# Effect of a localized impurity on soliton dynamics in the Bose–Einstein condensates*

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By using a multiple-scale method, we analytically study the effect of a localized impurity on the soliton dynamics in the Bose–Einstein condensates. It is shown that a dark soliton can be transmitted through a repulsive (or attractive) impurity, while at the position of the localized impurity the soliton can be quasitrapped by the impurity. Additionally, we find that the strength of the localized impurity has an important effect on the dark soliton dynamics. With increasing strength of the localized impurity, the amplitude of the dark soliton becomes bigger, while its width is narrower, and the soliton propagates slower.

**Keywords:** Bose–Einstein condensates, solitons, localized impurity

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## 1. Introduction

Since the experimental discovery of Bose–Einstein condensate (BEC) in dilute atomic alkali-metal gases, this opens the promising perspective for numerous applications of the nonlinear matter-wave physics.[1,2] A relevant interesting issue in the content is to learn how to control the motion of nonlinear excitations of the condensate.[1,2] In the realistic experimental setting, an external electromagnetic field is used to produce, trap, and manipulate the BEC.[1,2] Experimentally there exist several quantum phase-engineering techniques to generate dark solitons in a BEC.[3] Dark solitons can be excited in the BEC with repulsive interactions; such a soliton is characterized by a notch in the BEC density profile and a phase jump across its localized region.[3] Theoretically, the soliton dynamics in a trapping potential has been studied extensively.[4−25]

By far, related studies were focused on how one could influence or even guide the motion of the solitons. Particularly interesting in this content is that impurities could be used as elements of a matter-wave device that controls the motion of a network of nonlinear excitations.[3] The dynamics of the BEC in the presence of impurities has recently become a subject of growing interest.[26] The travelling impurity induces a BEC dynamics that leads to the expulsion of the impurity from BEC.[26] By investigating the interaction between a soliton and a single impurity, Frantzeskakais *et al.*[27] demonstrated that an immobile soliton may be captured and dragged by an adiabatically moving attractive impurity. However, the effect of a localized impurity on soliton dynamics in the BEC has not been studied in detail yet.

The purpose of this paper is twofold. First, we derive an effective equation for the motion of a dark soliton in a trapping potential, and in the presence of either repulsive or attractive impurities. Second, we study the effect of a localized impurity on soliton dynamics in BECs. It is shown that with increasing strength of the localized impurity, the amplitude of the soliton becomes bigger, while its width is narrower, and the soliton propagates slower.

## 2. Asymptotic expansion and variable coefficient KdV equation

### 2.1. Hydrodynamical model

A convenient model to study the mean-field dynamics of BECs is the Gross–Pitaevskii (GP) equation which has the form of a (3+1)-dimensional non-

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linear Schrödinger (NLS) equation with an external trapping potential.[28] In the case when the confinement for two of the three spatial dimensions is much stronger than that in the third dimension, the GP equation can be reduced to an effective quasi-(1+1)-dimensional GP equation.[17,29,30] For repulsive interatomic interactions, the latter equation can be expressed in the following dimensionless form[17,29,30]

$$\mathrm{i}\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + V(x)\,\psi + g_{1\mathrm{D}}\,|\psi|^2\,\psi, \qquad (1)$$

where the spatial coordinate $x$ and time $t$ are normalized to the harmonic-oscillator length $a_0 = \sqrt{\hbar/m\omega_\perp}$ and oscillation period $1/\omega_\perp$, respectively. The frequency $\omega_\perp$ belongs to the two dimensions with strong confinement. Here $g_{1\mathrm{D}} = 2\,|a_\mathrm{s}|\,/a_0$, with $a_\mathrm{s}$ being the s-wave scattering length. The normalized field $\psi$ describes the macroscopic wave function $\Psi$ of the condensate, according to the following scaling relation $\Psi(x,t) = \sqrt{m\omega_\perp/\,(4\pi a_\mathrm{s}\hbar)}\,\psi\,(x,t)$.

In order to study the effect of a localized impurity on dark soliton of a BEC in the framework of Eq. (1), it is convenient to decompose the external potential $V(x)$ as follows:[27]

$$V(x) = U_\mathrm{tr}(x) + b\delta(x), \qquad (2)$$

where $U_\mathrm{tr}(x)$ is the (conventional parabolic) time-independent trapping potential, which is assumed to be smooth and slowly varying on the soliton scale, and the additional sharp potential $\delta(x)$ accounts for an impurity localized in space at the point $x = 0$, and it is described by a Dirac $\delta$ function. The parameter $b$ in Eq. (2) which measures the impurity strength is assumed to be small and may take either positive or negative values for repulsive or attractive impurities, respectively. We here illustrate the shape of an external potential with a localized impurity in Fig. 1. As discussed in Ref. [27], the harmonic trapping potential $U_\mathrm{tr}(x) = (kx)^2$, with $k = 0.05$; this value is approximately twice as large the one used in the experimental studies of BEC dark solitons in Ref. [3]. We remark that both for the illustrations [see Fig. 1] and the numerical calculations the $\delta$ function for the localized impurity at $x = 0$ has been replaced by the steep function $10b\cosh^{-1}(x/0.05)$ with $b = \pm 0.15$ for repulsive or attractive impurities, respectively. It is interesting to observe that the repulsive impurity [Fig. 1(a)] creates a hump in the potential, while the attractive one [Fig. 1(b)] creates a hole.

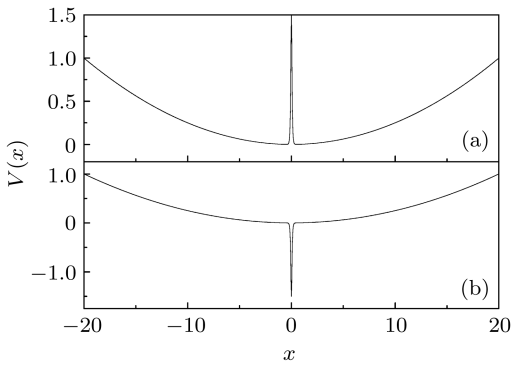



**Fig. 1.** An external potential is composed of a harmonic trapping potential $U_\mathrm{tr}(x) = (kx)^2$ with $k = 0.05$ and a localized impurity at $x = 0$ which is approximated by the steep function $10b\cosh^{-1}(x/0.05)$ with $b = \pm 0.15$. Both cases of repulsive (a) and attractive (b) impurities are shown respectively.

Expressing the wave function in terms of modulus and phase, we set $\psi\,(x,t) = A\,(x,t)\exp\left[-\mathrm{i}\mu t + \mathrm{i}\varphi\,(x,t)\right]$. Substituting it into Eq. (1) and then separating the real and imaginary parts, we obtain the following two equations

$$\frac{\partial A}{\partial t} + \frac{\partial A}{\partial x}\frac{\partial\varphi}{\partial x} + \frac{1}{2}A\frac{\partial^2\varphi}{\partial x^2} = 0, \qquad (3)$$

$$-\frac{1}{2}\frac{\partial^2 A}{\partial x^2} + \left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - \mu + \frac{\partial\varphi}{\partial t} + V(x)\right]A + g_{1\mathrm{D}}A^3 = 0. \qquad (4)$$

## 2.2. Linear stability condition of soliton formation

To obtain the effect of the localized impurity on the dynamics of the solitons in the condensates, we here study linear stability condition of soliton formation. Due to the strong confinement in the transverse direction, the system is similar to a wave guide, in which the excitation from ground state propagates in the elongated direction.[7,8] The strong confinement also ensures the dynamical stability of a soliton excitation.[7,8] Therefore, we set $A = u_0\,(x) + \alpha\,(x,t)$ (without loss of generality, we assume that $u_0\,(x)$, which characterizes the condensate background, is positive) with $\alpha\,(x,t) = \alpha_0\exp(\mathrm{i}\beta) + \text{c.c.}$ and $\varphi = \varphi_0\exp(\mathrm{i}\beta) + \text{c.c.}$ Here, c.c. is complex conjugate and $\beta = kx - \omega t$, where $k$ is wave number and $\omega$ indicates eigen-frequency. Considering that and $\varphi_0$ are small constants, we obtain

$$\mathrm{i}\omega\alpha_0 = \mathrm{i}k\varphi_0\frac{\partial u_0}{\partial x} - \frac{1}{2}k^2u_0\varphi_0, \qquad (5)$$

$$\mu\,u_0 = -\frac{1}{2}\frac{\partial^2}{\partial x^2}u_0 + V(x)\,u_0 + g_{1\mathrm{D}}u_0^3, \qquad (6)$$

$$\mu\,\alpha_0 = \frac{1}{2}k^2\alpha_0 - \mathrm{i}\omega\varphi_0 u_0 + V(x)\,\alpha_0 + 3g_{1\mathrm{D}}u_0^2\alpha_0, \quad (7)$$

from the linearization of Eqs. (3) and (4). For linear case, i.e., $g_{1\mathrm{D}} = 0$, equation (6) is turned into

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}u_0 + 2\left[\mu - U_{\mathrm{tr}}(x) - b\delta(x)\right]u_0 = 0. \quad (8)$$

Under the case of $g_{1\mathrm{D}} \neq 0$, and utilizing Eqs. (5), (6), and (7), we have

$$\omega^2 = \left(\frac{1}{2}k^2 + U_{\mathrm{tr}}(x) + b\delta(x) - \mu + 2g_{1\mathrm{D}}u_0^2\right) \times \left(\frac{1}{2}k^2 - \frac{\mathrm{i}k}{u_0}\frac{\partial u_0}{\partial x}\right). \quad (9)$$

Subsequently, we discuss stability problem of soliton formation. To demonstrate that the soliton is stable or unstable, we set $\omega = \omega_{\mathrm{r}} + \mathrm{i}\omega_{\mathrm{i}}$, where the subscripts denote the real and imaginary parts.[17] The main reason is that if $\omega_{\mathrm{i}} \neq 0$, it implies exponential growth of the mode and hence the state $\psi$ is in dynamical instability.[17] If the eigen-frequency is real, it means that the soliton in the BEC is stable. From Eq. (9), we see the dependence of the instability growth rate $\omega_{\mathrm{i}}$ on $k$ and $(1/u_0)\partial u_0/\partial x$. On the one hand, when $k = 0$, one finds $\omega = 0$, which is an inessential solution. On the other hand, we obtain that $(1/u_0)\partial u_0/\partial x = 0$. Combining Eq. (8) and Fig. 1, we find that at the localized impurity $x = 0$ and the region of the absence of localized impurity, the wave function $\psi$ possesses dynamical stability. We therefore conclude that there exhibit stable soliton at the localized impurity $x = 0$ and the region of the absence of the localized impurity.

## 2.3. Variable coefficient Korteweg–de Vries equation and its soliton solution

To understand the effect of the localized impurity on soliton dynamical characteristics in the BECs, we here make use of a multiple-scale method[31−33] to derive an explicit expression of the wave function of the BECs. First, we introduce that two "slow" variables $\xi = \varepsilon(x - V_{\mathrm{g}}t)$ and $\tau = \varepsilon^3 t$ characterize the slow variation of soliton dynamics and a "fast" variable $x_0 = x$ represents the direction of soliton propagation. Moreover, we assume that $A(x,t)$ and $\varphi(x,t)$ can be expanded into a polynomial form in terms of $\varepsilon$ in the forms of $A = u_0 + \varepsilon^2\left[a^{(0)}(\xi,\tau) + \varepsilon^2 a^{(1)}(\xi,\tau) + \cdots\right]$, and $\varphi = \varepsilon\left[\varphi^{(0)}(\xi,\tau) + \varepsilon^2\varphi^{(1)}(\xi,\tau) + \cdots\right]$, where $\varepsilon$ is a smallness parameter characterizing the relative amplitude of the excitation. Inserting them into Eqs. (3) and (4), and then separating each order in terms of the power of $\varepsilon$, respectively, from Eq. (3), we have

$$V_{\mathrm{g}}\frac{\partial a^{(0)}}{\partial\xi} - \frac{1}{2}u_0\frac{\partial^2\varphi^{(0)}}{\partial\xi^2} = 0, \quad (10)$$

$$V_{\mathrm{g}}\frac{\partial a^{(1)}}{\partial\xi} - \frac{1}{2}u_0\frac{\partial^2\varphi^{(1)}}{\partial\xi^2} = \frac{\partial a^{(0)}}{\partial\xi}\frac{\partial\varphi^{(0)}}{\partial\xi} + \frac{1}{2}a^{(0)}\frac{\partial^2\varphi^{(0)}}{\partial\xi^2} + \frac{\partial a^{(0)}}{\partial\tau}, \quad (11)$$

and from Eq. (4), we have

$$-\mu u_0 + V(x_0)\,u_0 + g_{1\mathrm{D}}u_0^3 = 0, \quad (12)$$

$$-\mu a^{(0)} + V(x_0)\,a^{(0)} + 3g_{1\mathrm{D}}u_0^2 a^{(0)} = V_{\mathrm{g}}u_0\frac{\partial\varphi^{(0)}}{\partial\xi}, \quad (13)$$

$$-\mu a^{(1)} + V(x_0)\,a^{(1)} + 3g_{1\mathrm{D}}u_0^2 a^{(1)} = \frac{1}{2}\frac{\partial^2 a^{(0)}}{\partial\xi^2} - \frac{1}{2}u_0\left(\frac{\partial\varphi^{(0)}}{\partial\xi}\right)^2 - 3g_{1\mathrm{D}}u_0\left(a^{(0)}\right)^2 + V_{\mathrm{g}}u_0\frac{\partial\varphi^{(1)}}{\partial\xi} + V_{\mathrm{g}}a^{(0)}\frac{\partial\varphi^{(0)}}{\partial\xi} - u_0\frac{\partial\varphi^{(0)}}{\partial\tau}. \quad (14)$$

From Eq. (12), we obtain that the chemical potential of the system is $\mu = U_{\mathrm{tr}}(x_0) + b\delta(x_0) + g_{1\mathrm{D}}u_0^2$. Comparing Eqs. (10) and (13), we gain $V_{\mathrm{g}} = \pm\sqrt{g_{1\mathrm{D}}}u_0$. Subsequently, from Eqs. (11) and (14), we obtain

$$\frac{\partial\varphi^{(0)}}{\partial\tau} + \frac{3}{4}\left(\frac{\partial\varphi^{(0)}}{\partial\xi}\right)^2 - \frac{1}{8V_{\mathrm{g}}}\frac{\partial^3\varphi^{(0)}}{\partial\xi^3} = 0. \quad (15)$$

Equation (15) can also be called variable coefficient Korteweg–de Vries (VCKdV) equation. Making transformation $\eta = \varepsilon\varphi^{(0)}$, $\xi = \varepsilon X = \varepsilon(x - V_{\mathrm{g}}t)$, and $\tau = \varepsilon^3 t$, equation (15) becomes

$$\frac{\partial\eta}{\partial t} + \frac{3}{4}\left(\frac{\partial\eta}{\partial X}\right)^2 - \frac{1}{8V_{\mathrm{g}}}\frac{\partial^3\eta}{\partial X^3} = 0. \quad (16)$$

The single-soliton solution of the VCKdV equation can be given by

$$\eta = -\frac{k}{V_{\mathrm{g}}}\exp(kX - \omega t + \delta_0)\,\mathrm{sech}(kX - \omega t + \delta_0), \quad (17)$$

where $k$ and $\delta_0$ are arbitrary constants. Here $\omega = -k^3/(2V_{\mathrm{g}})$. Setting $k^2 = 2V_{\mathrm{g}}^2 A_0$ ($A_0$ is a positive integrating constant) and using $\xi = \varepsilon X = \varepsilon(x - V_{\mathrm{g}}t)$, the single soliton solution of Eq. (1) is

$$\psi = u_0\left[1 - A_0\,\mathrm{sech}^2(\theta)\right] \times \exp[-\mathrm{i}\mu t - \sqrt{2A_0}\mathrm{i}\exp(\theta)\,\mathrm{sech}(\theta)], \quad (18)$$

where $\theta = V_{\mathrm{g}}\sqrt{2A_0}\,(X + A_0V_{\mathrm{g}}t - V_{\mathrm{g}}t - x_{00})$, in which $x_{00} = -V_{\mathrm{g}}\sqrt{2A_0}\delta_0$ is another constant denoting the initial position of soliton on the pedestal background.

## 3. Numerical results and discussion

To observe the effect of the localized impurity on the soliton dynamics in the BEC, we further give numerical calculation of the dynamical properties.

First, we probe the effect of a repulsive localized impurity on the soliton propagation. Figure 2 shows the density distribution at the different time of the BECs in the presence of a repulsive localized impurity ($b > 0$). We see that there exists a sinus in the amplitude function of the density distribution at $x = -5$ at the initial time, which can be associated with dark solitons [see Fig. 2(a)]. With the time going on, the dark soliton propagates rightward without attenuation and change in shapes (including its height and width) before it reaches the localized impurity at $x = 0$ [see Fig. 2(b)]. This illustrates that the dark soliton is a stable propagating solitary wave, which is the result of the interplay between the nonlinearity and atomic dispersion. When it approaches the impurity point $x = 0$, the soliton becomes unstable [see Fig. 2(c)]. It is possible that there exist interactions between dark soliton and the localized impurity. However, when it arrives at the centre of the localized impurity [see Fig. 2(d)], the dark soliton is "quasitrapped" by the impurity, so the amplitude of the dark soliton increases slightly. Nevertheless, at the next time ($t = 18$) the soliton is released by the impurity, i.e., when the soliton centre leaves the localized impurity [see Fig. 2(e)]. Compared with the case at $t = 18$, the dark soliton at $t = 25$ has the same amplitude as that at $t = 5$ [see Fig. 2(f)]. This phenomenon indicates that the dark soliton has transmitted through the repulsive localized impurity.

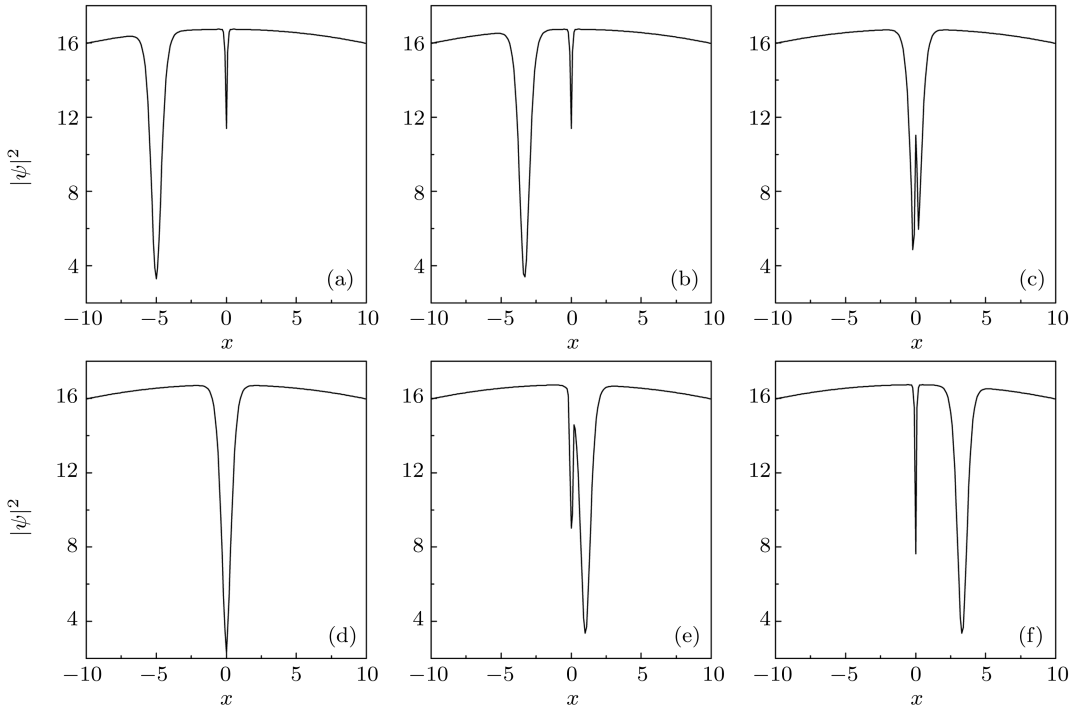



**Fig. 2.** In the presence of a repulsive localized impurity ($b = 0.15$), the variation of density distribution of the condensate with $x$ for different time (a) $t = 0$, (b) $t = 5$, (c) $t = 15$, (d) $t = 16$, (e) $t = 18$, and (f) $t = 25$. The other parameters used are $g_{1D} = 0.1$, $A_0 = 0.8$, $x_{00} = -5$, $\mu = 2.8$, and $k = 0.05$.

Subsequently, we study the effect of an attractive localized impurity on the propagation of the soliton as shown in Fig. 3. In the presence of an attractive impurity ($b < 0$), the dark soliton propagates stably rightward before it reaches the localized impurity at $x = 0$ [see Fig. 3(a)]. However, when it arrives at the centre of the localized impurity [see Fig. 3(b)], the dark soliton is also "quasitrapped" by the impurity. Different from the repulsive impurity, the amplitude of the dark soliton decreases slightly. The main reason is the attractive impurity-induced soliton. With the time going on, the dark soliton is released by the impurity and propagates stably on ward [see Fig. 3(c)].

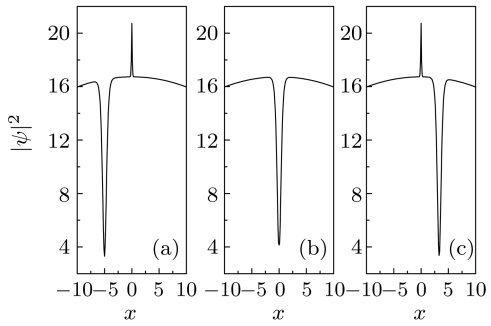



**Fig. 3.** In the presence of an attractive localized impurity ($b = -0.15$), the variation of density distribution of the condensate with $x$ for different time (a) $t = 0$, (b) $t = 16$, and (c) $t = 18$. The other parameters used are the same as those in Fig. 2.

From the case of both repulsive and attractive localized impurities, we may conclude that the dark soliton can be transmitted through a repulsive (or attractive) impurity, while at the centre of the localized impurity the soliton can be quasitrapped by the impurity. This result is in qualitative agreement with that of Ref. [27].

Finally, we discuss the effect of the strength of the localized impurity on the dynamical characteristics of the soliton in the condensates. Figure 4 shows the density distribution of the condensates at localized impurity $x = 0$, taking into consideration the different strength of a localized impurity. For a repulsive impurity, we find that the amplitude of the soliton increases with increasing strength of the localized impurity [see Fig. 4(a)]. The clearer picture can be seen in the inset of Fig. 4(a). For an attractive impurity, however, the amplitude of the soliton decreases as the absolute strength of a localized impurity increases [see Fig. 4(b)], which is clearer in the inset of Fig. 4(b).

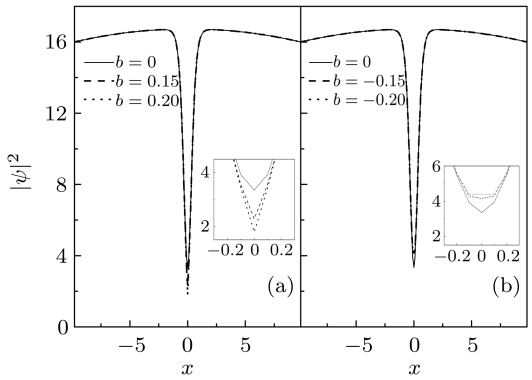



**Fig. 4.** At the localized impurity $x = 0$, the variation of density distribution of the condensate with $x$ for different $b$: (a) repulsive and (b) attractive impurity. The parameters used are the same as those in Fig. 2.

So, we conclude that the amplitude of the soliton increases when the strength $b$ of a localized impurity increases. Meanwhile, we depict the width of the soliton versus the strength $b$ of the localized impurity in Fig. 5. When the strength $b$ increases, the width of the soliton slowly decreases. In addition, the relation between the velocity of the soliton and the strength $b$ is shown in Fig. 6. One may see that the velocity of the soliton decreases with increasing strength of the localized impurity.

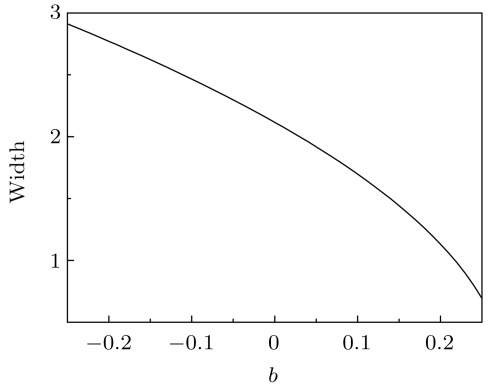



**Fig. 5.** At the localized impurity $x = 0$, the width of the soliton versus the strength $b$ of the localized impurity. The parameters used are the same as those in Fig. 2.

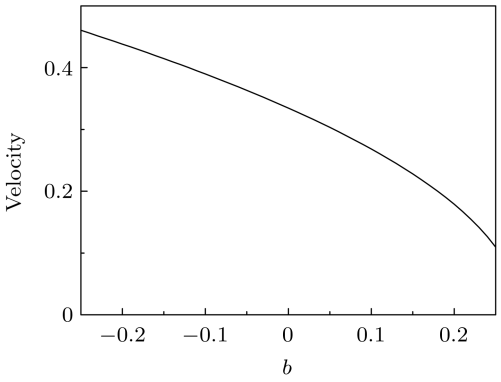



**Fig. 6.** At the localized impurity $x = 0$, the velocity of the soliton as a function of the strength $b$ of the localized impurity. The parameters used are the same as those in Fig. 2.

# 4. Conclusions

In summary, we have used multiple-scale method to study analytically the effect of the localized impurity on the dynamics of the BEC. In the linear case, we have analysed the soliton stability in the BEC in presence of a localized impurity. For the nonlinear case, the amplitude and phase of wave function of the condensates were governed by VCKdV equation. By solving the VCKdV equation, we obtained the equation of motion of the dark soliton in a trapping potential and in the presence of either repulsive or attractive potential. Our numerical calculation showed that a dark soliton can be transmitted through a repulsive

(or attractive) impurity, while at the position of the localized impurity the soliton can be quasitrapped by the impurity. Additionally, we find that the strength of the localized impurity had an important effect on the dark soliton dynamics. With increasing strength of the localized impurity, the amplitude of the dark soliton becomes bigger, while its width is narrower, and the soliton propagates slower.

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