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Nonlinear electrodynamics coupled to teleparallel theory of gravity

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Using nonlinear electrodynamics coupled to teleparallel theory of gravity, regular charged spherically symmetric solutions are obtained. The nonlinear theory is reduced to the Maxwell one in the weak limit and the solutions correspond to charged spacetimes. One of the obtained solutions contains an arbitrary function which we call general solution since we can generate from it the other solutions. The metric associated with these spacetimes is the same, i.e., regular charged static spherically symmetric black hole. In calculating the energy content of the general solution using the gravitational energy–momentum within the framework of the teleparallel geometry, we find that the resulting form depends on the arbitrary function. Using the regularized expression of the gravitational energy–momentum we obtain the value of energy.

Keywords: tetrad theory of gravitation, nonlinear electrodynamics, spherically symmetric black holes, gravitational energy–momentum

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1. Introduction

Energy–momentum, angular momentum and electric charge play central roles in modern physics. The conservations of the first two are related to the homogeneity and the isotropy of spacetime respectively while the charge conservation is related to the invariance of the action integral under internal $U(1)$ transformation. Local quantities such as energy–momentum, angular momentum and charge densities are well defined if gravitational field is not present in the system. However, in general relativity theory well-behaved energy–momentum and angular momentum densities have not yet been defined, although total energy–momentum and total angular momentum can be defined for an asymptotically flat spacetime surrounding an isolated finite system. The equality of the gravitational mass and the inertial mass holds within the framework of general relativity.[1−3] However, such an equality is not satisfied for the Schwarzschild metric when it is expressed in a certain coordinate system, i.e., spherical coordinate[4,5] using the energy–momentum complex like Landau–Lifschitz.[6]

In the last years an increasing revival of nonlinear electrodynamic (NLE) theories is observed.[7−9] Nonlinear electrodynamics was proposed first by Born and Infeld[10,11] in the 30's in order to obtain a finite-energy-electron model; they succeeded in determining an electron of finite radius. After these first achievements had been obtained, Plebánski mentioned in 1970 in the introduction of his monograph:[12] "If in recent times the interest in NLE cannot be said to be very popular, it is not due to the fact that one could rise some serious objections against this theory. It is simply rather difficult in its mathematical formulation. What causes that it is very unlikely to derive some concrete results in closed form." It is the aim of the present paper to find asymptotical flat solutions with spherical symmetry which are different from Schwarzschild solution in the teleparallel theory of gravity[13] coupled to nonlinear electrodynamics.[7−9] This can be achieved by inducing the teleparallel geometry in nonlinear electrodynamics. Using this philosophy, we obtain three different exact analytic solutions. We also use the superpotential given by Mikhail *et al.*[14] to calculate the energy.

In Section 2, we briefly review the teleparallel theory of gravitation coupled to nonlinear electrodynamics. The tetrad field with three unknown functions of the radial coordinate in spherical polar coordinates is applied to the field equations and three different exact asymptotically flat solutions are obtained in Section 3. The energy of the gravitation source is calculated using the superpotential method in Section 4. In Section 5 the energy is recalculated using the regularized expression of the gravitational

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energy–momentum. Discussion and conclusion of the obtained results are given in Section 6.[1)]

2. Tetrad theory of gravitation

In a spacetime with absolute parallelism the parallel vector fields $e_i{}^\mu$ define the nonsymmetric affine connection[2)]

$$\Gamma^\lambda{}_{\mu\nu} \stackrel{\text{def.}}{=} e_i{}^\lambda e^i{}_{\mu,\nu}, \tag{1}$$

where $e^i{}_{\mu,\,\nu} = \partial_\nu e^i{}_\mu$. The curvature tensor defined by $\Gamma^\lambda{}_{\mu\nu}$ is identically vanishing. However, the torsion tensor is defined by

$$T^a{}_{\mu\nu} \stackrel{\text{def.}}{=} e^a{}_\lambda T^\lambda{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu. \tag{2}$$

Møller constructed a gravitational theory based on this spacetime. In this theory the field variables are 16 tetrad components $e_i{}^\mu$, from which the metric tensor is defined by

$$g^{\mu\nu} \stackrel{\text{def.}}{=} \eta^{ij} e_i{}^\mu e_j{}^\nu, \tag{3}$$

where η^{ij} is the contravariant Minkowski metric of η_{ij} which is defined as

$$\eta_{ij} \stackrel{\text{def.}}{=} \text{diag}(+1\,,-1\,,-1\,,-1).$$

We note that for any tetrad field $e_i{}^\mu$ there is a metric field defined uniquely by Eq. (3), while a given metric $g^{\mu\nu}$ does not determine the tetrad field completely; for any local Lorentz transformation of the tetrads, $e_i{}^\mu$ leads to a new set of tetrads which also satisfy Eq. (3). The Lagrangian L is an invariant composed of $\gamma_{\mu\nu\rho}$ and $g^{\mu\nu}$, where $\gamma_{\mu\nu\rho}$ is the contorsion tensor given by

$$\gamma_{\mu\nu\rho} \stackrel{\text{def.}}{=} e_{i\ \mu} e_{i\nu;\ \rho}, \tag{4}$$

where the semicolon denotes covariant differentiation with respect to Christoffel symbol. The most general Lagrangian density invariant under the parity operation is given in the following form[13]

$$\begin{aligned}\mathscr{L}_G(e_{i\mu}, e_{i\mu,\ \nu}) \stackrel{\text{def.}}{=}\ & \sqrt{-g}(\alpha_1 \Phi^\mu \Phi_\mu + \alpha_2 \gamma^{\mu\nu\rho}\gamma_{\mu\nu\rho} \\ & + \alpha_3 \gamma^{\mu\nu\rho}\gamma_{\rho\nu\mu}),\end{aligned} \tag{5}$$

where

$$g \stackrel{\text{def.}}{=} \det(g_{\mu\nu}), \tag{6}$$

and Φ_μ is the basic vector field defined as

$$\Phi_\mu \stackrel{\text{def.}}{=} \gamma^\rho{}_{\mu\rho}. \tag{7}$$

Here α_1, α_2, and α_3 are constants determined by Møller such that the theory coincides with general relativity in a weak field:

$$\alpha_1 = -\frac{1}{\kappa}, \quad \alpha_2 = \frac{\lambda}{\kappa}, \quad \alpha_3 = \frac{1}{\kappa}(1-2\lambda), \tag{8}$$

where κ is the Einstein constant, and λ is a free dimensionless parameter.[3)] The same choice of the parameters was also made by Hayashi and Nakano.[15]

The NLE Lagrangian has the form[7–9]

$$\mathscr{H}(P) \stackrel{\text{def.}}{=} -\frac{1}{4} P_{\mu\nu} P^{\mu\rho} \tag{9}$$

with

$$\begin{aligned} & P_{\mu\nu} = \mathscr{L}(\mathscr{F})_F F_{\mu\nu}, \quad \mathscr{L}(\mathscr{F})_F = \frac{\partial \mathscr{L}(\mathscr{F})}{\partial F}, \\ & \mathscr{L}(\mathscr{F}) = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \end{aligned} \tag{10}$$

and $F_{\mu\nu}$ being given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where A_μ is the vector potential. We recall that there is more general Lagrangian depending also on the second invariant, $F^*_{\mu\nu} F^{\mu\nu}$ (the asterix denotes the Hodge duality), but for the objective of this work it is enough to consider only an action as that given in Eq. (9) to find a regular spherically symmetric solution. The dynamics of the theory is governed by

$$\mathscr{L} = \int \mathrm{d}v \left(\mathscr{L}_\mathscr{G}(e_{i\mu}, e_{i\mu,\ \nu}) + \mathscr{H}(P)\right).$$

From the above action the following field equations are obtained by taking the variation with respect to the parallel vector fields and the anti-symmetric tensor $P_{\mu\nu}$:[7–9,13]

$$\begin{aligned} & G_{\mu\nu} + H_{\mu\nu} = -\kappa T_{\mu\nu}\ , \\ & K_{\mu\nu} = 0\ , \quad \partial_\nu\left(\sqrt{-g} P^{\mu\nu}\right) = 0, \end{aligned} \tag{11}$$

where $G_{\mu\nu}$ is the Einstein tensor, $H_{\mu\nu}$ and $K_{\mu\nu}$ are defined as

$$\begin{aligned} H_{\mu\nu} \stackrel{\text{def.}}{=}\ & \lambda\Big[\gamma_{\rho\sigma\mu}\gamma^{\rho\sigma}{}_\nu + \gamma_{\rho\sigma\mu}\gamma_\nu{}^{\rho\sigma} + \gamma_{\rho\sigma\nu}\gamma_\mu{}^{\rho\sigma} \\ & + g_{\mu\nu}\left(\gamma_{\rho\sigma\tau}\gamma^{\tau\sigma\rho} - \frac{1}{2}\gamma_{\rho\sigma\tau}\gamma^{\rho\sigma\tau}\right)\Big], \end{aligned} \tag{12}$$

[1)] Computer algebra system Maple 6 is used to check some computations.

[2)] In this paper Latin indices $(i, j, \ldots)$ represent the vector numbers, and Greek indices $(\mu, \nu, \ldots)$ represent the vector components. All indices run from 0 to 3. The spatial parts of Latin indices are denoted by $(a, b, \ldots)$, while those of Greek indices by $(\alpha, \beta, \ldots)$.

[3)] Throughout this paper we use the relativistic units, $c = G = 1$ and $\kappa = 8\pi$.

and

$$K_{\mu\nu} \overset{\text{def.}}{=} \lambda[\Phi_{\mu,\nu} - \Phi_{\nu,\mu} - \Phi_\rho \left(\gamma^\rho{}_{\mu\nu} - \gamma^\rho{}_{\nu\mu}\right) + \gamma_{\mu\nu}{}^\rho{}_{;\rho}], \tag{13}$$

and they are symmetric and skew symmetric tensors, respectively. When the two tensors $H_{\mu\nu}$ and $K_{\mu\nu}$ are vanishing, then the symmetric part of Eq. (11) coincides with the Einstein field equations in teleparallel equivalent of general relativity.[16,17]

The energy–momentum tensor of the electromagnetic field depends on the parallel vector field e only through the metric tensor, $g_{\mu\nu}$ which is defined in Eq. (6) and hence it is locally Lorentz-invariant. Thus, the energy–momentum tensor of matter is locally invariant and therefore, the gravitational field equations of Eq. (11) are invariant under local Lorentz transformation.[18] Thus in this particular case, the energy–momentum tensor is identical with the energy–momentum of general relativity theory. Therefore, we define the energy–momentum tensor $T^{\mu\nu}$ as

$$T^{\mu\nu} \overset{\text{def.}}{=} 2(\mathscr{H}_P(P)\ P^\mu{}_\lambda P^{\nu\lambda} - \delta^{\mu\nu}\left[2P\mathscr{H}_P(P) - \mathscr{H}(P)\right]), \tag{14}$$

where

$$P = \frac{1}{4}(P_{\mu\nu}P^{\mu\nu}) \ \text{ and } \ \mathscr{H}_P(P) = \frac{\partial \mathscr{H}(P)}{\partial P}.$$

We should point out that the usual electromagnetic strength tensor is defined as

$$F_{\mu\nu} \overset{\text{def.}}{=} \mathscr{H}_P P_{\mu\nu}. \tag{15}$$

3. Spherically symmetric solutions

We begin with the tetrad having a spherical symmetry[19]

$$(e_l{}^\mu) = \begin{pmatrix} A & Dr & 0 & 0 \\ 0 & B\sin\theta\cos\phi & \dfrac{B}{r}\cos\theta\cos\phi & -\dfrac{B\sin\phi}{r\sin\theta} \\ 0 & B\sin\theta\sin\phi & \dfrac{B}{r}\cos\theta\sin\phi & \dfrac{B\cos\phi}{r\sin\theta} \\ 0 & B\cos\theta & -\dfrac{B}{r}\sin\theta & 0 \end{pmatrix}, \tag{16}$$

where A, D, and B, are unknown functions of the redial coordinate r. Applying Eq. (16) to the field equation (11) we obtain a system of nonlinear differential equations in Ref. [20]. Now we are interested in solving these system of differential equations.

First solution

A first non-trivial solution can be obtained by taking $D(r) = 0$, and solving $A(r)$ and $B(r)$, we obtain

$$A = \sqrt{1 - \frac{2m\left[1 - \tanh\left(\dfrac{q^2}{2mR}\right)\right]}{R}}, \quad B = \int \frac{1}{R}\left(1 - \sqrt{1 - \frac{2m}{R}\left[1 - \tanh\left(\frac{q^2}{2mR}\right)\right]}\right)\mathrm{d}R, \tag{17}$$

where R is a new radial coordinate defined as $R = r/B$.

The ansatz of the electromagnetic potential A_μ, the anti-symmetric fields $F_{\mu\nu}$, $P_{\mu\nu}$, the nonlinear electrodynamics source used to derive this solution and the energy–momentum tensor have the following forms:

$$A_t(R) = -\frac{q^2\mathrm{e}^{q^2/MR} - 3MR\left(\mathrm{e}^{q^2/MR} + 1\right)}{qR(\mathrm{e}^{q^2/MR} + 1)^2},$$

$$\boldsymbol{F} = q\frac{\mathrm{e}^{q^2/MR}\left(4MR\left\{\mathrm{e}^{q^2/MR} + 1\right\} - q^2\left\{\mathrm{e}^{q^2/MR} - 1\right\}\right)}{MR^3\left(\mathrm{e}^{q^2/MR} + 1\right)^3}\mathrm{d}t \wedge \mathrm{d}R,$$

$$\boldsymbol{P} = \frac{q}{R^2}\mathrm{d}t \wedge \mathrm{d}R, \quad \mathscr{H} = -2\frac{q^2\mathrm{e}^{q^2/MR}}{R^4\left(\mathrm{e}^{q^2/MR} + 1\right)^2}, \quad T_0{}^0 = T_1{}^1 = \frac{q^2\mathrm{e}^{(q^2/mR)}}{2\pi R^4\left(1 + \mathrm{e}^{(q^2/mR)}\right)^2},$$

$$T_2{}^2 = T_3{}^3 = \frac{q^2 \mathrm{e}^{(q^2/mR)} \left(q^2 (\mathrm{e}^{(q^2/mR)} - 1) - 2mR(1 + \mathrm{e}^{(q^2/mR)}) \right)}{4\pi m R^5 (1 + \mathrm{e}^{(q^2/mR)})^3}. \tag{18}$$

Using expression (17), the tetrad (16) turns into the following form:

$$(e_i{}^\mu) = \begin{pmatrix} \dfrac{1}{\sqrt{1 - \dfrac{2m\left[1 - \tanh\left(\frac{q^2}{2mR}\right)\right]}{R}}} & 0 & 0 & 0 \\ 0 & \sin\theta\cos\phi\sqrt{1 - \dfrac{2m\left[1 - \tanh\left(\frac{q^2}{2mR}\right)\right]}{R}} & \dfrac{\cos\theta\cos\phi}{R} & -\dfrac{\sin\phi}{R\sin\theta} \\ 0 & \sin\theta\sin\phi\sqrt{1 - \dfrac{2m\left[1 - \tanh\left(\frac{q^2}{2mR}\right)\right]}{R}} & \dfrac{\cos\theta\sin\phi}{R} & \dfrac{\cos\phi}{R\sin\theta} \\ 0 & \cos\theta\sqrt{1 - \dfrac{2m\left[1 - \tanh\left(\frac{q^2}{2mR}\right)\right]}{R}} & -\dfrac{\sin\theta}{R} & 0 \end{pmatrix}, \tag{19}$$

with the relevant Riemannian metric

$$\mathrm{d}s^2 = -\eta_1 \mathrm{d}t^2 + \mathrm{d}R^2/\eta_1 + R^2 \mathrm{d}\Omega^2, \quad \eta_1 = \left[1 - \frac{2m\left[1 - \tanh\left(q^2/2mR\right)\right]}{R}\right] \cong 1 - \frac{2m}{R} + \frac{q^2}{R^2}, \tag{20}$$

where $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2$. Equation (20) represents a static spherically symmetric regular black hole solution.[7−9] Using Eq. (2), the non-vanishing components of the torsion tensor of solution (17) are given by

$$\begin{aligned} T^0{}_{10} &= \frac{2q^2 \mathrm{e}^{q^2/MR} - MR\left(\mathrm{e}^{q^2/MR} + 1\right)}{R^2\left(\mathrm{e}^{q^2/MR} + 1\right)\left\{R\left(\mathrm{e}^{q^2/MR} + 1\right) - 4M\right\}}, \\ T^2{}_{21} = T^3{}_{31} &= \frac{R\mathrm{e}^{q^2/MR} - \sqrt{R}\mathrm{e}^{q^2/4MR}\sqrt{\mathrm{e}^{q^2/MR} + 1}\sqrt{R\mathrm{e}^{q^2/2MR} + \mathrm{e}^{-q^2/2MR}(R - 4M)} + R - 4M}{R\left(R\mathrm{e}^{q^2/MR} + R - 4M\right)}. \end{aligned} \tag{21}$$

Second solution

A second non-trivial solution can be obtained by taking the unknown functions $A(r) = 1$, $B(r) = 1$ and solving the unknown function $D(r)$. In this case the resulting field equations[20] can be integrated directly to give

$$D(r) = \sqrt{\frac{2m\left[1 - \tanh\left(q^2/2mr\right)\right]}{r^3}}. \tag{22}$$

Substituting $D(r)$ given in Eq. (22) into Eq. (16), we obtain

$$(e_i{}^\mu) = \begin{pmatrix} 1 & \sqrt{\dfrac{2m\left[1 - \tanh\left(q^2/2mr\right)\right]}{r}} & 0 & 0 \\ 0 & \sin\theta\cos\phi & \dfrac{\cos\theta\cos\phi}{r} & -\dfrac{\sin\phi}{r\sin\theta} \\ 0 & \sin\theta\sin\phi & \dfrac{\cos\theta\sin\phi}{r} & \dfrac{\cos\phi}{r\sin\theta} \\ 0 & \cos\theta & -\dfrac{\sin\theta}{r} & 0 \end{pmatrix}, \tag{23}$$

with the relevant metric

$$\mathrm{d}s^2 = -\left[1 - \frac{2m\left[1 - \tanh\left(q^2/2mR\right)\right]}{r}\right]\mathrm{d}t^2 - 2\sqrt{\frac{2m\left[1 - \tanh\left(q^2/2mR\right)\right]}{r}}\mathrm{d}r\mathrm{d}t + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2. \tag{24}$$

It should be noted that m and q appearing in metrics (20) and (24) are constants of integration that will play the role of mass and charge producing the field in the calculations of energy. Also the ansatz of the electromagnetic potential A_μ, the anti-symmetric fields $F_{\mu\nu}$, $P_{\mu\nu}$, the nonlinear electrodynamics source used to derive solution (22) and the energy–momentum tensor have the form shown in Eq. (18) with $R = r$.

Using the coordinate transformation

$$\mathrm{d}T = \mathrm{d}t + \frac{Dr}{1 - D^2r^2}\mathrm{d}r, \tag{25}$$

we can eliminate the cross terms of Eq. (24) to obtain

$$\mathrm{d}s^2 = -\eta_1 \mathrm{d}T^2 + \frac{\mathrm{d}r^2}{\eta_1} + r^2\mathrm{d}\Omega^2, \tag{26}$$

where η_1 is defined in Eq. (20) with $R = r$. Using Eq. (2), the non-vanishing components of the torsion tensor of the second solution are given by

$$T^0{}_{10} = 2\frac{\mathrm{e}^{q^2/Mr}\left\{Mr\left(\mathrm{e}^{q^2/Mr}+1\right) - q^2\right\}}{r^2\left(\mathrm{e}^{q^2/Mr}+1\right)\left\{4M - r\left(\mathrm{e}^{q^2/Mr}+1\right)\right\}},$$

$$T^1{}_{01} = \frac{Mr\left(\mathrm{e}^{q^2/Mr}+1\right) - q^2\mathrm{e}^{q^2/Mr}}{\sqrt{M}r^{5/2}\left(\mathrm{e}^{q^2/Mr}+1\right)^{3/2}},$$

$$T^2{}_{20} = T^3{}_{30} = 2\frac{\sqrt{M}}{r^{3/2}\sqrt{\mathrm{e}^{q^2/M1}+1}},$$

$$T^2{}_{12} = T^3{}_{13} = \frac{4M}{r\left(r\left\{\mathrm{e}^{q^2/Mr}+1\right\} - 4M\right)}. \tag{27}$$

Third solution

It is our purpose to find a general solution for tetrad (16) when the stress-energy momentum tensor is not vanishing and has the form given by[21]

$$T_0{}^0 = T_1{}^1, \quad T_2{}^2 = T_3{}^3, \tag{28}$$

where all the other mixed spatial components are equal to zero.[21] We can find solutions of unknown functions D and A of unknown function B in the form[20]

$$D = \frac{1}{\left(1 - \dfrac{rB'}{B}\right)}\sqrt{\frac{k_1B^3}{r^3}\left[1 - \tanh\left(\frac{q^2}{2mr}\right)\right] + \frac{BB'}{r}\left(\frac{rB'}{B} - 2\right)}, \quad A = \frac{k_2}{\left(1 - \dfrac{rB'}{B}\right)}, \tag{29}$$

where k_1 and k_2 are constants of integration.

The line-element squared of the tetrad field given by Eq. (16) takes the form

$$\mathrm{d}s^2 = -\frac{(B^2 - D^2r^2)}{A^2B^2}\mathrm{d}t^2 - \frac{2Dr}{AB^2}\mathrm{d}r\mathrm{d}t + \frac{1}{B^2}(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2). \tag{30}$$

We assume $B(r)$ to be nonvanishing so that the surface area of the sphere of a constant r is finite. We also assume that $A(r)$ and $B(r)$ satisfy the asymptotic conditions, $\lim_{r\to\infty} A(r)$=$\lim_{r\to\infty} B(r) = 1$ and $\lim_{r\to\infty} rB' = 0$. Then, we can show from Eqs. (29) and (30) that $k_2 = 1$, $B(r) > 0$, $\lim_{r\to\infty} rD(r) = 0$, and if $B - rB'$ vanishes at some point, then $\left(1 - \dfrac{B(r)k_1\left[1 - \tanh\left(q^2/2mr\right)\right]}{r}\right) < 0$ at that point.

Using the coordinate transformation

$$\mathrm{d}T = \mathrm{d}t + \frac{ADr}{B^2 - D^2r^2}\mathrm{d}r, \tag{31}$$

we can eliminate the cross terms of Eq. (30) to obtain

$$\mathrm{d}s^2 = -\eta_2\mathrm{d}T^2 + \frac{1}{\eta_2}\frac{\mathrm{d}r^2}{A^2B^2} + \frac{r^2}{B^2}\mathrm{d}\Omega^2 \tag{32}$$

with $\eta_2 = (B^2 - D^2r^2)/A^2B^2$. Taking the new radial coordinate $R = r/B$, we finally obtain

$$\mathrm{d}s^2 = -\eta_2\mathrm{d}T^2 + \frac{\mathrm{d}R^2}{\eta_2} + R^2\mathrm{d}\Omega^2, \tag{33}$$

where

$$\eta_2(R) = \left(1 - \frac{k_1\left[1 - \tanh\left(q^2/2mR\right)\right]}{R}\right) \cong 1 - \frac{k_1}{R} + \frac{q^2}{R^2}. \tag{34}$$

Then, equation (33) coincides with the charged regular black hole solution given in Refs. [10] and [11] with the mass, $m = k_1/2$, hence the general solution in the case of the spherically symmetric tetrad when the stress-energy momentum tensor is nonvanishing, gives no more than the regular charged black hole solution when $1 - rB'/B$ has no zero and R is monotonically increasing function of r. If $1 - rB'/B$ has zeroes, the line-element (30) is singular at these zeroes which lie inside the event horizon as seen from property (4) mentioned above. We shall study in the future whether this singularity at zero-points of $(1 - rB'/B)$ is physically acceptable or not.

Using the above transformation given by Eq. (31) and also using the new redial coordinate, the tetrad (16) turns into the following form:

$$(e_i{}^{\mu}) = \begin{pmatrix} \dfrac{\mathscr{A}}{1-\mathscr{D}^2R^2} & \mathscr{D}R(1-R\mathscr{B}') & 0 & 0 \\ \dfrac{\mathscr{A}\mathscr{D}R\sin\theta\cos\phi}{1-\mathscr{D}^2R^2} & (1-R\mathscr{B}')\sin\theta\cos\phi & \dfrac{\cos\theta\cos\phi}{R} & -\dfrac{\sin\phi}{R\sin\theta} \\ \dfrac{\mathscr{A}\mathscr{D}R\sin\theta\sin\phi}{1-\mathscr{D}^2R^2} & (1-R\mathscr{B}')\sin\theta\sin\phi & \dfrac{\cos\theta\sin\phi}{R} & \dfrac{\cos\phi}{R\sin\theta} \\ \dfrac{\mathscr{A}\mathscr{D}R\cos\theta}{1-\mathscr{D}^2R^2} & (1-R\mathscr{B}')\cos\theta & \dfrac{-\sin\theta}{R} & 0 \end{pmatrix}. \tag{35}$$

Here $\mathscr{A}$ and $\mathscr{D}$ are given in terms of the unknown function $B(R)$ as

$$\mathscr{A}(\mathscr{R}) = \frac{1}{1-R\mathscr{B}'}, \quad \mathscr{D}(\mathscr{R}) = \frac{1}{1-R\mathscr{B}'}\sqrt{\frac{2m\left[1-\tanh\left(q^2/2mr\right)\right]}{R^3} + \frac{\mathscr{B}'}{R}\left(R\mathscr{B}'-2\right)}, \tag{36}$$

where $\mathscr{B}' = \mathrm{d}\mathscr{B}(\mathscr{R})/\mathrm{d}R$. It is of interest to note that general solution (36) satisfies the field equation (11) when the anastz of the electromagnetic potential A_μ, the anti-symmetric fields $F_{\mu\nu}$, $P_{\mu\nu}$, the nonlinear electrodynamics source and the energy–momentum tensor are in the forms of Eq. (18). The two tensors $H_{\mu\nu}$ and $F_{\mu\nu}$ related to solutions (17), (22) and (36) are vanishing identically. Therefore, Møller tetrad theory of gravitation coincides with teleparallel theory equivalent to general relativity. Using Eq. (2), the non-vanishing components of the torsion tensor of the third solution are given by

$$\begin{aligned}
T^0{}_{01} &= 2\frac{\mathrm{e}^{q^2/MR}\left\{MR\left(\mathrm{e}^{q^2/MR}+1\right)-q^2\right\}}{R^2\left(\mathrm{e}^{q^2/MR}+1\right)\left\{4M-R\left(\mathrm{e}^{q^2/MR}+1\right)\right\}}, \\
T^1{}_{01} &= -\frac{\mathrm{e}^{-q^2/4MR}}{R^{5/2}\left(\mathrm{e}^{q^2/MR}+1\right)^{3/2}\sqrt{\mathrm{e}^{-q^2/2MR}\left(4M+R^3\mathscr{B}'^2-2R^2\mathscr{B}'\right)+\mathrm{e}^{q^2/2MR}\left(R^3B'^2-2R^2B'\right)}} \\
&\quad\times\{6MR^2\mathscr{B}'-R^3\mathscr{B}'-2MR-R^4\mathscr{B}''+4MR^3\mathscr{B}''+\mathrm{e}^{q^2/MR}(2q^2-2R^3\mathscr{B}'-2MR-2R^4\mathscr{B}'' \\
&\quad-2q^2R\mathscr{B}'+6MR^2\mathscr{B}'+4MR^3\mathscr{B}'')-\mathrm{e}^{2q^2/MR}\left(R^3\mathscr{B}'+R^4\mathscr{B}''\right)\}, \\
T^2{}_{20} &= T^3{}_{30} = \frac{\mathrm{e}^{q^2/4MR}\sqrt{\mathrm{e}^{-q^2/2MR}\left(4M+R^3\mathscr{B}'^2-2R^2\mathscr{B}'\right)+\mathrm{e}^{q^2/2MR}\left(R^3\mathscr{B}'^2-2R^2\mathscr{B}'\right)}}{R^{3/2}\sqrt{\mathrm{e}^{q^2/MR}+1}}, \\
T^2{}_{12} &= T^3{}_{13} = \frac{\mathrm{e}^{q^2/2MR}\left(\mathrm{e}^{-q^2/2MR}\left(4M-R^2\mathscr{B}'\right)-R^2\mathscr{B}'\mathrm{e}^{q^2/2MR}\right)}{R\left(R\left\{\mathrm{e}^{q^2/MR}+1\right\}-4M\right)}.
\end{aligned} \tag{37}$$

The previously obtained solutions can be verified as special cases of the general solution (36). The choice

$$\mathscr{B}(\mathscr{R}) = 1, \tag{38}$$

reproduces solution (22). On the other hand, the choice

$$\mathscr{B}(\mathscr{R}) = \int\frac{1}{R}\left(1-\sqrt{1-\frac{2m(1-\tanh\left(q^2/2mR\right))}{R}}\right)\mathrm{d}R, \tag{39}$$

reproduces solution (17). It is of interest to note that if $q=0$ then the general solution (36) reduces to that obtained before by Mikhail *et al.*[22] and the two choices (38) and (39) will give the Schwarzschild solution in its standard form.

4. Energy associated with the third solution

To make the consequence of computations clearer, we calculate the energy associated with solution (36). The superpotential is given by[14]

$$\mathscr{U}_\mu{}^{\nu\lambda} = \frac{(-g)^{1/2}}{2\kappa}P_{\chi\rho\sigma}{}^{\tau\nu\lambda}[\Phi^\rho g^{\sigma\chi}g_{\mu\tau} - \lambda g_{\tau\mu}\gamma^{\chi\rho\sigma} - (1-2\lambda)g_{\tau\mu}\gamma^{\sigma\rho\chi}], \tag{40}$$

where $g_{\mu\nu}$, $(-g)^{1/2}$, $\gamma^{\sigma\rho\chi}$ and Φ^{ρ} are the metric tensor defined by Eq. (3), the determinate of the metric tensor defined by Eq. (6), the contorsion tensor defined by Eq. (4), the basic vector defined by Eq. (7) respectively; $P_{\chi\rho\sigma}{}^{\tau\nu\lambda}$ is a tensor defined as

$$P_{\chi\rho\sigma}{}^{\tau\nu\lambda} \stackrel{\text{def.}}{=} \delta_{\chi}{}^{\tau} g_{\rho\sigma}{}^{\nu\lambda} + \delta_{\rho}{}^{\tau} g_{\sigma\chi}{}^{\nu\lambda} - \delta_{\sigma}{}^{\tau} g_{\chi\rho}{}^{\nu\lambda} \quad (41)$$

with $g_{\rho\sigma}{}^{\nu\lambda}$ being a tensor defined as

$$g_{\rho\sigma}{}^{\nu\lambda} \stackrel{\text{def.}}{=} \delta_{\rho}{}^{\nu}\delta_{\sigma}{}^{\lambda} - \delta_{\sigma}{}^{\nu}\delta_{\rho}{}^{\lambda}, \quad (42)$$

where $\delta_{\rho}{}^{\nu}$ is the Kronecker delta

$$\delta_{\rho}{}^{\nu} = \begin{cases} 1 & \mu = \nu, \\ 0 & \mu \neq \nu. \end{cases}$$

The energy is expressed by the surface integral[23]

$$E = \lim_{r\to\infty} \int_{r=\text{constant}} \mathscr{U}_0{}^{0\alpha} n_{\alpha} \mathrm{d}S, \quad (43)$$

where n_{α} is the unit 3-vector normal to the surface element $\mathrm{d}S$.

Now we are in a position to calculate the energy associated with solution (36) using the superpotential (40). It is clear from Eq. (43) that only component which makes contribution to the energy is $\mathscr{U}_0{}^{0\alpha}$. Thus substituting solution (36) into Eq. (40) yields the following non-vanishing value:

$$\mathscr{U}_0{}^{0\alpha} = \frac{2X^{\alpha}}{\kappa R^3}\left(2m - 2m\tanh\left(\frac{q^2}{2mR}\right) - R^2\mathscr{B}(\mathscr{R})'\right). \quad (44)$$

Substituting Eq. (44) into Eq. (43) we have

$$E(R) = 2m\left[1 - \tanh\left(\frac{q^2}{2mR}\right)\right] - R^2\mathscr{B}(\mathscr{R})', \quad (45)$$

which depends on the arbitrary function $\mathscr{B}(\mathscr{R})$.

5. Regularized expression for gravitational energy–momentum

An important property of the tetrad field that satisfies the condition

$$e_{i\mu} \cong \eta_{i\mu} + 1/2h_{i\mu}(1/r), \quad (46)$$

is that in the flat space–time limit $e^{i}{}_{\mu}(t,x,y,z) = \delta^{i}{}_{\mu}$, and therefore the torsion tensor defined by Eq. (2) is vanishing, i.e., $T^{\lambda}{}_{\mu\nu} = 0$. Hence for the flat space–time normally a set of tetrad fields is considered such that $T^{\lambda}{}_{\mu\nu} = 0$ in any coordinate system. However, in general an arbitrary set of tetrad fields, which yields the metric tensor for the asymptotically flat space–time, does not satisfy the asymptotic condition given by Eq. (46). Moreover for such tetrad fields the torsion $T^{\lambda}{}_{\mu\nu} \neq 0$ for the flat space–time.[24–27] It might be argued, therefore, that the expression for the energy given by Eq. (43) is restricted to a particular class of tetrad fields, namely, the class of frames such that $T^{\lambda}{}_{\mu\nu} = 0$ if $e^{i}{}_{\mu}$ represents the flat space–time tetrad field.[25] To explain this, we calculate the flat space–time tetrad field of Eq. (35) using Eq. (36) which is given by

$$(E_i{}^{\mu}) = \begin{pmatrix} (1 - R\mathscr{B}') & \sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'} & 0 & 0 \\ \sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'}\sin\theta\cos\phi & (1 - R\mathscr{B}')\sin\theta\cos\phi & \dfrac{\cos\theta\cos\phi}{R} & \dfrac{-\sin\phi}{R\sin\theta} \\ \sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'}\sin\theta\sin\phi & (1 - R\mathscr{B}')\sin\theta\sin\phi & \dfrac{\cos\theta\sin\phi}{R} & \dfrac{\cos\phi}{R\sin\theta} \\ \sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'}\cos\theta & (1 - R\mathscr{B}')\cos\theta & \dfrac{\sin\theta}{R} & 0 \end{pmatrix}. \quad (47)$$

Expression (47) yields the following non-vanishing torsion components:

$$T_{001} = \mathscr{B}', \quad T_{112} = -r\cos(\theta)\cos\phi\mathscr{B}', \quad T_{113} = \sin(\theta)\sin\phi\mathscr{B}', \quad T_{114} = -\frac{\sin(\theta)\cos\phi\sqrt{\mathscr{B}'}(1 - \mathscr{B}')}{\sqrt{R^2\mathscr{B}' - 2R}},$$

$$T_{124} = \cos\theta\cos\phi\sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'}, \quad T_{134} = -\sin(\theta)\sin\phi\sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'}, \quad T_{212} = -R\cos(\theta)\sin\phi\mathscr{B}',$$

$$T_{213} = -R\sin(\theta)\cos\phi\mathscr{B}', \quad T_{214} = -\frac{\sin\theta\sin\phi\sqrt{\mathscr{B}'}(1 - \mathscr{B}')}{\sqrt{R^2\mathscr{B}' - 2R}}, \quad T_{224} = \cos(\theta)\sin\phi\sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'},$$

$$T_{234} = \sin(\theta)\cos\phi\sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'}, \quad T_{312} = R\sin(\theta)\mathscr{B}', \quad T_{314} = -\frac{\cos(\theta)\sqrt{\mathscr{B}'}(1 - \mathscr{B}')}{\sqrt{R^2\mathscr{B}' - 2R}},$$

$$T_{324} = -\sin(\theta)\sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'}. \tag{48}$$

The tetrad field (47) when written in the Cartesian coordinate has the following form:

$$(E_i{}^{\mu}(t,x,y,z)) = \begin{pmatrix} 1 - R\mathscr{B}' & n^a\sqrt{R^2\mathscr{B}'^2 - 2R\mathscr{B}'} \\ n^{\alpha}\sqrt{R^2\mathscr{B}' - 2R\mathscr{B}'} & \delta_a{}^{\alpha} - n_a n^{\alpha} R\mathscr{B}' \end{pmatrix}. \tag{49}$$

In view of the geometric structure of Eq. (49), we see that equation (35) obtained using Eq. (36) does not display the asymptotic behaviour required by Eq. (46). Moreover, in general the tetrad field (49) is adapted to accelerated observers.[24−27] To explain this, we consider a boost in the x direction of Eq. (49). We find

$$\left(E^i{}_{\mu}(t,x,y,z)\right) = \begin{pmatrix} \frac{vx\sqrt{R\mathscr{B}'^2-2\mathscr{B}'}+\sqrt{R}(1-R\mathscr{B}')}{\gamma^{3/2}\sqrt{R}} & \frac{x\sqrt{R\mathscr{B}'^2-2\mathscr{B}'}+v\sqrt{R}(1-R\mathscr{B}')}{\gamma^{3/2}\sqrt{R}} & \frac{y\sqrt{R\mathscr{B}'^2-2\mathscr{B}'}}{R} & \frac{z\sqrt{R\mathscr{B}'^2-2\mathscr{B}'}}{R} \\ \frac{x\sqrt{R\mathscr{B}'^2-2R\mathscr{B}'}+v\sqrt{R}(R-x^2\mathscr{B}')}{(\gamma R)^{3/2}} & \frac{xv\sqrt{R\mathscr{B}'^2-2R\mathscr{B}'}+\sqrt{R}(R-x^2\mathscr{B}')}{(\gamma R)^{3/2}} & -\frac{xy\mathscr{B}'}{R} & -\frac{xz\mathscr{B}'}{R} \\ y\frac{\sqrt{R\mathscr{B}'^2-2R\mathscr{B}'}-vx\sqrt{R}\mathscr{B}'}{(R\gamma)^{3/2}} & y\frac{v\sqrt{R\mathscr{B}'^2-2R\mathscr{B}'}-x\sqrt{R}\mathscr{B}')}{(\gamma R)^{3/2}} & \frac{1-y^2\mathscr{B}'}{R} & -\frac{yz\mathscr{B}}{R} \\ z\frac{\sqrt{R\mathscr{B}'^2-2R\mathscr{B}'}-vx\sqrt{R}\mathscr{B}'}{(\gamma R)^{3/2}} & z\frac{v\sqrt{R\mathscr{B}'^2-2R\mathscr{B}'}-x\sqrt{R}\mathscr{B}'}{(\gamma R)^{3/2}} & -\frac{zy\mathscr{B}'}{R} & \frac{1-z^2\mathscr{B}'}{R} \end{pmatrix}, \tag{50}$$

where v is the speed of the observer and $\gamma = \sqrt{1-v^2}$. It can be shown that along an observer trajectory whose velocity is determined by

$$u^{\mu} = E_0^{\mu} = \left(\frac{vx\sqrt{R\mathscr{B}'^2 - 2\mathscr{B}'} + \sqrt{R}(1-R\mathscr{B}')}{\gamma^{3/2}\sqrt{R}}, \frac{x\sqrt{R\mathscr{B}'^2 - 2\mathscr{B}'} + v\sqrt{R}(1-R\mathscr{B}')}{\gamma^{3/2}\sqrt{R}}, \frac{y\sqrt{R\mathscr{B}'^2 - 2\mathscr{B}'}}{R}, \frac{z\sqrt{R\mathscr{B}'^2 - 2\mathscr{B}'}}{R}\right), \tag{51}$$

the quantities $\phi_a{}^b = u^{\alpha}\left(E^b{}_{\beta}\partial_{\alpha}E_a{}^{\beta}\right)$, are non vanishing. This fact indicates that along the observer path the spatial axis $E_i{}^{\mu}$ rotates.[24−27] In spite of the above problems discussed for the tetrad field of Eq. (35) using (36) a satisfactory value for the total gravitational energy–momentum is yielded as will be discussed below.

Maluf *et al.*[24−27] discussed the above problems in the framework of TEGR and constructed a regularized expression for the gravitational energy–momentum in this frame. They checked this expression for a tetrad field that suffers the above problems and obtained a very satisfactory result.[25]. In this section we will follow the same procedure to derive a regularized expression for the gravitational energy–momentum defined by Eq. (43). It can be shown that the gravitational energy–momentum contained within an arbitrary volume V of the three-dimensional space-like hypersurface has the form[13,14]

$$P_{\mu} = \int_V \mathrm{d}^3x\partial_{\alpha}\mathscr{U}_{\mu}{}^{0\alpha}, \tag{52}$$

where $\mathscr{U}_{\mu}{}^{\nu\lambda}$ is given by Eq. (40). Expression (52) bears no relationship to the ADM energy–momentum.[26] P_{μ} transforms as a vector under the global $SO(3,1)$ group. It describes the gravitational energy–momentum with respect to observers adapted to $e^i{}_{\mu}$. These observers are characterized by the velocity field $u^{\mu} = e_0{}^{\mu}$ and the acceleration f^{μ} given by[25,26]

$$f^{\mu} = \frac{Du^u}{\mathrm{d}s} = \frac{De_0{}^{\mu}}{\mathrm{d}s} = u^a\nabla_a e_0{}^{\mu}. \tag{53}$$

Our assumption is that the space–time is asymptotically flat. In this case the total gravitational energy–momentum is given by

$$P_{\mu} = \oint_{S\to\infty} \mathrm{d}S_{\alpha}\ \mathscr{U}_{\mu}{}^{0\alpha}. \tag{54}$$

The field quantities are evaluated on a surface S in the limit $r \to \infty$.

In Eqs. (52) and (54) it is implicitly assumed that the reference space is determined by a set of tetrad fields $e^i{}_{\mu}$ for flat space–time such that the condition $T^{\lambda}{}_{\mu\nu} = 0$ is satisfied. However, in general there exist flat space–time tetrad fields for which $T^a{}_{\mu\nu} \neq 0$. In this case equation (52) may be generalized[25,26] by adding a suitable reference space subtraction term, exactly like in the Brown–York formalism.[28−30]

We will denote $T^{a}{}_{\mu\nu}(E) = \partial_\mu E^{a}{}_{\nu} - \partial_\nu E^{a}{}_{\mu}$ and $\mathscr{U}_{\mu}{}^{0\alpha}(E)$ as the expression of $\mathscr{U}_{\mu}{}^{0\alpha}$ constructed out of the flat tetrad $E^{i}{}_{\mu}$. The regularized form of the gravitational energy–momentum P_μ is determined by

$$P_\mu = \int_V \mathrm{d}^3x \partial_\alpha \left[\mathscr{U}_{\mu}{}^{0\alpha}(e) - \mathscr{U}_{\mu}{}^{0\alpha}(E)\right]. \qquad (55)$$

This condition guarantees that the energy–momentum of the flat space–time always vanishes. The reference space–time is determined by tetrad fields $E^{i}{}_{\mu}$, obtained from $e^{i}{}_{\mu}$ by requiring the vanishing of the physical parameters like mass, angular momentum, etc. Assuming that the space–time is asymptotically flat then equation (55) can have the form

$$P_\mu = \oint_{S\to\infty} \mathrm{d}S_\alpha \left[\mathscr{U}_{\mu}{}^{0\alpha}(e) - \mathscr{U}_{\mu}{}^{0\alpha}(E)\right], \qquad (56)$$

where the surface S is established at space-like infinity. Equation (56) transforms as a vector under the global $SO(3,1)$ group.[13] Now we are in a position to prove that the tetrad field (35) obtained using Eq. (36) yields a satisfactory value for the total gravitational energy–momentum.

We integrate Eq. (56) over a surface of constant radius $x^1 = R$ and requires $R \to \infty$. Therefore, the index α in Eq. (56) takes the value $\alpha = 1$. We need to calculate the quantity $\mathscr{U}_{0}{}^{01}$ which has the form

$$\mathscr{U}_{0}{}^{01}(e) \cong -\frac{1}{4\pi} R\sin(\theta)\left[\frac{2m}{R}\left\{1 - \tanh\left(\frac{q^2}{2mR}\right)\right\} - R\mathscr{B}' + R^2\mathscr{B}'^2\right], \qquad (57)$$

and the expression of $\mathscr{U}_{0}{}^{01}(E)$ is obtained by just making $m = 0$ in Eq. (57), and it is given by

$$\mathscr{U}_{0}{}^{01}(E) \cong -\frac{1}{4\pi} R\sin(\theta)(R^2\mathscr{B}'^2 - R\mathscr{B}'). \qquad (58)$$

Thus the gravitational energy contained within a sphere of radius R_1 is given by

$$\begin{aligned} P_0 &\cong \int_{R\to R_1} \mathrm{d}\theta\mathrm{d}\phi \frac{1}{4\pi}\sin(\theta)\left\{-R\left(\frac{2m}{R}\left[1 - \tanh\left(\frac{q^2}{2mR}\right)\right] - R\mathscr{B}' + R^2\mathscr{B}'^2\right) + (R^3\mathscr{B}'^2 - R^2\mathscr{B}')\right\} \\ &= 2m\left[1 - \tanh\left(\frac{q^2}{2mR_1}\right)\right]. \end{aligned} \qquad (59)$$

6. Main results and discussion

We have studied the nonlinear electrodynamics coupled to the teleparallel theory of gravitation. We show that it is possible to find a regular charged solution within the tetrad theory of gravitation by coupling to Møller's field equations an appropriate nonlinear electrodynamics, which in the weak field approximation becomes the usual Maxwell theory. The extra tensors of Møller's field equations, i.e., $H_{\mu\nu}$ and $K_{\mu\nu}$, of these solutions are identically vanishing and the theory is reduced to the teleparallel equivalent of general relativity coupled to nonlinear electrodynamics. It is well known that the only nonlinear electrodynamics that was experimentally successful is the Heisenberg–Euler electrodynamics.[31] However, the solutions obtained in this paper are not satisfied for the nonlinear Heisenberg–Euler electrodynamics.

Three different exact analytic solutions of the field equations are obtained for the case of spherical symmetry. These solutions give rise to the same Riemannian metric spacetime, i.e., "regular charged spacetime spherically symmetric spacetime". The exact solutions (17), (22) and (36) represent a regular charged which contain the Reissner–Nordström black hole. Solution (36) is a general solution, and we can generate from it other solutions (17) and (22) by giving the arbitrary function $\mathscr{B}(r)$ the values (38) and (39).

It was shown by Møller[32] that a tetrad description of a gravitational field equation allowed a more satisfactory treatment of the energy–momentum complex than general relativity. Therefore, we have used the superpotential method given by Mikhail *et al.*[14] to calculate the energy of the central gravitating body. Calculating the energy associated with solution (36), we obtain formula (45) which depends on the arbitrary function $\mathscr{B}(R)$. This formula is not accepted in physics because it depends on the arbitrary function.

Maluf *et al.*[25–27] derived a simple expression for the energy–momentum flux of the gravitational field. This expression was obtained on the assumption that

equation (52) represented the energy–momentum of the gravitational field in a volume V of the three-dimensional space-like hypersurface. They[25,26] gave this definition for the gravitational energy–momentum tensor in the framework of TEGR, which required $T^{\lambda}{}_{\mu\nu}(E) = 0$ for the flat space–time. They extended this definition to the case where the flat space–time tetrad fields $E^{a}{}_{\mu}$ yielded $T^{\lambda}{}_{\mu\nu}(E) \neq 0$. They showed that[26] in the context of the regularized gravitational energy–momentum definition it is not strictly necessary to stipulate asymptotic boundary conditions for tetrad fields that describe asymptotically flat space–times.

Following the same procedure as that given by Maluf *et al.*[24−26] to derive a regularized expression for the gravitational energy–momentum in the framework of TEGR, we derive a similar expression of the regularized energy–momentum in the framework of tetrad theory of gravitation. Then we use definition (2) of the torsion tensor and apply it to the tetrad field (35) obtained using Eq. (36). We have shown that the flat space–time associated with this tetrad field has non-vanishing torsion components shown in Eq. (48) and it is adapted to an accelerated observer (49). However, using Eq. (56) and calculating all the necessary components we finally obtained Eq. (59) which shows that the total gravitational mass does not depend on the arbitrary function.

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