i(t)$ solves the second-order consensus problem for the system (1) and (2) if $x_i(t) - x_j(t) \to 0$ and $v_i(t) - v_j(t) \to 0$, as $t \to \infty$, for any $1 \leq i < j \leq n$. A commonly used consensus algorithm is

$$u_i = -\alpha \sum_{j\in\mathscr{N}_i} a_{ij}[x_i(t) - x_j(t)] - \beta \sum_{j\in\mathscr{N}_i} a_{ij}[v_i(t) - v_j(t)]. \tag{3}$$

It is shown in Ref. [23] that using Eq. (3), the second-order consensus for the system described by Eqs. (1) and (2) if and only if the network contains a directed spanning tree and

$$\frac{\beta^2}{\alpha} > \max_{2\leq i\leq n} \frac{\mathrm{Im}^2(\mu_i)}{\mathrm{Re}(\mu_i)\sqrt{\mathrm{Re}^2(\mu_i) + \mathrm{Im}^2(\mu_i)}}, \tag{4}$$

where $\mu_i$ is the nonzero values of the Laplacian matrix $L$. In addition, when the second-order consensus is achieved, the final values of position and the velocity of the agents are $(\gamma^\tau \otimes I_m)\bar{x}(0) + t(\gamma^\tau \otimes I_m)\bar{v}(0)$ and $(\gamma^\tau \otimes I_m)\bar{v}(0)$, where $\gamma$ is the left eigenvector of $L$ corresponding to eigenvalue 0 satisfying $\gamma^\tau \mathbf{1}_n = 1$, $\mathbf{1}_n \in R^n = [\,1, \ldots, 1\,]^\tau$. It can be seen that before choosing $\alpha$ and $\beta$ in Eq. (3), one should know the eigenvalues of the Laplacian matrix $L$. However, because each agent can communicate with its neighbours

only, these eigenvalues are not available in practice. To solve this problem, we propose the following reference model based on consensus algorithm.

**Algorithm 1**

$$\dot{\bar{x}}_i(t) = \bar{v}_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}[\bar{x}_i(t) - \bar{x}_j(t)], \quad (5)$$

$$\dot{\bar{v}}_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}[\bar{v}_i(t) - \bar{v}_j(t)], \quad (6)$$

$$u_i(t) = -\alpha_i[x_i(t) - \bar{x}_i(t)] - \beta_i[v_i(t) - \bar{v}_i(t)], \quad (7)$$

where $\bar{x}_i(t)$ and $\bar{v}_i(t)$ are states of the reference model of agent $i$, $\alpha_i, \beta_i > 0$ are constant scalars, $\bar{x}_i(0)$ and $\bar{v}_i(0)$ can be chosen arbitrarily. Usually we choose $\bar{x}_i(0) = x_i(0)$ and $\bar{v}_i(0) = v_i(0)$. As is shown in Fig. 1, each agent has a reference model with states $\bar{x}_i(t)$ and $\bar{v}_i(t)$. The agents send $\bar{x}_i(t)$ and $\bar{v}_i(t)$ to there neighbours. The main thought of Algorithm 1 is that consensus can be achieved if the reference models can reach consensus and each agent can track its reference model. Because the dynamics of the reference models are properly chosen, the reference models can reach consensus easily. So Algorithm 1 can solve the consensus problem with less restrictive conditions.

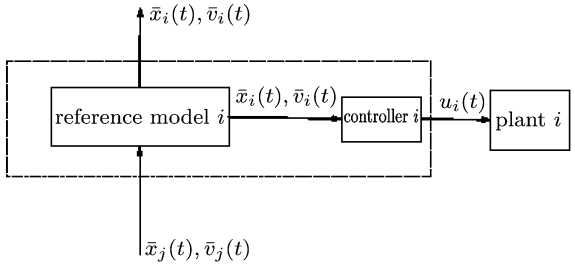



**Fig. 1.** The structure of agent $i$.

## 3. Main results

This section gives the main results of this paper. First, we will show that the second-order consensus can be achieved under Algorithm 1 with less restrictive conditions than the algorithm described by Eq. (3). Then, an algorithm for the consensus problem without velocity measurement will be given. Finally, we will investigate the effectiveness of input delay and communication delay on Algorithm 1.

Using Algorithm 1, we can describe the dynamics of the reference model by the following equations:

$$\dot{\bar{x}}(t) = \bar{v}(t) - (L \otimes I_m)\bar{x}(t), \quad (8)$$

$$\dot{\bar{v}}(t) = -(L \otimes I_m)\bar{v}(t), \quad (9)$$

where

$$\bar{x}(t) = [\,\bar{x}_1^\tau(t), \ldots, \bar{x}_n^\tau(t)\,]^\tau,$$

$$\bar{v}(t) = [\,\bar{v}_1^\tau(t), \ldots, \bar{v}_n^\tau(t)\,]^\tau.$$

Equations (8) and (9) can be rewritten in a compact form as

$$\dot{\xi}(t) = (\bar{A} \otimes I_m)\xi(t), \quad (10)$$

where $\xi(t) = [\,\bar{x}^\tau(t), \bar{v}^\tau(t)\,]^\tau$ and $\bar{A} = \begin{bmatrix} -L & I_n \\ 0 & -L \end{bmatrix}$. The solution of Eq. (10) is

$$\xi(t) = \mathrm{e}^{(\bar{A}\otimes I_m)t}\xi(0). \quad (11)$$

When the communication graph $\mathcal{G}$ has a spanning tree, we know that $L$ has a simple eigenvalue 0 and all the other eigenvalues have positive real parts.[10] By some calculations we know that $\det(\lambda I_{2n} - \bar{A}) = [\det(\lambda I_n + L)]^2$. It is easy to see that $\bar{A}$ has a zero eigenvalue with algebraic degree 2 and all the other eigenvalues have negative real parts. By some algebraic calculations we can obtain that $\mathrm{e}^{\bar{A}t} \to \begin{bmatrix} \mathbf{1}_n\gamma^\tau & t\mathbf{1}_n\gamma^\tau \\ 0 & \mathbf{1}_n\gamma^\tau \end{bmatrix}$, as $t \to \infty$. From Eq. (11) we obtain that $\xi(t)$ will converge to $[(p_1\nu_1 + p_2\nu_2 + tp_1\nu_2) \otimes I_m]\xi(0)$, which implies that $\bar{x}_i(t)$ and $\bar{v}_i(t)$ will converge to $(\gamma^\tau \otimes I_m)\bar{x}(0) + t(\gamma^\tau \otimes I_m)\bar{v}(0)$ and $(\gamma^\tau \otimes I_m)\bar{v}(0)$, respectively. Therefore, the reference models can reach consensus.

Since the states of the reference models will converge to the same value, the consensus will be achieved for the system described by Eqs. (1) and (2) if $x_i(t) - \bar{x}_i(t)$ and $v_i(t) - \bar{v}_i(t)$ can converge to zero. It is easy to see that

$$\dot{x}_i(t) - \dot{\bar{x}}_i(t) = v_i(t) - \bar{v}_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}[\bar{x}_i(t) - \bar{x}_j(t)], \quad (12)$$

$$\dot{v}_i(t) - \dot{\bar{v}}_i(t) = -\alpha_i[x_i(t) - \bar{x}_i(t)] - \beta_i[v_i(t) - \bar{v}_i(t)] - \sum_{j\in\mathcal{N}_i} a_{ij}[\bar{v}_i(t) - \bar{v}_j(t)], \quad (13)$$

Note that $\sum_{j\in\mathcal{N}_i} a_{ij}[\bar{x}_i(t) - \bar{x}_j(t)]$ and $\sum_{j\in\mathcal{N}_i} a_{ij}[\bar{v}_i(t) - \bar{v}_j(t)]$ will converge to zero. We only need to ensure that the system described by the following equations is stable:

$$\dot{x}_i(t) - \dot{\bar{x}}_i(t) = v_i(t) - \bar{v}_i(t), \quad (14)$$

$$\dot{v}_i(t) - \dot{\bar{v}}_i(t) = -\alpha_i[x_i(t) - \bar{x}_i(t)] - \beta_i[v_i(t) - \bar{v}_i(t)]. \quad (15)$$

Choose $\alpha_i$ and $\beta_i$ such that all roots of $[\prod_{i=1}^n(\lambda^2 + \beta_i\lambda + \alpha_i)] = 0$ have negative real parts, $i = 1, \ldots, n$.

It is easy to verify that system described by Eqs. (14) and (15) is stable.

It is worth while mentioning that the final values of the positions and the velocities of the agents depend on the initial values of the reference models, but not $x_i(0)$ and $v_i(0)$. In fact, if we let $\bar{x}_i(0) = x_i(0)$ and $\bar{v}_i(0) = v_i(0)$, we have $x_i(t) \to (\gamma^\tau \otimes I_m)x(0) + t(\gamma^\tau \otimes I_m)v(0)$, $v_i(t) \to (\gamma^\tau \otimes I_m)v(0)$ as $t \to \infty$.

From the above analysis we know that conditions on the parameters in Algorithm 1 under which the consensus is achieved do not relate to the eigenvalues of the Laplacian matrix. So we need not to know these eigenvalues when we choose the parameters in Protocol 1. This is the main advantage of Algorithm 1.

In real world multi-agent systems, it is more difficult to measure the velocities than the positions. Sometimes, the velocities of the agents cannot be measured. In this case, we can use the following algorithm.

**Algorithm 2**

$$\dot{\hat{x}}_i(t) = \hat{v}_i(t) - \varepsilon_i[\hat{x}_i(t) - x_i(t)], \tag{16}$$

$$\dot{\hat{v}}_i(t) = -\delta_i[\hat{x}_i(t) - x_i(t)] + u_i(t), \tag{17}$$

$$\dot{\bar{x}}_i(t) = \bar{v}_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}[\bar{x}_i(t) - \bar{x}_j(t)], \tag{18}$$

$$\dot{\bar{v}}_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}[\bar{v}_i(t) - \bar{v}_j(t)], \tag{19}$$

$$u_i(t) = -\alpha_i[\hat{x}_i(t) - \bar{x}_i(t)] - \beta_i[\hat{v}_i(t) - \bar{v}_i(t)], \tag{20}$$

where $\varepsilon_i$, $\delta_i$, $\alpha_i$, $\beta_i$ are positive constant scalars.

Since the velocities of the agents are not known, we need to estimate them based on the position measurements. In Algorithm 2, equations (16) and (17) are used to estimate the velocities of agent $i$. From Eqs. (1), (2), (16) and (17) we know that

$$\dot{\hat{x}}_i(t) - \dot{x}_i(t) = \hat{v}_i(t) - v_i(t) - \varepsilon_i[\hat{x}_i(t) - x_i(t)], \tag{21}$$

$$\dot{\hat{v}}_i(t) - \dot{v}_i(t) = -\delta_i[\hat{x}_i(t) - x_i(t)]. \tag{22}$$

Note that $\hat{x}_i(t) - x_i(t)$ and $\hat{v}_i(t) - v_i(t)$ will converge to zero if $\delta_i$ and $\gamma_i$ satisfy the condition that all roots of $[\prod_{i=1}^n(\lambda^2+\delta_i\lambda+\gamma_i)] = 0$ have negative real parts. Substituting $\hat{x}_i(t) = x_i(t)$ and $\hat{v}_i(t) = v_i(t)$ into Eq. (20) we can obtain Eq. (7). Choose $\alpha_i$ and $\beta_i$ such that all roots of $[\prod_{i=1}^n(\lambda^2 + \beta_i\lambda + \alpha_i)] = 0$ have negative real parts. By the above analysis we can see that, when the directed communication graph has a spanning tree, the positions and the velocities of all agents will converge to $(\gamma^\tau \otimes I_m)\bar{x}(0) + t(\gamma^\tau \otimes I_m)\bar{v}(0)$ and $(\gamma^\tau \otimes I_m)\bar{v}(0)$, respectively.

Now, we consider the effectiveness of time-delays on Algorithm 1. Real world multi-agent systems are often effected by the input delay and the communication delay. If we take the input delay and the communication delay into account, Algorithm 1 becomes

**Algorithm 3**

$$\dot{\bar{x}}_i(t) = \bar{v}_i(t) - \sum_{j\in\mathcal{N}_i} a_{ij}[\bar{x}_i(t) - \bar{x}_j(t - \tau_{ij}^c)], \tag{23}$$

$$\dot{\bar{v}}_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}[\bar{v}_i(t) - \bar{v}_j(t - \tau_{ij}^c)], \tag{24}$$

$$u_i(t) = -\alpha_i[x_i(t - \tau_i^u) - \bar{x}_i(t - \tau_i^u)] - \beta_i[v_i(t - \tau_i^u) - \bar{v}_i(t - \tau_i^u)]. \tag{25}$$

where $\tau_i^u \geq 0$ is the input delay caused by the response time of the actuator in agent $i$, $\tau_{ij}^c \geq 0$ is the communication delay caused by the communication channel between agents $i$ and $j$.

Based on the above analysis we know that all agents will have same positions and velocities, if the reference models can reach consensus and each agent can track its reference model.

We consider the reference model Eqs. (23) and (24) first, which is affected by the communication delay. Equations (23) and (24) can be rewritten in a compact form as

$$\dot{\bar{x}}(t) = \bar{v}(t) - (D \otimes I_m)\bar{x}(t) + \sum_{i=1}^n \sum_{j\in\mathcal{N}_i} (A_{ij}^c \otimes I_m)\bar{x}(t - \tau_{ij}^c), \tag{26}$$

$$\dot{\bar{v}}(t) = -(D \otimes I_m)\bar{v}(t) + \sum_{i=1}^n \sum_{j\in\mathcal{N}_i} (A_{ij}^c \otimes I_m)\bar{v}(t - \tau_{ij}^c), \tag{27}$$

where $A_{ij}^c \in R^{n\times n}$ is a matrix with the $j$-th element in the $i$-th row being $a_{ij}$ and all the other elements being zero. From the above analysis we know that consensus can be achieved between the reference models if the characteristic equation of Eqs. (23) and (24) has a zero root of multiplicity two and all the other roots have negative real parts. When the communication graph $\mathcal{G}$ has a spanning tree, we know that the equation $\det[\lambda I_n + D - \sum_{i=1}^n \sum_{j\in\mathcal{N}_i}(A_{ij}^c e^{-\tau_{ij}^c\lambda})] = 0$ has a simple zero root and all the other roots have negative real parts, where $D$ is defined in Section 2.[13] Notice that the characteristic equation of Eqs. (23) and (24) is $\det[\lambda I_n + D - \sum_{i=1}^n \sum_{j\in\mathcal{N}_i}(A_{ij}^c e^{-\tau_{ij}^c\lambda})]^2 = 0$, which has a zero root of multiplicity two and all the other roots have negative real parts.

From Eqs. (1), (2) and (23)–(25) we know that

$$\dot{x}_i(t) - \dot{\bar{x}}_i(t) = v_i(t) - \bar{v}_i(t)$$

$$- \sum_{j\in\mathcal{N}_i} a_{ij}[\bar{x}_i(t) - \bar{x}_j(t-\tau_{ij}^{\rm c})], \quad (28)$$

$$\begin{aligned}\dot{v}_i(t) - \dot{\bar{v}}_i(t) = & -\alpha_i[x_i(t-\tau_i^u) - \bar{x}_i(t-\tau_i^u)] \\ & - \beta_i[v_i(t-\tau_i^u) - \bar{v}_i(t-\tau_i^u)] \\ & - \sum_{j\in\mathcal{N}_i} a_{ij}[\bar{v}_i(t) - \bar{v}_j(t-\tau_{ij}^{\rm c})]. \quad (29)\end{aligned}$$

If the reference models can reach consensus, then $\sum_{j\in\mathcal{N}_i} a_{ij}[\bar{x}_i(t) - \bar{x}_j(t-\tau_{ij}^{\rm c})]$ and $\sum_{j\in\mathcal{N}_i} a_{ij}[\bar{v}_i(t) - \bar{v}_j(t-\tau_{ij}^{\rm c})]$ will converge to zero. So each agent can track its reference model if the following system is stable

$$\dot{x}_i(t) - \dot{\bar{x}}_i(t) = v_i(t) - \bar{v}_i(t), \quad (30)$$

$$\begin{aligned}\dot{v}_i(t) - \dot{\bar{v}}_i(t) = & -\alpha_i[x_i(t-\tau_i^u) - \bar{x}_i(t-\tau_i^u)] \\ & - \beta_i[v_i(t-\tau_i^u) - \bar{v}_i(t-\tau_i^u)]. \quad (31)\end{aligned}$$

The characteristic equation of Eqs. (30) and (31) is $\lambda^2 + \beta_i e^{-\tau_i^{\rm c}\lambda}\lambda + \alpha_i e^{-\tau_i^{\rm c}\lambda} = 0$. From Lemmas 3, 4 and 5 in Ref. [23] we know that all the roots of $\lambda^2 + \beta_i e^{-\tau_i^{\rm c}\lambda}\lambda + \alpha_i e^{-\tau_i^{\rm c}\lambda} = 0$ have negative real parts, if $0 \le \tau_i^{\rm c} < \tau_{0i}$, where $\tau_{0i} = \theta_i/\omega_i$ with $0 \le \theta_i < 2\pi$ satisfying $\cos(\theta_i) = \alpha_i/\omega_i^2$, $\sin(\theta_i) = \beta_i/\omega_i$, $\omega_i = \sqrt{(\beta_i^2 + \sqrt{\beta_i^4 + 4\alpha_i^2})/2}$.

It can be seen that the allowable upper bound of $\tau_i^u$ is determined by $\alpha_i$ and $\beta_i$. Therefore, agent $i$ can change $\alpha_i$ and $\beta_i$ to improve the robustness against the input delay. On the other hand, if $\tau_i^u$ is bigger than the allowable bound, the left agents can also achieve consensus. This is because the reference models run dependently beside the plants of the agents, so whether consensus can be achieved among the reference models is not affected by the plants. And the left agents can achieve consensus by tracking their reference models. This will be illustrated in an example in the next section.

## 4. Numerical simulations

This section gives simulation results to illustrate the theoretical results in the last section. Consider a group of 4 agents. The Laplacian matrix is chosen as

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}. \quad (32)$$

For Algorithm 1, let $\alpha_1 = -1$, $\beta_1 = -1$, $\alpha_2 = -1.2$, $\beta_2 = -1.2$, $\alpha_3 = -1.2$, $\beta_3 = -1.1$, $\alpha_4 = -1.1$, $\beta_4 = -1.2$. For Algorithm 3, let $\tau_i^u = 0.3$, $i = 1, \ldots, 4$. $\tau_{ij}^{uc} = 1$, $i = 1 \le i < j \le 4$. Figures 2 and 3 show the convergence result of agents 1–4 under Algorithms 1 and 3, respectively.

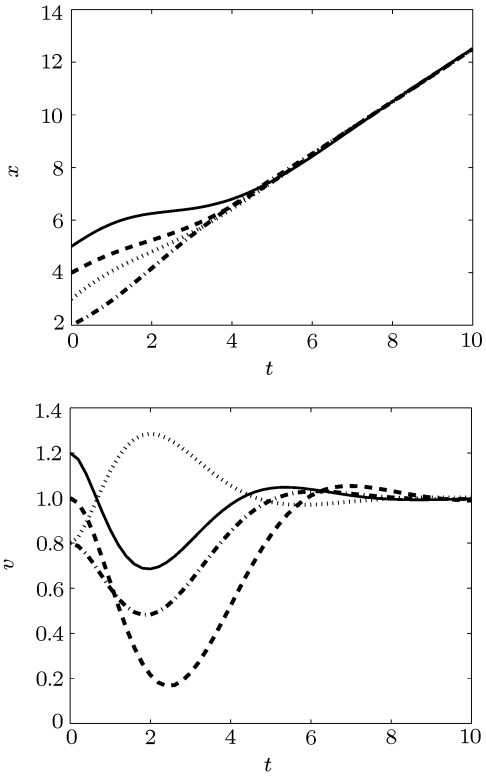



**Fig. 2.** Convergence result of agents 1–4 under Algorithm 1.

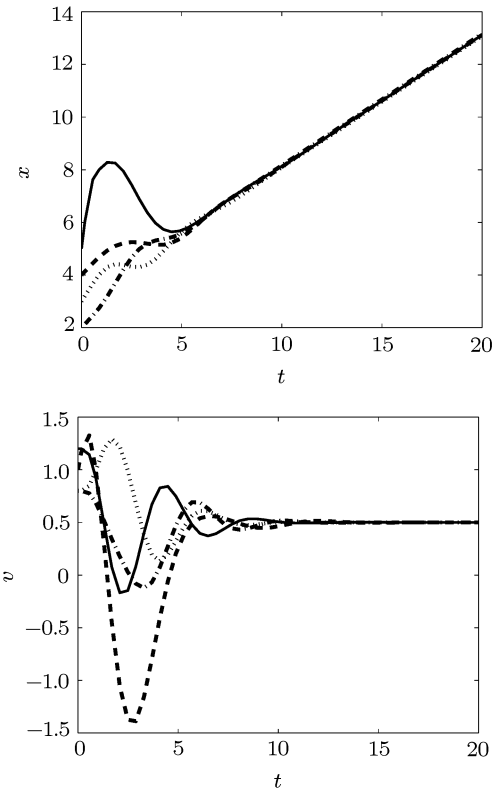



**Fig. 3.** Convergence result of agents 1–4 under Algorithm 3 with $\tau_i^u = 0.3$, $i = 1, 2, 3, 4$.

For Algorithm 2, let $\alpha_i$ and $\beta_i$ be the same as stated above, and $\varepsilon_i = \alpha_i$, $\delta_i = \beta_i$, $i = 1, \ldots, 4$. Figure 4 shows the convergence result of agents 1–4 under Algorithm 2. It can be seen that consensus is achieved in each case.

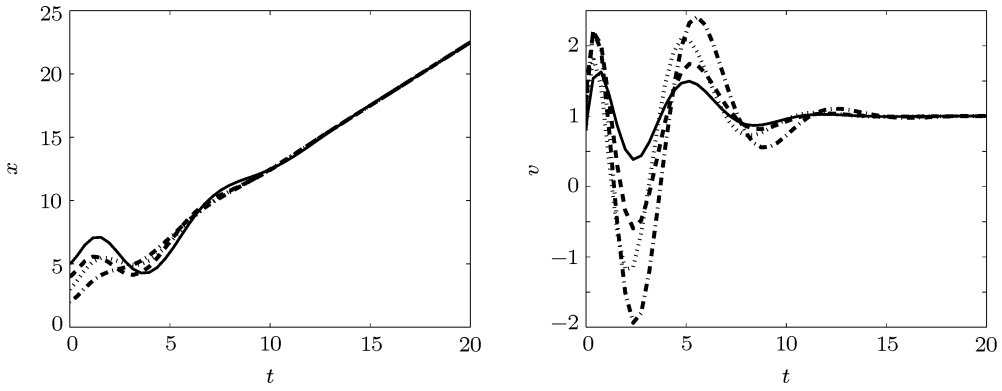



**Fig. 4.** Convergence result of agents 1–4 under Algorithm 3.

If $\tau_i^u = 0.3$, $i = 1, 2, 4$, $\tau_3^u = 0.8$, it can be seen that $\tau_3^u > \tau_{03}$. Figure 5 shows the convergence result of agents 1–4 under Algorithm 3, where the dot lines denote the state and velocity of agent 3. It can be seen that agents 1, 2 and 4 can reach consensus.

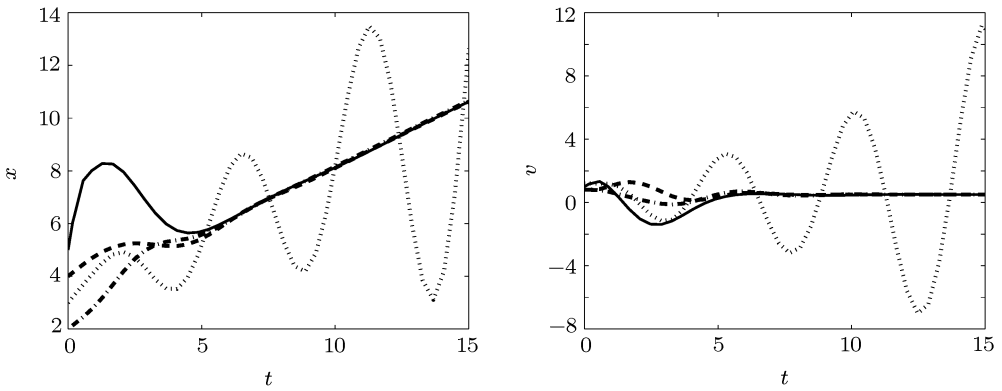



**Fig. 5.** Convergence result of agents 1–4 under Algorithm 3 with $\tau_1^u = \tau_2^u = \tau_4^u = 0.3$, $\tau_3^u = 0.8$.

## 5. Conclusions

In this paper, the consensus problem of multi-agent systems with second-order dynamics has been studied. A reference model based on consensus algorithm is proposed with sufficient conditions under which consensus can be achieved. A consensus algorithm without velocity measurement is also given. Different from most of existing results, the parameters in the protocols are different among agents. We have shown that consensus can be achieved under these protocols if the communication graph has a spanning tree. The effectiveness of the input delay and the communication delay has also been considered. Numerical simulations illustrate the proposed results.

## References

[1] Vicsek T, Czirok A, Jacob E B, Cohen I and Schochet O 1995 *Phys. Rev. Lett.* **75** 1226

[2] Olfati-Saber R 2006 *IEEE Trans. Automat. Control* **51** 401

[3] Fax J A and Murray R M 2004 *IEEE Trans. Automat. Control* **49** 1465

[4] Su H, Wang X and Lin Z 2009 *Automatica* **45** 2286
[5] Su H, Wang X and Chen G 2010 *Syst. Control Lett.* **59** 313
[6] Olfati-Saber R, Fax J A and Murray R M 2007 *Proc. IEEE* **95** 215
[7] Ren W, Beard R W and Atkins E M 2007 *IEEE Control Syst. Mag.* **27** 71
[8] Jadbabaie A, Lin J and Morse A S 2003 *IEEE Trans. Automat. Control* **48** 9880
[9] Olfati-Saber R and Murray R M 2004 *IEEE Trans. Automat. Control* **49** 1520
[10] Ren W and Beard R W 2005 *IEEE Trans. Automat. Control* **50** 655
[11] Chen F, Chena Z, Xiang L, Liu Z and Yuan Z 2009 *Automatica* **45** 1215
[12] Moreau L 2005 *IEEE Trans. Automat. Control* **50** 169
[13] Liu Y, Zhu J and Ding D 2009 *J. Shanghai Jiaotong University* **14** 321
[14] Xiao F and Wang L 2008 *IEEE Trans. Automat. Control* **53** 1804
[15] Hu J and Yuan H 2009 *Chin. Phys.* B **18** 3777
[16] Carli R, Bullo F and Zampieri S 2010 *Int. J. Robust Nonlinear Control* **20** 156
[17] Li T, Fu M, Xie L and Zhang J *IEEE Trans. Automat. Control* (in press)
[18] Li T and Zhang J 2010 *IEEE Trans. Automat. Control* (in press)
[19] Ren W and Atkins E 2007 *Int. J. Robust Nonlinear Control* **17** 1002
[20] Li H, Lin P and Zhang C 2008 *Chin. Phys.* B **17** 4458
[21] Zhang W, Zeng D and Guo Z 2010 *Chin. Phys.* B **19** 070518
[22] Lin P and Jia Y 2009 *Automatica* **45** 2154
[23] Yu W, Chen G and Cao M 2010 *Automatica* **46** 1089
[24] Liu H, Xie G and Wang L *Int. J. Robust Nonlinear Control* (in press)
[25] Lin P and Jia Y 2009 *IET Control Theory and Appllcations* **3** 957
[26] Gao Y, Wang L and Jia Y 2009 *Proceeding of American Control Conference*, June, 2009 p. 4464 (St, Louis, MO, USA) IEEE