Stability properties of vector solitons in two-component Bose–Einstein condensates with tunable interactions*

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By using a unified theory of the formation of various types of vector-solitons in two-component Bose–Einstein condensates with tunable interactions, we obtain a family of exact vector-soliton solutions for the coupled nonlinear Schrödinger equations. Moreover, the Bogoliubov equation shows that there exists stable dark soliton in specific situations. Our results open up new ways in considerable experimental interest for the quantum control of multi-component Bose–Einstein condensates.

Keywords: Bose–Einstein condensates, vector soliton, Bogoliubov equation

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1. Introduction

The experimental realization of Bose–Einstein condensates (BECs) has stimulated great interest in vector solitons in multi-component BECs due to their novel properties not seen in individual components.[1−4] Theoretical[5−12] and experimental[13,14] studies have shown that the properties of the nonlinear excitation are determined by the interaction between atoms, which is typically characterized by the s-wave scattering length (SL). The main advantage of BEC systems is that the interactions between atoms can be effectively tuned by magnetic or optical Feshbach resonance. For example, by tuning the inter-component interaction via the Feshbach resonance technique, different types of vector-solitons can be produced and transformed to each other.[15−17]

It is interesting because in conventional wisdom, dark solitons are associated with self-repulsive waves while bright solitons with self-attractive waves.[18−20] This picture is only limited to single-component BECs which possess only one interaction parameter, and becomes too simplified.[21,22] In the case of multi-component BECs, previous studies showed that it is possible to realize stable dark soliton, or even stable bright soliton in BECs with repulsive interaction. Motivated by our previous study on two-component BECs, the aim of this paper is to study the stability properties of vector solitons in two-component Bose–Einstein condensates with tunable interactions.

2. Model and vector-soliton solutions

2.1. Model

At the mean-field level, the coupled Gross–Pitaevskii (GP) equation governs the evolution of the macroscopic wave functions of two-component BECs.[7,8,10] In the physically important case of the cigar-shaped BECs, it is reasonable to reduce the coupled GP equation into a one-dimensional dimensionless coupled nonlinear Schrödinger equations[23]

$$\mathrm{i}\frac{\partial\psi_1}{\partial t}=\left[-\frac{1}{2}\frac{\partial^2}{\partial x^2}+V_1+g_{11}|\psi_1|^2+g_{12}|\psi_2|^2\right]\psi_1, \quad (1)$$

$$\mathrm{i}\frac{\partial\psi_2}{\partial t}=\left[-\frac{\mu}{2}\frac{\partial^2}{\partial x^2}+V_2+g_{21}|\psi_1|^2+g_{22}|\psi_2|^2\right]\psi_2, \quad (2)$$

where the coordinate x and time t are measured in units $\sqrt{\hbar/(m_1\omega_{1\perp})}$ and $2\pi/\omega_{1\perp}$. The interactions between atoms are described by self-interactions

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$g_{11} = 2a_{11}N_1$, $g_{22} = 2a_{22}m_1N_1\gamma/m_2$, and the interaction between different components $g_{12} = g_{21} = 2m_1a_{12}N_1/[(1+\gamma)m]$, where a_{ii} are the scattering lengths of component i and a_{12} is that between components 1 and 2. The trapping potentials are $V_1 = \lambda_1^2x^2/2$ and $V_2 = \lambda_2^2x^2/(2\mu)$. Finally, $\psi_{1,2}$ is normalized as $\int|\psi_1|^2\mathrm{d}x = 1$ and $\int|\psi_2|^2\mathrm{d}x = N_2/N_1$. Other parameters are defined as: $\gamma = \omega_{2\perp}/\omega_{1\perp}$, $\lambda_1 = \omega_{1x}/\omega_{1\perp}$, $\lambda_2 = \omega_{2x}/\omega_{1\perp}$, and $\mu = m_1/m_2$. Typically, in ^{7}Li, $N = 5000$, $a_\mathrm{s} = -0.21$ nm, the time unit is 0.224 ms, length unit is 1.42 μm.[24,25]

2.2. Vector-soliton solutions

When the soliton size is small enough compared to the harmonic trap, we can find the vector-soliton solutions for this system.[16,26]

The bright–bright (BB) solution is found to be

$$\begin{aligned}\psi_1 &= \sqrt{\frac{g_{22}-g_{12}\mu}{g_{21}g_{12}-g_{11}g_{22}}}\eta_1\mathrm{sech}(\eta_1x-\eta_1v_1t+d_0)\\ &\quad\times\exp\left\{\mathrm{i}\left[v_1x+\left(\frac{\eta_1^2}{2}-\frac{v_1^2}{2}\right)t\right]\right\},\\ \psi_2 &= \sqrt{\frac{-(g_{21}-g_{11}\mu)}{g_{21}g_{12}-g_{11}g_{22}}}\eta_1\mathrm{sech}(\eta_1x-\eta_1v_1t+d_0)\\ &\quad\times\exp\left\{\mathrm{i}\left[\frac{v_1}{\mu}x+\left(\frac{\mu}{2}\eta_1^2-\frac{1}{2\mu}v_1^2\right)t\right]\right\}\end{aligned}\tag{3}$$

with v_1 being the speed of the soliton, and η_1 the phase parameter of the solion.

The dark–bright (DB) solution is found to be

$$\begin{aligned}\psi_1 &= \eta_1\sqrt{\frac{g_{22}-\mu g_{12}}{g_{12}g_{21}-g_{11}g_{22}}}\mathrm{sech}(\eta_1x-\eta_1v_1t+d_0)\\ &\quad\times\exp[\mathrm{i}(v_1x+f_1t)],\\ \psi_2 &= \exp[\mathrm{i}(f_2t)]\left[\mathrm{i}\frac{v_1}{\mu}\sqrt{\frac{g_{21}-\mu g_{11}}{g_{21}g_{12}-g_{22}g_{11}}}\right.\\ &\quad+\eta_1\sqrt{\frac{g_{21}-\mu g_{11}}{g_{21}g_{12}-g_{22}g_{11}}}\\ &\quad\left.\times\tanh(\eta_1x-\eta_1v_1t+d_0)\right]\end{aligned}\tag{4}$$

with

$$\begin{aligned}f_1 &= \frac{\eta_1^2-v_1^2}{2}-g_{12}\frac{g_{21}-\mu g_{11}}{g_{21}g_{12}-g_{22}g_{11}}\left[\eta_1^2+\left(\frac{v_1}{\mu}\right)^2\right],\\ f_2 &= -g_{22}\frac{g_{21}-\mu g_{11}}{g_{21}g_{12}-g_{22}g_{11}}\left[\eta_1^2+\left(\frac{v_1}{\mu}\right)^2\right].\end{aligned}$$

The dark–dark (DD) solution is found to be

$$\begin{aligned}\psi_1 &= \exp(\mathrm{i}f_1t)\left[-\mathrm{i}\sqrt{\frac{\mu g_{12}-g_{22}}{g_{12}g_{21}-g_{11}g_{22}}}\frac{v_1}{\eta_1}+\eta_1\sqrt{\frac{\mu g_{12}-g_{22}}{g_{12}g_{21}-g_{11}g_{22}}}\tanh(\eta_1x+v_1t+d_0)\right],\\ \psi_2 &= \exp(\mathrm{i}f_2t)\left[-\mathrm{i}\sqrt{\frac{g_{21}-\mu g_{11}}{g_{21}g_{12}-g_{22}g_{11}}}\frac{v_1}{\mu\eta_1}+\eta_1\sqrt{\frac{g_{21}-\mu g_{11}}{g_{21}g_{12}-g_{22}g_{11}}}\tanh(\eta_1x+v_1t+d_0)\right]\end{aligned}\tag{5}$$

with

$$\begin{aligned}f_1 &= -\left(\frac{v_1^2}{\eta_1^2}+\eta_1^2\right)-g_{12}\frac{v_1^2}{\eta_1^2}\frac{g_{21}-\mu g_{11}}{g_{21}g_{12}-g_{22}g_{11}}\left(\frac{1}{\mu^2}-1\right),\\ f_2 &= -\mu\left(\frac{v_1^2}{\eta_1^2}+\eta_1^2\right)-g_{22}\frac{v_1^2}{\eta_1^2}\frac{g_{21}-\mu g_{11}}{g_{21}g_{12}-g_{22}g_{11}}\left(\frac{1}{\mu^2}-1\right),\end{aligned}$$

where the parameter η_1 determines the width of the soliton and can be found by the normalization condition, and v_1 gives the velocity of the soliton. Here we would like to point out that the exact vector-soliton solutions (3), (4), and (5) are obtained under the assumption that the quantities under a radical sign must be a number greater than zero, thus giving the constrains for the existence of the exact vector-soliton solutions. For example, when the intracomponent interactions are fixed, we can tuned the intercomponent interaction by means of the magnetic Feshbach-resonance (FR) technique to a special region to obtain the corresponding vector solitons.

Now we try to solve the stability properties of these solutions with Bogoliubov equation. The process is quite clear: consider the system governed by the equation of $\mathrm{i}(\partial/\partial t)\psi = -(1/2)\partial_{xx}\psi + g_{11}|\psi|^2\psi$, with the solution of $\psi_0 = r_1\exp(-\mathrm{i}\mu t)$. Then we introduce $\psi = \psi_0 + \phi$, with ϕ a small perturbation, as $\phi = u\beta\exp(\mathrm{i}\omega_1t) + v^*\beta^+\exp(-\mathrm{i}\omega_1t)$, where ω_1 is the eigenvalue of the perturbation, which means that if there exists an imaginary value of ω_1, then the perturbation will increase with time evolution. And it will finally lead to unstable phenomena of the system. Whereas, if all the solution of ω_1 is real, the system is dynamically stable.

3. Stability properties of vector solitons

3.1. Bogoliubov equation of the bright–bright vector-soliton solution

Introducing $\psi_1 = (r_1 + \phi_1)\exp(\mathrm{i}kt), \psi_2 = (r_2 + \phi_2)\exp(\mathrm{i}\mu kt)$ with $\phi_1 = u_1\beta + v_1^*\beta^+, \phi_2 = u_2\beta + v_2^*\beta^+$, the Bogoliubov equation is written as

$$
\begin{aligned}
&\mathrm{i}\frac{\partial}{\partial t}u_1 = ku_1 - \frac{1}{2}\frac{\partial^2}{\partial x^2}u_1 + g_{11}(2r_1^2u_1 + r_1^2v_1) + g_{12}(r_2^2u_1 + r_1r_2u_2 + r_1r_2v_2),\\
&-\mathrm{i}\frac{\partial}{\partial t}v_1 = kv_1 - \frac{1}{2}\frac{\partial^2}{\partial x^2}v_1 + g_{11}(2r_1^2v_1 + r_1^2u_1) + g_{12}(r_2^2v_1 + r_1r_2v_2 + r_1r_2u_2),\\
&\mathrm{i}\frac{\partial}{\partial t}u_2 = \mu ku_2 - \frac{\mu}{2}\frac{\partial^2}{\partial x^2}u_2 + g_{21}(r_1^2u_2 + r_1r_2u_1 + r_1r_2v_1) + g_{22}(2r_2^2u_2 + r_2^2v_2),\\
&-\mathrm{i}\frac{\partial}{\partial t}v_2 = \mu kv_2 - \frac{1}{2}\frac{\partial^2}{\partial x^2}v_2 + g_{21}(r_1^2v_2 + r_1r_2v_1 + r_1r_2u_1) + g_{22}(2r_2^2v_2 + r_2^2u_2).
\end{aligned}
$$

Let $u_1 = u_1(x)\exp(-\mathrm{i}Et)$ and so on, and define $m_1 = u_1 - v_1$, $n_1 = u_1 + v_1$, $m_2 = u_2 - v_2$, $n_2 = u_2 + v_2$, then we have

$$
\begin{aligned}
&km_1 - \frac{1}{2}\frac{\partial^2}{\partial x^2}m_1 + g_{11}r_1^2m_1 + g_{12}r_2^2m_1 = En_1,\\
&kn_1 - \frac{1}{2}\frac{\partial^2}{\partial x^2}n_1 + 3g_{11}r_1^2n_1 + g_{12}r_2^2n_1 + 2g_{12}r_1r_2n_2 = Em_1,\\
&\mu km_2 - \frac{\mu}{2}\frac{\partial^2}{\partial x^2}m_2 + g_{21}r_1^2m_2 + g_{22}r_2^2m_2 = En_2,\\
&k\mu n_2 - \frac{\mu}{2}\frac{\partial^2}{\partial x^2}n_2 + g_{21}r_1^2n_2 + 2g_{21}r_1r_2n_1 + 3g_{22}r_2^2n_2 = Em_2.
\end{aligned}
$$

Now we can write the Bogliubov matrix for bright–bright vector soliton:

$$
M = \begin{pmatrix}
0 & -\frac{1}{2}\partial_{xx} + \frac{\eta^2}{2} + g_{11}r_1^2 + g_{12}r_2^2 & 0 & 0\\
-\frac{1}{2}\partial_{xx} + \frac{\eta^2}{2} + 3g_{11}r_1^2 + g_{12}r_2^2 & 0 & 2g_{12}r_1r_2 & 0\\
0 & 0 & 0 & -\frac{\mu}{2}\partial_{xx} + \frac{\mu\eta^2}{2} + g_{21}r_1^2 + g_{22}r_2^2\\
2g_{21}r_1r_2 & 0 & -\frac{\mu}{2}\partial_{xx} + \frac{\mu\eta^2}{2} + g_{21}r_1^2 + 3g_{22}r_2^2 & 0
\end{pmatrix}.
$$

It is difficult to solve the eigenvalue of this matrix. For the system with N grads, we need to solve the eigenvalue of $4N \times 4N$-dimensional matrix. Now the transformation skill is used to simplify the question

$$
A = \begin{pmatrix}
-\frac{1}{2}\partial_{xx} + \frac{\eta^2}{2} + 2g_{11}r_1^2 + g_{12}r_2^2 & g_{12}r_1r_2\\
g_{21}r_1r_2 & -\frac{\mu}{2}\partial_{xx} + \mu\frac{\eta^2}{2} + 2g_{22}r_2^2 + g_{21}r_1^2
\end{pmatrix},\quad
B = \begin{pmatrix}
g_{11}r_1^2 & g_{12}r_1r_2\\
g_{21}r_1r_2 & g_{22}r_2^2
\end{pmatrix}.
$$

Then the matrix becomes the problem of equation solution of $(A + B)(A - B)X = \lambda X$. Define X as $2N$-dimensional vector: $(x, y) = (x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_N)$,

$$
MX = (A + B)(A - B)X = \lambda \begin{pmatrix} x\\ y \end{pmatrix}. \tag{6}
$$

For $i \le N$ (the i-th row of matrix M), we have

$$
\left(-\frac{1}{2}\partial_{xx} + \frac{\eta^2}{2} + 3g_{11}r_1^2 + g_{12}r_2^2\right)\left(-\frac{1}{2}\partial_{xx} + \frac{\eta^2}{2} + g_{11}r_1^2 + g_{12}r_2^2\right)x_i
$$

$$= \left(-\frac{1}{2}\partial_{xx} + \frac{\eta^2}{2} + 3g_{11}r_1^2 + g_{12}r_2^2\right)\left[\frac{x_{i+1}+x_{i-1}-2x_i}{2\Delta x^2} + \left(\frac{\eta^2}{2} + g_{11}r_1^2 + g_{12}r_2^2\right)x_i\right].$$

For the term of $(1/2)\{[(\eta^2/2) + g_{11}r_1^2 + g_{12}r_2^2]x_i\}$, we have

$$\begin{aligned}
&\partial_{xx}\left[\left(\frac{\eta^2}{2} + g_{11}r_1^2 + g_{12}r_2^2\right)x_i\right] \\
&= \partial_{xx}\left(\frac{\eta^2}{2} + g_{11}r_1^2 + g_{12}r_2^2\right)x_i + 2\partial_x\left(\frac{\eta^2}{2} + g_{11}r_1^2 + g_{12}r_2^2\right)\partial_x x_i + \left(\frac{\eta^2}{2} + g_{11}r_1^2 + g_{12}r_2^2\right)\partial_{xx}x_i,
\end{aligned}$$

with $r_1 = \eta\sqrt{(g_{22} - g_{12}\mu)/(g_{21}g_{12} - g_{11}g_{22})}\,\mathrm{sech}(\eta x)$, $r_2 = \eta\sqrt{-(g_{21} - g_{11}\mu)/(g_{21}g_{12} - g_{11}g_{22})}\,\mathrm{sech}(\eta x)$.

And the term

$$\begin{aligned}
&\left(-\frac{1}{2}\partial_{xx} + \frac{\eta^2}{2} + 3g_{11}r_1^2 + g_{12}r_2^2\right)\left(-\frac{1}{2}\partial_{xx} + \frac{\eta^2}{2} + g_{11}r_1^2 + g_{12}r_2^2\right)x_i \\
&= \left(-\frac{1}{2}\partial_{xx} + \frac{\eta^2}{2} + 3g_{11}r_1^2 + g_{12}r_2^2\right)\left[-\frac{x_{i+1}+x_{i-1}-2x_i}{2\Delta x^2} + \left(\frac{\eta^2}{2} + g_{11}r_1^2 + g_{12}r_2^2\right)x_i\right], \\
&\left(-\frac{\mu}{2}\partial_{xx} + \mu\frac{\eta^2}{2} + 3g_{22}r_2^2 + g_{21}r_1^2\right)\left(-\frac{\mu}{2}\partial_{xx} + \mu\frac{\eta^2}{2} + g_{22}r_2^2 + g_{21}r_1^2\right)x_i \\
&= \left(-\frac{\mu}{2}\partial_{xx} + \mu\frac{\eta^2}{2} + 3g_{22}r_2^2 + g_{21}r_1^2\right)\left[-\mu\frac{x_{i+1}+x_{i-1}-2x_i}{2\Delta x^2} + \left(\mu\frac{\eta^2}{2} + g_{22}r_2^2 + g_{21}r_1^2\right)x_i\right].
\end{aligned}$$

Now we can solve the Bogoliubov matrix for the bright–bright soliton.

3.2. Bogoliubov equation of the BD and DD vector-soliton solutions

Using a similar process, we can find the Bogoliubov matrix for bright–dark (BD) soliton

$$M = \begin{pmatrix}
0 & -\frac{1}{2}\partial_{xx} + f_1 + g_{11}r_1^2 + g_{12}r_2^2 & 0 & 0 \\
-\frac{1}{2}\partial_{xx} + f_1 + 3g_{11}r_1^2 + g_{12}r_2^2 & 0 & 2g_{12}r_1r_2 & 0 \\
0 & 0 & 0 & -\frac{\mu}{2}\partial_{xx} + f_2 + g_{21}r_1^2 + g_{22}r_2^2 \\
2g_{21}r_1r_2 & 0 & -\frac{\mu}{2}\partial_{xx} + f_2 + g_{21}r_1^2 + 3g_{22}r_2^2 & 0
\end{pmatrix}$$

with

$$\begin{aligned}
f_1 &= \frac{\eta_1^2}{2} - g_{12}\frac{g_{21} - \mu g_{11}}{g_{21}g_{12} - g_{22}g_{11}}\eta_1^2, \\
f_2 &= -g_{22}\frac{g_{21} - \mu g_{11}}{g_{21}g_{12} - g_{22}g_{11}}\eta_1^2.
\end{aligned} \tag{7}$$

The matrix for the dark–dark vector soliton is similar, so we can deal with it in the same manner. The environment here is different from the bright–bright (BB) soliton, because for dark soliton, the boundary is nonzero and there exists the phase difference π. The space difference method is not accurate enough. So we introduce the cosine transform to solve the problem. Define $F = \mathrm{dct}(f(x))$, then $f(j) = \sum_k \omega(k)F(k)\cos(\pi(2j-1)(k-1)/2N)$. Notice that the $F(k)$ is already constant here, so

$$\frac{\partial^2}{\partial x^2}n_1(j) = \sum_k \omega(k)N_1(k)\cos(\pi(2j-1)(k-1)/2N) - \left(\frac{\pi(k-1)}{N\Delta x}\right)^2, \tag{8}$$

then

$$\begin{aligned}
&\omega \sum_k \omega(k) N_1(k) \cos(\pi(2j-1)(k-1)/2N) \\
&= \sum_{k'} \omega(k') F(k') \cos(\pi(2j-1)(k'-1)/2N) \sum_{\dot{k}} \omega(\dot{k}) N_1(\dot{k}) \cos(\pi(2j-1)(\dot{k}-1)/2N) \\
&- \frac{1}{2} \left(\sum_k \omega(k) N_1(k) \cos(\pi(2j-1)(k-1)/2N) - \left(\frac{\pi(k-1)}{N\Delta x} \right)^2 \right). \qquad (9)
\end{aligned}$$

Now we have the numerical matrix

$$\omega_{\text{eigenvalue}} \begin{pmatrix} \omega(1)N_1(1) \\ \omega(2)N_1(2) \\ \cdots \\ \cdots \\ \omega(N)N_1(N) \end{pmatrix} = \begin{pmatrix} A_{11} & \ldots & A_{1N} \\ & & \\ & & \\ A_{N1} & \cdots & A_{NN} \end{pmatrix} \begin{pmatrix} N_1(1) \\ N_1(2) \\ \cdots \\ \cdots \\ N_1(N) \end{pmatrix}.$$

By solving the corresponding Bogoliubov equation, our calculations show that, the excitation frequencies of the BB and BD vector solitons are all real, while those of the dark–dark (DD) vector soliton contain complex values. Since the presence of complex excitation frequencies is an indication that the system is dynamically unstable, we come to the concludsion that for this system, the BB and BD vector solitons are dynamically stable, while the DD vector-soliton is dynamically unstable. This stability analysis is confirmed by numerical simulations. For the BD and DD vector solitons, we add a soft harmonic trapping potential. For the initial condition, we use the exact solution for the bright component, while we multiply the dark soliton by a Thomas–Fermi profile.[27]

4. Conclusion

In conclusion, we have found the Bogoliubov matrix for the vector solitons in two-component BECs with tunable interactions and the stability properties for different cases. It is quite interesting that the two-component BECs help to stabilize the dark soliton in one component. Besides its fundamental interest in quantum dynamics, our study may also find applications in the quantum control of multi-component BECs. We hope that these results can be useful in the study of the novel phenomena in multi-component BECs.

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