Performance characteristics and optimal analysis of a nonlinear diode refrigerator*

Wang Xiu-Mei(王秀梅)[a], He Ji-Zhou(何济洲)[a]†, and Liang Hong-Ni(梁红妮)[b]

[a] *Department of Physics, Nanchang University, Nanchang* 330031, *China*
[b] *CALIS Information Service Center of Jiangxi Province, Nanchang University, Nanchang* 330031, *China*

(Received 7 July 2010; revised manuscript received 19 August 2010)

This paper establishes a model of a nonlinear diode refrigerator consisting of two diodes switched in the opposite directions and located in two heat reservoirs with different temperatures. Based on the theory of thermal fluctuations, the expressions of the heat flux absorbed from the heat reservoirs are derived. After the heat leak between the two reservoirs is considered, the cooling rate and the coefficient of performance are obtained analytically. The influence of the heat leak and the temperature ratio on the performance characteristics of the refrigerator is analysed in detail.

Keywords: nonlinear diode refrigerator, thermal fluctuations, heat leak, performance characteristics

PACS: 05.40.–a, 05.70.Ln, 07.20.–n **DOI:** 10.1088/1674-1056/20/2/020503

1. Introduction

As the size of a system approaches that of molecules, thermal fluctuations begin to play a significant role. It would be rather difficult to operate a nanoscale machine against strong thermal fluctuations. Instead, the nanomachine must be able to work harmoniously or collaboratively with the fluctuations. In order to design such machines, taking into account large thermal fluctuations, we need to understand thermodynamics of small systems. Recently, the study of microscopic heat engines and refrigerators rectifying thermal fluctuations has attracted considerable attention.[1−12] One typical model of the microscopic engines rectifying thermal fluctuations is Feynman's ratchet and pawl engine. Feynman *et al.* attempted to investigate the thermodynamics of the Feynman–Smoluchowski (FS) motor and concluded that it can reach Carnot efficiency.[13] But later it was shown that the heat transfer between two heat reservoirs never ceases even when the motor moves quasistatically due to the effect of thermal fluctuations. Thus it is not possible to reach the Carnot efficiency.[14−16] When the heat leak between two heat reservoirs was identified as an unavoidable source of irreversibility, Velasco *et al.* analysed how this irreversibility affected the performance of the FS motor and found more realistic bounds for its efficiency based on the finite-time thermodynamics (FTT).[17] The thermodynamic performance of similar microscopic heat engines and refrigerators was investigated.[18−22]

Corresponding to the model of Feynman's ratchet and pawl engine, a diode engine, consisting of a diode and a linear resistor kept at different temperatures always cannot reach the Carnot efficiency.[23] But in the model with two diodes switched in the opposite directions, it was shown that the Carnot efficiency can be achieved when backward resistances of the diodes tend to infinity.[24] Moreover, Salas *et al.* investigated and analysed the behaviour of the power-efficiency curves of the diode engine in the regimes of maximum power and maximum efficiency.[25] It was found that the power-efficiency curves are loop shapes in general case, similar to those in real heat engines.[26] In this paper we will extend the above-mentioned work of the diode engine to a nonlinear diode refrigerator, namely, the reverse process of the diode engine. The comparison of the results here presented for the cooling rate and the coefficient of performance (COP) and those reported for classical and real refrigerators seems to show that some qualitative behaviours are similar for classical and real refrigerators.[27]

This paper is arranged as follows. In Section 2, we introduce the model of the nonlinear diode refrigerator after the heat leak between the two heat reservoirs is considered. Based on the theory of thermal fluctuations, the expressions of heat flux absorbed from two reservoirs are derived. The cooling rate and the

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10765004 and 11065008).
†Corresponding author. E-mail: hjzhou@ncu.edu.cn
© 2011 Chinese Physical Society and IOP Publishing Ltd

coefficient of performance (COP) are obtained analytically. In Section 3, the performance characteristics of the diode refrigerator are analysed in general case. The optimal criteria of some performance parameters are determined. In Section 4, the performance characteristics are discussed in ideal case. In Section 5, the influence of the temperature ratio on the maximum cooling rate and the maximum COP is analysed. Lastly, we summarize and conclude in Section 6.

2. Model and theory

The model of the diode refrigerator is presented in Fig. 1. It is made up of two identical and nonlinear diodes D_1 and D_2, switched in the arbitrary opposite directions and kept at different temperatures T_1 and T_2 ($T_1 > T_2$) respectively. The two diodes are parallel to a capacitor C. If u is the voltage of the capacitor, $p(u)$ is probability distribution of the voltage u of the capacitor, i_1 and i_2 are currents through two nonlinear resistors, then

$$i_1 R_1(u) + u = \nu_1(t) \quad \text{and} \quad i_2 R_2(u) + u = \nu_2(t), \tag{1}$$

where $\nu_1(t)$ and $\nu_2(t)$ are Nyquist stochastic electromotive forces. Due to thermal agitation in the corresponding resistors, we have

$$\langle \nu_i(t) \rangle = 0, \quad \langle \nu_i(t)\nu_j(t') \rangle = 2kT_i R_i \delta_{ij} \delta(t - t').$$

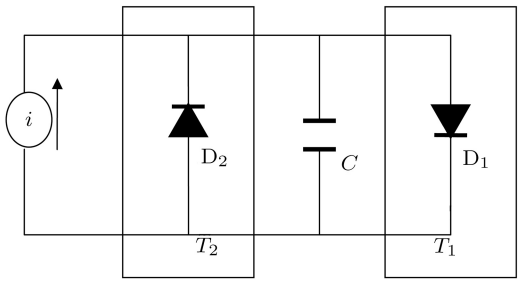


Fig. 1. Schematic diagram of a diode refrigerator: two diodes D_1 and D_2 at different temperatures T_1 and T_2 ($T_1 > T_2$), and a capacitor C on which an ideal current generator is supplied to do work.

But $i_1 + i_2 = \dot{q}$, where $q = Cu$ is the charge of the capacitor with capacitance C. Therefore, equation (1) indicates the following Langevin equation

$$\dot{u} = -\left[\left(\frac{1}{R_1(u)C} + \frac{1}{R_2(u)C}\right)u\right] + \frac{\nu_1(t)}{R_1(u)C} + \frac{\nu_2(t)}{R_2(u)C},$$

the corresponding Fokker–Planck equation is given by

$$\frac{\partial p(u)}{\partial t} = \frac{\partial}{\partial u}\left\{\left[\left(\frac{1}{R_1(u)C} + \frac{1}{R_2(u)C}\right)u\right]p(u) + \left(\frac{kT_1}{R_1(u)C^2} + \frac{kT_2}{R_2(u)C^2}\right)\frac{\partial p(u)}{\partial u}\right\}. \tag{2}$$

Additionally, when an ideal current generator i is applied to the system, it takes i unit charges per unit time from the upper plate of the capacitor and puts them back to its lower plate. This corresponds to an additional systematic drift i, therefore the Fokker–Planck equation is changed as[24]

$$\frac{\partial p(u)}{\partial t} = \frac{\partial}{\partial u}\left\{\left[\left(\frac{1}{R_1(u)C} + \frac{1}{R_2(u)C}\right)u - \frac{i}{C}\right]p(u) + \left(\frac{kT_1}{R_1(u)C^2} + \frac{kT_2}{R_2(u)C^2}\right)\frac{\partial p(u)}{\partial u}\right\}. \tag{3}$$

The stationary solution of Eq. (2) is

$$p(u) = A\exp\left(-\int \mathrm{d}u\left[\frac{(1/R_1(u) + 1/R_2(u))\,u - i}{kT_1/R_1(u)C + kT_2/R_2(u)C}\right]\right), \tag{4}$$

where A is the normalization constant. Once $p(u)$ is known, it is straightforward to obtain other thermodynamic quantities of the system. The heat flux absorbed from the heat reservoir at temperature T_j is

$$\dot{Q}_j = -\int_{-\infty}^{\infty} u\left[\frac{kT_j}{R_j(u)C}\frac{\partial p(u)}{\partial u} + \frac{u}{R_j(u)}p(u)\right]\mathrm{d}u. \tag{5}$$

The power input is

$$P = \mathrm{i}\int_{-\infty}^{+\infty} u\,p(u)\,\mathrm{d}u. \tag{6}$$

We assume the following piecewise-linear volt–ampere characteristics

$$R_j(u) = R_j^+\theta(u) + R_j^-\theta(u), \quad j = 1, 2, \tag{7}$$

where $\theta(u)$ is the step-wise function. Substituting Eq. (7) into Eq. (4), we obtain

$$p(u) = A\begin{cases} \exp\left[-(\alpha u^2 + \beta u)\right], & u > 0, \\ \exp\left[-(\alpha' u^2 + \beta' u)\right], & u < 0, \end{cases} \tag{8}$$

where

$$\alpha = \frac{1}{2}(R_1^+ + R_2^+)/(\bar{T}_1 R_2^+ + \bar{T}_2 R_1^+),$$
$$\alpha' = \frac{1}{2}(R_1^- + R_2^-)/(\bar{T}_1 R_2^- + \bar{T}_2 R_1^-),$$
$$\beta = -(\mathrm{i}R_2^+ R_1^+)/(\bar{T}_1 R_2^+ + \bar{T}_2 R_1^+),$$
$$\beta' = -(\mathrm{i}R_2^- R_1^-)(\bar{T}_1 R_2^- + \bar{T}_2 R_1^-)$$

and $\bar{T}_j = (kT_j)/(C)$.

Substituting Eq. (8) into Eq. (5), we obtain the heat flux absorbed from the heat reservoir at temperature T_j,

$$\begin{aligned} A^{-1}\dot{Q}_j =& C_1^j \left\{ \sqrt{\pi} \mathrm{e}^{\beta'^2/4\alpha'} \left[1 + \frac{\beta'^2}{2\alpha'}\right] \left[1 + \mathrm{erf}\Big(\frac{\beta'}{2\sqrt{\alpha'}}\Big)\right] + \frac{\beta'}{\sqrt{\alpha'}} \right\} - C_2^j \left\{ \sqrt{\frac{\pi}{2}} \frac{\beta'}{2\alpha'} \mathrm{e}^{\beta'^2/4\alpha'} \left[1 + \mathrm{erf}\Big(\frac{\beta'}{2\sqrt{\alpha'}}\Big)\right] + 1 \right\} \\ &+ C_3^j \left\{ \sqrt{\pi} \mathrm{e}^{\beta^2/4\alpha} \left[1 + \frac{\beta^2}{2\alpha}\right] \left[1 - \mathrm{erf}\Big(\frac{\beta}{2\sqrt{\alpha}}\Big)\right] - \frac{\beta}{\sqrt{\alpha}} \right\} \left(\frac{\alpha'}{\alpha}\right)^{3/2} \\ &+ C_4^j \left\{ 1 - \sqrt{\frac{\pi}{2}} \frac{\beta}{\sqrt{2\alpha}} \mathrm{e}^{\beta^2/4\alpha} \left[1 - \mathrm{erf}\Big(\frac{\beta}{2\sqrt{\alpha}}\Big)\right] \right\} \frac{\alpha'}{\alpha}, \end{aligned} \tag{9}$$

where $\mathrm{erf}(z) = \int_0^z \mathrm{e}^{y^2} \mathrm{d}y$ is the error function,

$$A^{-1} = \frac{1}{\sqrt{2\alpha'}} \sqrt{\frac{\pi}{2}} \left\{ \sqrt{\frac{\alpha'}{\alpha}} e^{\beta^2/4\alpha} \left[1 - \mathrm{erf}\Big(\frac{\beta}{2\sqrt{\alpha}}\Big)\right] + \mathrm{e}^{\beta'^2/4\alpha'} \left[1 + \mathrm{erf}\Big(\frac{\beta'}{2\sqrt{\alpha'}}\Big)\right] \right\},$$

and

$$C_1^j = \frac{1}{\sqrt{2\alpha'}} \frac{2\bar{T}_j\alpha' - 1}{2\sqrt{2}\alpha' R_j^-},$$
$$C_2^j = \frac{1}{\sqrt{2\alpha'}} \frac{\bar{T}_j\beta'}{\sqrt{2\alpha'} R_j^-},$$
$$C_3^j = \frac{1}{\sqrt{2\alpha'}} \frac{2\bar{T}_j\alpha - 1}{2\sqrt{2}\alpha' R_j^+},$$
$$C_4^j = \frac{1}{\sqrt{2\alpha'}} \frac{\bar{T}_j\beta}{\sqrt{2\alpha'} R_j^+}.$$

Due to the electrical link between the two heat reservoirs, the heat leak induced by temperature difference occurs. We assume that the heat leak obeys the Newton linear heat transfer law, i.e.

$$\dot{Q}_\mathrm{L} = \sigma(T_1 - T_2), \tag{10}$$

where σ is the heat conductance. Therefore, the cooling rate and the COP can be obtained by

$$\dot{Q}_\mathrm{C} = \dot{Q}_2 - \lambda(\bar{T}_1 - \bar{T}_2), \tag{11}$$

and

$$\varepsilon = \frac{\dot{Q}_2 - \lambda(\bar{T}_1 - \bar{T}_2)}{\left|\dot{Q}_1 + \dot{Q}_2\right|}, \tag{12}$$

respectively, where $\lambda = \sigma C/k$ is called irreversible parameter induced by the heat leak between the two heat reservoirs. Obviously, the cooling rate and the COP closely depend on the parameters $R_j^\pm$, $\bar{T}_j$ and λ.

3. Performance characteristics in general case

In order to generate performance characteristic curves of the refrigerator in the form of numerical simulation, we can take the current i as a dimensionless independent variable and R^+, R^-, $\bar{T}_1$, $\bar{T}_2$ and λ as a set of dimensionless external parameters. Let $R_1^+ = R_2^- = R^+$ and $R_1^- = R_2^+ = R^-$ for the two identical diodes switched in opposite directions and the temperature ratio of the two reservoirs $\tau = \bar{T}_2/\bar{T}_1$.

According to Eq. (11), the curves of the dimensionless cooling rate $\dot{Q}_\mathrm{C}$ versus the dimensionless current i are plotted at different irreversible parameter λ for given parameters $R^+ = 1$, $R^- = 100$, $\bar{T}_1 = 2$ and $\tau = 0.5$, as shown in Fig. 2. It is shown that the cooling rate $\dot{Q}_\mathrm{C}$ increases firstly and then decreases as i increases and presents a maximum value at special current i_Q. The maximum $\dot{Q}_\mathrm{m}$ decreases but the corresponding current i_Q increases slightly as λ increases. The initial value of current i is bigger than zero.

According to Eq. (12), the curves of the COP ε versus the dimensionless current i are plotted at different irreversible parameter λ for given parameters $R^+ = 1$, $R^- = 100$, $\bar{T}_1 = 2$ and $\tau = 0.5$, as shown in Fig. 3. It can be seen that the COP ε is not a monotonous function of i. There exists a maximum value at special current i_ε. The maximum COP ε_m decreases while the corresponding current i_ε increases

as λ increases. Obviously, in this case, the maximum COP ε_m is far lower than Carnot value $\varepsilon_{\rm C} = \tau/(1-\tau)$ even if $\lambda = 0$.

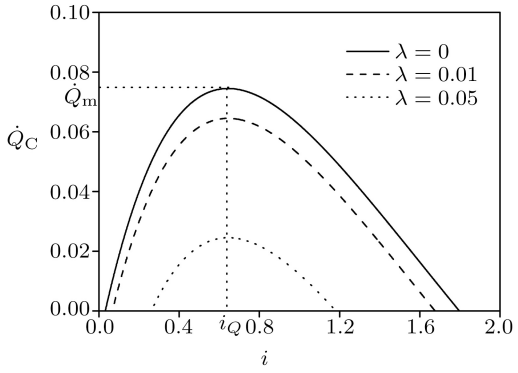


Fig. 2. The dimensionless cooling rate $\dot{Q}_{\rm C}$ versus the dimensionless current i for $R^+ = 1$, $R^- = 100$, $\bar{T}_1 = 2$ and $\tau = 0.5$.

Furthermore, the performance characteristic curves of the dimensionless cooling rate $\dot{Q}_{\rm C}$ versus the COP ε are plotted at different irreversible parameter λ for $R^+ = 1$, $R^- = 100$, $\bar{T}_1 = 2$ and $\tau = 0.5$, as shown in Fig. 4. It can be found that all curves are loop-shaped ones. The maximum cooling rate $\dot{Q}_{\rm m}$ and the corresponding COP ε_Q, as well as the maximum $\varepsilon_{\rm m}$ and the corresponding $\dot{Q}_\varepsilon$ are observed here. For a refrigerator, we always want to obtain the cooling rate and the COP as large as possible. From Fig. 4, we find that the cooling rate should be located in the range: $\dot{Q}_\varepsilon < \dot{Q}_{\rm C} < \dot{Q}_{\rm m}$ and the COP should be located in the range: $\varepsilon_Q < \varepsilon < \varepsilon_{\rm m}$. These two ranges make a region with a negative slope, which is referred to as the optimal region of the refrigerator. The $\dot{Q}_\varepsilon$ and $\dot{Q}_{\rm m}$ as well as $\varepsilon_{\rm Q}$ and $\varepsilon_{\rm m}$ are four important performance parameters which respectively determine the lower and upper bounds of the optimal cooling rate and the optimal COP.

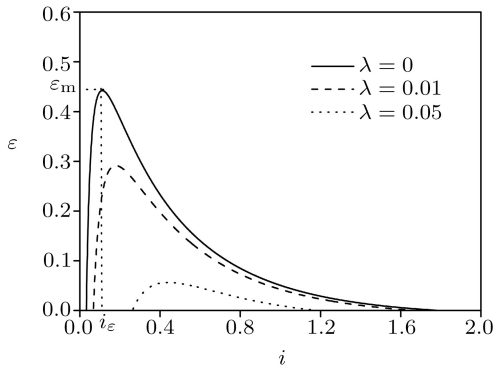


Fig. 3. The COP ε versus the dimensionless current i with the same parameters given in Fig. 2.

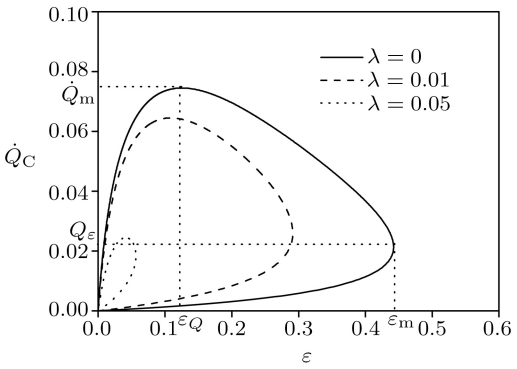


Fig. 4. The dimensionless cooling rate $\dot{Q}_C$ versus the COP ε with the same parameters given in Fig. 2.

4. Performance characteristics in ideal case

Especially, for given parameters $R^+ = 1$, $R^- \to \infty$, $\bar{T}_1 = 2$ and $\tau = 0.5$, using Eqs. (11) and (12), the curves of the dimensionless cooling rate $\dot{Q}_{\rm C}$ and the COP ε of the ideal diode refrigerator versus the dimensionless current i at different irreversible parameter λ are respectively plotted in Figs. 5 and 6. In Fig. 5, it can be seen that the cooling rate $\dot{Q}_{\rm C}$ is far bigger than that in general case and the initial current is zero when $\lambda = 0$. The cooling rate $\dot{Q}_{\rm C}$ decreases as λ increases.

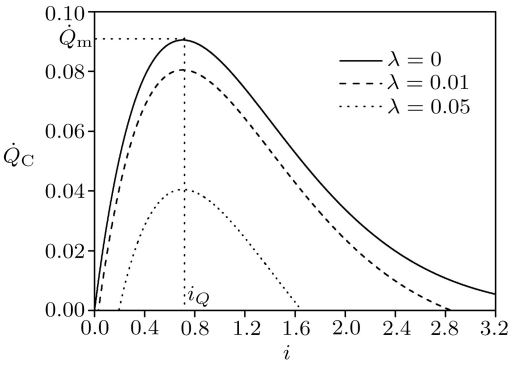


Fig. 5. The dimensionless cooling rate $\dot{Q}_{\rm C}$ versus the dimensionless current i for $R^+ = 1$, $R^- \to \infty$, $\bar{T}_1 = 2$ and $\tau = 0.5$.

In Fig. 6, it can be seen that if $\lambda = 0$, the maximum COP can attain Carnot value, i.e. $\varepsilon_{\rm m} = \varepsilon_{\rm C}$ under no current. The COP ε also decreases as λ increases. Similarly, the curves of the dimensionless cooling rate $\dot{Q}_{\rm C}$ versus the COP ε at different irreversible parameter λ are plotted in Fig. 7. It is obviously found that when $\lambda = 0$, the curve is an opened-loop one. The maximum $\varepsilon_{\rm m}$ equals Carnot value $\varepsilon_{\rm C}$ and the corresponding cooling rate $\dot{Q}_\varepsilon$ is zero. With the same

irreversible parameter λ, the optimal region of the refrigerator with infinite backward resistance is bigger than that with finite backward resistance. The bigger λ is, the smaller the optimal region is.

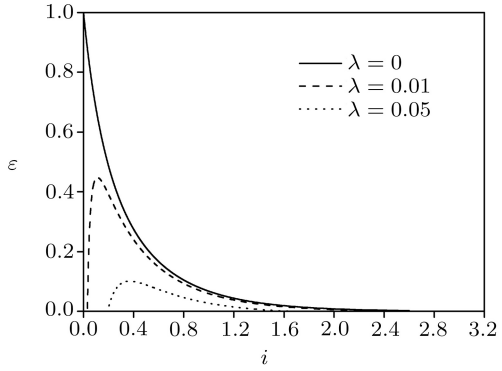


Fig. 6. The COP ε versus the dimensionless current i with the same parameters given in Fig. 5.

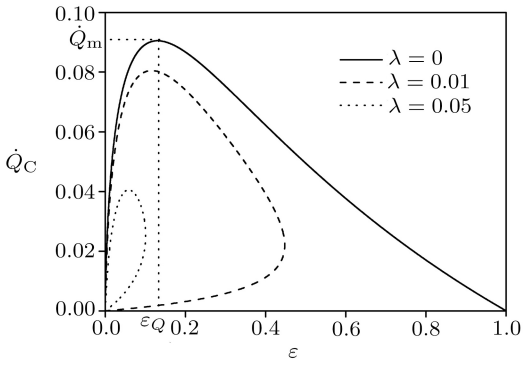


Fig. 7. The dimensionless cooling rate $\dot{Q}_{\rm C}$ versus the COP ε with the same parameters given in Fig. 5.

5. The influence of the temperature ratio on the optimal performance parameters

The influence of the temperature ratio τ on the optimal performance parameters $\dot{Q}_{\rm m}$, $\dot{Q}_\varepsilon$, $\varepsilon_{\rm m}$ and ε_Q is analysed for given parameters $R^+ = 1$, $R^- \to \infty$, $\lambda = 0.01$ and $\bar{T}_1 = 2$. The curves of the optimal performance parameters versus τ are plotted by numerical simulation, as shown in Fig. 8. It is found that the maximum cooling rate $\dot{Q}_{\rm m}$ is a monotonous function of τ, while the cooling rate $\dot{Q}_\varepsilon$ at the maximum COP ε_m is not a monotonous function of τ. The maximum cooling rate $\dot{Q}_{\rm m}$ lineally increases as τ increases, while the cooling rate $\dot{Q}_\varepsilon$ at maximum COP $\varepsilon_{\rm m}$ increases firstly and then decreases till to zero at $\tau = 1$. It also can be seen that the initial value of the temperature ratio is much bigger than zero, which implies that the refrigerator works within a temperature range. On the other hand, both maximum COP $\varepsilon_{\rm m}$ and COP ε_Q at maximum cooling rate $\dot{Q}_{\rm m}$ increase as τ increases. The maximum COP ε_m increases fast as τ increases. When $\tau = 1$, $\varepsilon_{\rm m} \to \infty$. While the COP ε_Q at maximum cooling rate $\dot{Q}_{\rm m}$ increases slowly.

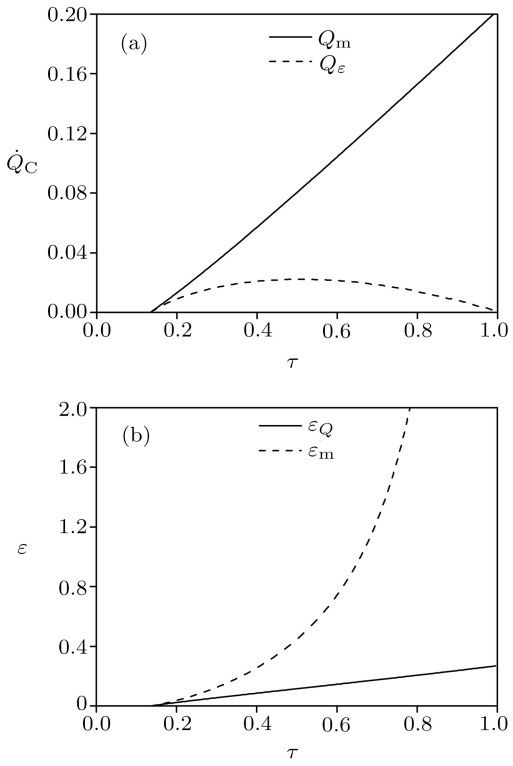


Fig. 8. (a) The $\dot{Q}_{\rm m}$ (full curve) and $\dot{Q}_\varepsilon$ (dashed curve) versus temperature ratio τ; (b) ε_Q (full curve) and ε_m (dashed curve) versus the temperature ratio τ for $R^+ = 1$, $R^- \to \infty$, $\lambda = 0.01$ and $\bar{T}_1 = 2$.

6. Conclusions

The performance characteristics of the diode refrigerator have been investigated based on the theory of thermal fluctuations. By using numerical calculation, the various performance characteristic curves are plotted. The influence of the heat leak and the temperature ratio of two reservoirs on performance characteristics of the refrigerator has been analysed in detail. The major results are concluded as follows: (i) The cooling rate and the COP decrease as the heat leak increases. The characteristic curves of the cooling rate versus the COP are loop-shaped ones, similar to that of real irreversible refrigerators.[27] (ii) The optimal performance parameters, such as the maximum cooling rate, the maximum COP, and the cor-

responding COP at the maximum cooling rate, are monotonous functions of the temperature ratio. But the corresponding cooling rate at the maximum COP is a non-monotonous function of the temperature ratio. (iii) When the backward resistance increases up to infinity and the heat leak is neglected, the COP of the refrigerator can reach Carnot value $\varepsilon_{\rm C}$. The characteristic curve of the cooling rate versus the COP is an open-shaped one similar to that for an endoreversible refrigerator.[28] In general, the diode refrigerator is always irreversible and its COP is less than the Carnot value.

References

[1] Reimann P and Häanggi P 2002 *Appl. Phys.* A: *Mater. Sci. & Proc.* **75** 169

[2] Parrondo J M R and De Cisneros B J 2002 *Appl. Phys.* A: *Mater. Sci. & Proc.* **75** 179

[3] Rozenbaum V M, Korochkova T Ye and Liang K K 2007 *Phys. Rev.* E **75** 06111

[4] Ai B Q, Wang L Q and Liu L G 2006 *Phys. Lett.* A **352** 286

[5] Ai B Q, Xie H Z, Wen D H, Liu X M and Liu L G 2005 *Euro. Phys. J.* B **48** 101

[6] Zhang Y, Lin B H and Chen J C 2006 *Euro. Phys. J.* B **53** 481

[7] Lin B H and Chen J C 2009 *J. Phys.* A*: Math. Theor.* **42** 075006

[8] Gao T F, Zhang Y and Chen J C 2009 *Chin. Phys.* B **18** 3279

[9] Liu J L and He J Z 2010 *Chin. Phys.* B **19** 030504

[10] Li D, Zheng W W and Wang Z S 2008 *Chin. Phys.* B **17** 1916

[11] He D H, Yang T, Li W H, Zhang Q L and Ma H R 2007 *Chin. Phys.* **16** 3138

[12] Lin F and Bao J D 2008 *Acta Phys. Sin.* **57** 696 (in Chinese)

[13] Feynman R P, Leighton R B and Sands M 1966 *The Feynman Lectures on Physics* (Reading MA: Addison-Wesley)

[14] Parrondo J M R and Español P 1996 *Am. J. Phys.* **64** 112

[15] Sekimoto K 1997 *J. Phys. Soc. Jpn.* **66** 1234

[16] Hondou T and Takagi F 1998 *J. Phys. Soc. Jpn.* **67** 2974

[17] Velasco S, Roco J M M, Medina A and Calvo Hernandez A 2001 *J. Phys.* D*: Appl. Phys.* **34** 1000

[18] Van den Broeck C, Kawai R and Meurs P 2004 *Phys. Rev. Lett.* **93** 090601

[19] Van den Broeck C, Kestemont E and Malek Mansour M 2001 *Europhys. Lett.* **56** 771

[20] Van den Broeck C and Kawai R 2006 *Phys. Rev. Lett.* **96** 210601

[21] Nakagawa N and Komatsu T S 2006 *Europhys. Lett.* **75** 22

[22] Benjamin R and Kawai R 2008 *Phys. Rev.* E **77** 051132

[23] Sokolov I M 1998 *Europhys. Lett.* **44** 278

[24] Sokolov I M 1999 *Phys. Rev.* E **60** 4946

[25] Sanchez Salas N and Calvo Hernandez A 2002 *J. Phys.* D: *Appl. Phys.* **35** 1442

[26] Gordon J M and Huleihil M 1992 *J. Appl. Phys.* **72** 829

[27] Chen J C and Yan Z J 1996 *J. Phys.* D: *Appl. Phys.* **29** 987

[28] Karl H H, Josef M B and Sven S 1997 *J. Non-equilib. Thermodyn.* **22** 311